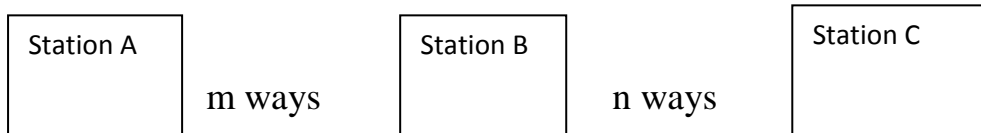


CHAPTER 7

PERMUTATIONS (Arrangements) AND COMBINATIONS (selections)

In permutation **order is important**, since 27 & 72 are different numbers(arrangements). In combination order is not important.

- **Fundamental principle of counting (FPC)**



then by FPC there are mn ways to go from station A to station C

- The number of permutations of n different things taken r at a time, where repetition is not allowed is given by ${}^n P_r = n(n-1)(n-2)\dots(n-r+1)$ where $0 < r \leq n$.

eg ${}^5 P_2 = 5 \times 4 = 20$

${}^7 P_3 = 7 \times 6 \times 5 = 210$

- Factorial notation: $n! = 1 \times 2 \times 3 \times \dots \times n$, where n is a natural number

eg $5! = 1 \times 2 \times 3 \times 4 \times 5$

we define $0! = 1$

also $n! = n(n-1)!$

$= n(n-1)(n-2)!$

- ${}^n P_r = \frac{n!}{(n-r)!}$ Where $0 \leq r \leq n$

- Number of permutations of n different things, taken r at a time, where repetition is allowed is n^r

- Number of permutations of n objects taken all at a time, where P_1 objects are of first kind, P_2 objects are of second kind..... P_k objects are of the k^{th} kind and rest, if any, are all different is $\frac{n!}{P_1! \cdot P_2! \cdot \dots \cdot P_k!}$ (eg 9)

- The number of combinations of n different things taken r at a time is given by

${}^n C_r = \frac{n(n-1)(n-2)\dots(n-r+1)}{1 \cdot 2 \cdot 3 \cdot \dots \cdot r}$, where $0 < r \leq n$

$1 \cdot 2 \cdot 3 \cdot \dots \cdot r$

eg ${}^5 C_3 = \frac{5 \times 4 \times 3}{1 \times 2 \times 3} = {}^5 C_2$

$1 \times 2 \times 3$

- ${}^n C_r = {}^n C_{n-r}$
eg ${}^5 C_3 = {}^5 C_2$
 ${}^7 C_5 = {}^7 C_2$
- ${}^n C_r = \frac{n!}{r!(n-r)!}$, where $0 \leq r \leq n$.
- ${}^n C_r = {}^n C_s$ implies $r = s$ or $n = r+s$ (eg 17*) 1 mark
- ${}^n C_n = {}^n C_0 = 1$
- ${}^n C_1 = n$
eg ${}^5 C_1 = 5$
- ${}^n C_r + {}^n C_{r-1} = {}^{n+1} C_r$

Ex 7.1

1, 2, 4

Ex 7.2

4*, 5* (1 mark)

eg 8* (1 mark), eg 11*, 12**, 13**, 14**, 16** (4 marks)

Ex 7.3

7*, 8*, 9**, 10**, 11**

Theorem 6 to prove (4 marks)*

eg 17* (1 mark) use direct formula $n = 9+8 = 17$ since ${}^n C_r = {}^n C_s$ implies $r = s$ or $n = r+s$

eg 19**

Ex 7.4

2**, 3*, 5*, 6*, 7**, 8*, 9*

eg 21**, eg 23*(HOT), eg 24*

Misc Ex

1**, 2**, 3**, 4*, 5*, 7**, 10**, 11**

EXTRA/HOT QUESTIONS

- 1) How many permutations can be made with letters of the word MATHEMATICS ? In how many of them vowels are together?
- 2) In how many ways can 9 examination papers be arranged so that the best and the worst papers are never together. (HOT)
- 3) How many numbers greater than 56000 can be formed by using the digits 4,5,6,7,8; no digit being repeated in any number.
- 4) Find the number of ways in which letters of the word ARRANGEMENT can be arranged so that the two A's and two R's do not occur together. (HOT)
- 5) If $C(2n,3) : C(n,3) :: 11:1$ find n.
- 6) If $P(11,r) = P(12,r-1)$ find r.

- 7) Five books, one each in Physics, Chemistry, Mathematics, English and Hindi are to be arranged on a shelf. In how many ways can this be done?
- 8) If ${}^n P_r = {}^n P_{r+1}$ and ${}^n C_r = {}^n C_{r-1}$ find the values of n and r .
- 9) A box contains five red balls and six black balls. In how many ways can six balls be selected so that there are at least two balls of each color.
- 10) A group consist of 4 girls and 7 boys in how many ways can a committee of five members be selected if the committee has i) no girl
ii) atleast 1 boy and 1 girl
iii) atleast 3 girls.

Note : atleast means \geq

Answers

- 1) 4989600, 120960
2) 282240 Hint (consider the best and the worst paper as one paper)
3) 90
4) 1678320
5) 6
6) 9
7) 120
8) $n = 3, r = 2$
9) 425
10) i) 21
ii) 441
iii) 91