Chapter-6

Application of Derivatives

- If a quantity y varies with another quantity x, satisfying some rule ( ) y f x = ,then $\frac{dx}{dy}$ (or f '(x) represents the rate of change of y with respect to x and $\left.\frac{dy}{dx}\right]_{x=x_0}$ (or f '(x_0) represents the rate of change of y with respect to x at0 x x = .
- If two variables x and y are varying with respect to another variable t, i.e., if $x=f\left(t\right)$ *and* $y=g\left(t\right)$ then by Chain Rule
- $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$, if $\frac{dx}{dt}\neq 0$

(a) A function f is said to be increasing on an interval (a, b) if

$x_1 < x_2 \, in \, (a, b) \Rightarrow f(x_1) \le f(x_2) \, for \, all \, x_1, x_2 \in (a, b).$

Alternatively, if $f'(x) \ge 0$ *for each x in* (a, b)

(b) decreasing on (a,b) if

$x_1 < x_2 \, in \, (a, b) \Rightarrow f(x_1) \ge f(x_2) \, for \, all \, x_1, x_2 \in (a, b).$

Alternatively, if $f'(x) \le 0$ for each x in (a, b)

- The equation of the tangent at $\left(x_{0,} \, y_0\right)$ to the curve y = f (x) is given by
- $y-y_o=\left.\frac{dy}{dx}\right]_{(x_0,y_0)}(x-x_o)$
- If $\frac{dy}{dx}$ does not exist at the point $\left(x_0,y_0\right)$, then the tangent at this point is parallel to the y-axis and its equation is $x = x_0$.

- If tangent to a curve $y = f(x)$ at $x = x_0$ is parallel to x-axis, then $\left.\frac{dy}{dx}\right]_{x=x_0}$

- Equation of the normal to the curve y = f (x) at a point (x_o, y_o) is given by

$$y - y_o = \frac{-1}{\left.\frac{dy}{dx}\right]_{(x_0, y_0)}}(x - x_0)$$

- If $\frac{dy}{dx}$ at the point (x_o, y_o) is zero, then equation of the normal is $x = x^o$.

- If $\frac{dy}{dx}$ at the point (x_o, y_o) does not exist, then the normal is parallel to x-axis and its equation is $y = y_0$.

- Let y = f (x), Δx be a small increment in x and Δy be the increment in y corresponding to the increment in x, i.e., Δy = f (x + Δx) – f (x). Then dy given by $dy\, f'(x)\, dx$ or $dy = \left(\frac{dx}{dx}\right)$Δx is a good approximation of Δy when dx x = Δ is relatively small and we denote it by dy ≈ Δy.

- A point c in the domain of a function f at which either f ′(c) = 0 or f is not differentiable is called a critical point of f.

- **First Derivative** Test Let f be a function defined on an open interval I. Let f be continuous at a critical point c in I. Then,
  - If f ′(x) changes sign from positive to negative as x increases through c, i.e., if f ′(x) > 0 at every point sufficiently close to and to the left of c, and f ′(x) < 0 at every point sufficiently close to and to the right of c, then c is a point of local maxima.
  - If f ′(x) changes sign from negative to positive as x increases through c, i.e., if f ′(x) < 0 at every point sufficiently close to and to the left of c, and f ′(x) > 0 at every point sufficiently close to and to the right of c, then c is a point of local minima.
  - If f ′(x) does not change sign as x increases through c, then c is neither a point of local maxima nor a point of local minima. In fact, such a point is called point of inflexion.

- **Second Derivative Test** Let f be a function defined on an interval I and $c \in I$. Let f be twice differentiable at c. Then, $x = c$ is a point of local maxima if $f'(c) = 0$ and $f''(c) < 0$

  The values $f(c)$ is local maximum value of f.

  (i) $x = c$ is a point of local minima if $f'(c) = 0$ and $f''(c) > 0$

  In this case, $f(c)$ is local minimum value of f.

  (ii) The test fails if $f'(c) = 0$ and $f''(c) = 0$.

  In this case, we go back to the first derivative test and find whether c is a point of maxima, minima or a point of inflexion.

- Working rule for finding absolute maxima and/or absolute minima

  Step 1: Find all critical points of f in the interval, i.e., find points x where either $f'(x) = 0$ or f is not differentiable.

  Step 2: Take the end points of the interval.

  Step 3: At all these points (listed in Step 1 and 2), calculate the values of f.

  Step 4: Identify the maximum and minimum values of f out of the values calculated in Step

- This maximum value will be the absolute maximum value of f and the minimum value will be the absolute minimum value of f .