

Key Notes

Chapter-7

Integrals

- Integration is the inverse process of differentiation. In the differential calculus, we are given a function and we have to find the derivative or differential of this function, but in the integral calculus, we are to find a function whose differential is given. Thus, integration is a process which is the inverse of differentiation. Let $\frac{d}{dx}F(x) = f(x)$. Then we write $\int f(x)dx = F(x) + C$. These integrals are called indefinite integrals or general integrals, C is called constant of integration. All these integrals differ by a constant.
- From the geometric point of view, an indefinite integral is collection of family of curves, each of which is obtained by translating one of the curves parallel to itself upwards or downwards along the y-axis.
- Some properties of indefinite integrals are as follows:

$$1. \int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$$

$$2. \text{ For any real number } k, \int kf(x)dx = k \int f(x)dx$$

More generally, if $f_1, f_2, f_3, \dots, f_n$ are functions and k_1, k_2, \dots, k_n are real numbers. Then

$$\int [k_1f_1(x) + k_2f_2(x) + \dots + k_nf_n(x)]dx = k_1 \int f_1(x)dx + k_2 \int f_2(x)dx + \dots + k_n \int f_n(x)dx$$

- **Some standard integrals:**

$$(i) \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1. \quad \text{Particularly, } \int dx = x + C$$

$$(ii) \int \cos x dx = \sin x + C$$

$$(iii) \int \sin x dx = -\cos x + C$$

$$(iv) \int \sec^2 x dx = \tan x + C$$

$$(v) \int \operatorname{cosec}^2 x dx = -\cot x + C$$

Key Notes

$$(vi) \quad \int \sec x \tan x \, dx = \sec x + C$$

$$(vii) \quad \int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + C$$

$$(viii) \quad \int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$$

$$(ix) \quad \int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1} x + C$$

$$(x) \quad \int \frac{dx}{\sqrt{1+x^2}} = \tan^{-1} x + C$$

$$(xi) \quad \int \frac{dx}{\sqrt{1+x^2}} = -\cot^{-1} x + C$$

$$(xii) \quad \int e^x dx = e^x + C$$

$$(xiii) \quad \int a^x dx = \frac{a^x}{\log a} + C$$

$$(xiv) \quad \int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C$$

$$(xv) \quad \int \frac{dx}{x\sqrt{x^2-1}} = -\operatorname{cosec}^{-1} x + C$$

$$(xvi) \quad \int \frac{1}{x} dx = \log |x| + C$$