Chapter-4

Determinant

- Determinant of a matrix $A = [a_{11}]_{1\times1}$ is given by $|a_{11}| = a_{11}$
- Determinant of a matrix A $\begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{matrix}$ is given by

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}\,a_{22} - a_{12}\,a_{21}$$

- Determinant of a matrix A $\begin{matrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{matrix}$ is given by (expanding along (R_1)

$$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

- **For any square matrix A, the |A| satisfy following properties.**
- |A′| = |A|, where A′ = transpose of A.
- If we interchange any two rows (or columns), then sign of determinant changes.
- If any two rows or any two columns are identical or proportional, then value of determinant is zero.
- If we multiply each element of a row or a column of a determinant by constant k, then value of determinant is multiplied by k.
- Multiplying a determinant by k means multiply elements of only one row (or one column) by k.
- *If* $A = \left[a_{ij}\right]_{3\times3}$ *, then* $|k.\,A| = k^3\,|A|$
- If elements of a row or a column in a determinant can be expressed as sum of two or more elements, then the given determinant can be expressed as sum of two or more determinants.

Key Notes

- If to each element of a row or a column of a determinant the equimultiples of corresponding elements of other rows or columns are added, then value of determinant remains same.
- Area of a triangle with vertices $(x_1, y_1), (x_2, y_2)$ *and* (x_3, y_3) is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

- Minor of an element aij of the determinant of matrix A is the determinant obtained by deleting i^{th} row and j^{th} column and denoted by M_{ij}
- Cofactor of aij of given by Aij = (– 1)i+ j Mij
- Value of determinant of a matrix A is obtained by sum of product of elements of a row (or a column) with corresponding cofactors. For example, $|A| = a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13}$.
- If elements of one row (or column) are multiplied with cofactors of elements of any other row (or column), then their sum is zero. For example, $a_{11} A_{21} + a_{12} A_{22} + a_{13} A_{23} = 0$
- $A\,(adj\,A) = (adj\,A)\,A = |A|\,I$, where A is square matrix of order n.
- A square matrix A is said to be singular or non-singular according as $|A|| = 0$ *or* $|A| \neq 0$.
- If AB = BA = I, where B is square matrix, then B is called inverse of A. Also $A^{-1} = B$ *or* $B^{-1} = A$ *and hence* $\left(A^{-1}\right)^{-1} = A$.
- A square matrix A has inverse if and only if A is non-singular.
- $A^{-1} = \frac{1}{|A|}(\text{adj } A)$
- If $a_1x + b_1y + c_1z = d_1$
- $a_2x + b_2y + c_2z = d_2$
- $a_3x + b_3y + c_3z = d_3$
- then these equations can be written as A X = B, where

- $A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = X \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$
- Unique solution of equation AX = B is given by $X = A^{-1}B$, *where* $|A|\ 0\ \neq 0$.
- A system of equation is consistent or inconsistent according as its solution exists or not.
- For a square matrix A in matrix equation AX = B
- |$|A| \neq 0$, there exists unique solution
- |A| = 0 and (adj A) B ≠ 0, then there exists no solution
- $|A| = 0$ *and* $(adj\ A)\ B = 0$, then system may or may not be consistent.