

# Key Notes

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## Chapter-5

### Continuity and Differentiability

- A real valued function is continuous at a point in its domain if the limit of the function at that point equals the value of the function at that point. A function is continuous if it is continuous on the whole of its domain.

- Sum, difference, product and quotient of continuous functions are continuous. i.e., if  $f$  and  $g$  are continuous functions, then

$$(f \pm g)(x) = f(x) \pm g(x) \text{ is continuous.}$$

$$(f \cdot g)(x) = f(x) \cdot g(x) \text{ is continuous.}$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \text{ (wherever } g(x) \neq 0) \text{ is continuous.}$$

- Every differentiable function is continuous, but the converse is not true.
- Chain rule is rule to differentiate composites of functions. If  $f = v \circ u$ ,  $t = u(x)$  and if both

$$\text{and if both } \frac{dt}{dx} \text{ and } \frac{dv}{dt} \text{ exist then } \frac{df}{dx} = \frac{dv}{dt} = \frac{dt}{dx}$$

- Following are some of the standard derivatives (in appropriate domains):

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{1-x^2}}$$

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$$\frac{d}{dx}(\operatorname{cosec}^{-1}x) = \frac{-1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\log x) = \frac{1}{x}$$

- Logarithmic differentiation is a powerful technique to differentiate functions of the form  $f(x) = [u(x)]^{v(x)}$ . Here both  $f(x)$  and  $u(x)$  need to be positive for this technique to make sense.
- **Rolle's Theorem:** If  $f: [a, b] \rightarrow \mathbb{R}$  is continuous on  $[a, b]$  and differentiable on  $(a, b)$  such that  $f(a) = f(b)$ , then there exists some  $c$  in  $(a, b)$  such that  $f'(c) = 0$ .
- Mean Value Theorem: If  $f: [a, b] \rightarrow \mathbb{R}$  is continuous on  $[a, b]$  and differentiable on  $(a, b)$ . Then there exists some  $c$  in  $(a, b)$  such that  $f'(c) = \frac{f(b) - f(a)}{b - a}$