Chapter-02

Inverse Trigonometric Functions

- The domains and ranges (principal value branches) of inverse trigonometric functions are given in the following table:

Functions	**Domain**	**Range (Principal Value Branches)**
$y = sin^{-1}x$	[-1, 1]	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1} x$	[-1, 1]	$[0, \pi]$
$y = \operatorname{cosec}^{-1}x$	R- [-1, 1]	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ -{0}
$y = \sec^{-1} x$	R-[-1, 1]	$[0, \pi] - \{\frac{\pi}{2}\}$
$y = \tan^{-1} x$	R	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \cot^{-1} x$	R	$[0, \pi]$

$sin^{-1}x$ should not be condused with $(\sin x)^{-1}$. In fact $(\sin x)^{-1} = \frac{1}{\sin x}$ And similarly for other trigonometric functions

- The value of an inverse trigonometric functions which lies in its principal value branch is called the principal value of that inverse trigonometric functions.

For suitable values of domain, we have

- $y = sin^{-1} x \Rightarrow x = sin\, y$
- $x = sin\ y \Rightarrow y = sin^{-1}\, x$
- $\sin\ (\sin^{-1} x) = x$
- $\sin^{-1}\ (\sin x) = x$

Key Notes

- $\sin^{-1}\frac{1}{x} = \operatorname{cosec}^{-1} x$
- $\cos^{-1}(-x) = \pi - \cos^{-1} x$
- $\cos^{-1}\frac{1}{x} = \sec^{-1} x$
- $\cot^{-1}(-x) = \pi - \cot^{-1} x$
- $\tan^{-1}\frac{1}{x} = \cot^{-1} x$
- $\sec^{-1}(-x) = \pi - \sec^{-1} x$
- $\sin^{-1}(-x) = -\sin^{-1} x$
- $\tan^{-1}(-x) = -\tan^{-1} x$
- $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$
- $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$
- $\operatorname{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}$
- $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$
- $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\frac{x+y}{1-xy}$
- $2\tan^{-1} x = \tan^{-1}\frac{2x}{1-x^2}$
- $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\frac{x-y}{1+xy}$
- $2\tan^{-1} x = \sin^{-1}\frac{2x}{1+x^2} = \frac{1-x^2}{1+x^2}$