Chapter 13

LIMITS AND DERIVATIVES

Some standard results on limits

1) $\lim_{x\to c} f(x) = l$ iff $\lim_{x\to c-} f(x) = \lim_{x\to c+} f(x) = l$

2) $\lim_{x\to a} k = k$ where k is a fixed real number.

3) $\lim_{x\to c} f(x) = f(c)$ where f(x) is a real polynomial in x.

4) $\lim_{x\to c} x^n = c^n$ for all n $\in$ N

5) $\lim_{x\to c} |x| = |c|$

Algebra of limits

Refer : NCERT Textbook page no: 292.

Evaluation of algebraic limits

Type I (Direct substitution method)

Eg: Evaluate $\lim_{x\to 1} (x^3 - x^2 + 1) = 1^3 - 1^2 + 1 = 1$

Evaluate the following limits:

NCERT Textbook Exercise 13.1 Q 1,2,3,4,5,9,11,12.

Additional Questions and HOT Questions

** 1) $\lim_{x\to -1} (1 + x + x^2 + \cdots + x^{10})$ Ans: 1

** 2) $\lim_{x\to -1} (x^5 + 2x^8)^{60}$ Ans: 1

** 3) $\lim_{x\to 0} \frac{\sqrt{1+x} + \sqrt{1-x}}{1+x}$ Ans:2

Type II (Factorisation method)

Eg : Evaluate : $\lim_{x\to 1} \frac{x^3-1}{x-1}$

$\lim_{x\to 1} \frac{x^3-1}{x-1}$ ($\frac{0}{0}$ form) $= \lim_{x\to 1} \frac{(x-1)(x^2+x+1)}{x-1}$

$= \lim_{x\to 1} (x^2 + x + 1)$

$= 1^2 + 1 + 1 = 3$

Evaluate the following limits.

NCERT Textbook Exercise 13.1 Q 7,8 , Example 2.

Evaluate:

** 1) $\lim_{x \to 1} \frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3}$ ans: $\frac{-1}{10}$

** 2) $\lim_{x \to \sqrt{2}} \frac{x^4-4}{x^2+3\sqrt{2}\,x-8}$ ans: $\frac{8}{5}$

** 3) $\lim_{x \to 2} \frac{x^3-4x^2+4x}{x^2-4}$ ans: 0

Type III

For any positive integer n, $\lim_{x \to a} \frac{x^n-a^n}{x-a} = n\,a^{n-1}$ (This is applicable for any rational number n and for a> 0.

Evaluate : $\lim_{x \to 2} \frac{x^4-16}{x-2}$

Ans: $\lim_{x \to 2} \frac{x^4-16}{x-2} = \lim_{x \to 2} \frac{x^4-2^4}{x-2} = 4.\,2^{4-1} = 4.2^3 = 32$

Evaluate the following limits.

NCERT Textbook Exercise 13.1 Q 10 , Example 3.

1) Evaluate : $\lim_{x \to 2} \frac{x^{10}-1024}{x^5-32}$ Ans: 64

2) If $\lim_{x \to -a} \frac{x^9+a^9}{x+a}$ = 9 , find the value of a.

Type IV

Evaluate $\lim_{x \to 0} \frac{\sqrt{1+x}-1}{x}$

Ans: $\lim_{x \to 0} \frac{\sqrt{1+x}-1}{x} = \lim_{x \to 0} \frac{\sqrt{1+x}-1}{1+x-1}$

Put y = 1+x when x→ 0 , y→ 1

∴ value = $\lim_{y \to 0} \frac{\sqrt{y}-1}{y-1}$

$= \lim_{y \to 0} \frac{y^{1/2}-1^{1/2}}{y-1}$

$= \frac{1}{2}$

Evaluate

1) NCERT Ex 13.1 **Q** 6

2) $\lim_{x \to 0} \frac{(1-x)^n - 1}{x}$ ans = – n

3) $\lim_{x \to 0} \frac{(1+x)^6 - 1}{(1+x)^5 - 1}$ ans = $\frac{6}{5}$

4) $\lim_{x \to 0} \frac{(x+8)^{1/3} - 2}{x}$ ans = $\frac{1}{12}$

Type V

Concepts : $\lim_{x \to 0} \frac{\sin x}{x} = 1$

$\lim_{x \to 0} \cos x = 1$

$\lim_{x \to 0} \frac{\tan x}{x} = 1$

NCERT Exercise 13.1 : Q 13, 14, 16, 18, 19, 20, 21

Evaluate : $\lim_{x \to 0} \frac{\tan 8x}{\sin 3x}$ ans: $\frac{8}{3}$

Evaluate : $\lim_{x \to 0} \frac{\sin 2x + 3x}{2x + \sin 3x}$ ans: 1

Type VI

NCERT Exercise 13.1 : Q 17

Evaluate : $\lim_{x \to 0} \frac{1-\cos 5x}{1-\cos 6x}$

Type VII

Evaluate : $\lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi\,(\pi - x)}$

Let $x = \pi + h$

$\therefore \lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi\,(\pi - x)} = \lim_{h \to 0} \frac{\sin[\pi - (\pi + h)]}{\pi[\pi - (\pi + h)]}$

$= \frac{1}{\pi} \lim_{h \to 0} \frac{\sin h}{h} = \frac{1}{\pi}$

Textbook Q no: 22

Evaluate 1) $\lim_{x \to \pi/2} \frac{\tan 2x}{x - \frac{\pi}{2}}$ ans: 2

2) $\lim_{x \to 1} \frac{\cos \frac{\pi}{2} x}{1-x}$ ans : $\frac{\pi}{2}$

3) $\lim_{x \to \pi} \frac{\sin x}{x-\pi}$ ans : (–1)

4) $\lim_{x \to \pi/2} \frac{1+\cos 2x}{(\pi-2x)^2}$ ans : ½

Type VIII

Applying sin C ± sin D formulae

Evaluate 1) $\lim_{x \to 0} \frac{\sin 2x+\sin 6x}{\sin 5x-\sin 3x}$

2) $\lim_{x \to a} \frac{\sin x-\sin a}{x-a}$ ans : cos a

Type IX

$\lim_{x \to c-} f(x) = \lim_{x \to c+} f(x) = l$

** NCERT Exercise 13.1 -

1) Q 23

2) Q 24

3) Q 25

4) Q 26

** 5) If $f(x) = \begin{cases} \frac{|x-2|}{2-x}, & x \neq 2 \\ -1, & x = 2 \end{cases}$, find $\lim_{x \to 2} f(x)$

6) If $f(x) = \begin{cases} x, & x > 0 \\ 1, & x = 0 \\ -x, & x < 0 \end{cases}$, find $\lim_{x \to 0} f(x)$

Type X

** NCERT Exercise 13.1 -

1) Q 28

2) Q 30

3) Q 32

4) Let $f(x) = \begin{cases} 4x-5, & x \le 2 \\ x-k, & x > 2 \end{cases}$. Find k if $\lim_{x \to 2} f(x)$ exists. **Ans : k = –1**

Type XI

Exponential Limits

$$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

NCERT Exercise 13.2 Q1 , Q2, Q4, Q6.

Type XII

NCERT Exercise 13.2 Q3 , Q5

Type XIII

$$\lim_{x \to 0} \frac{\log_e(1+x)}{x} = 1$$

NCERT Exercise 13.2 Q7 , Q8

DIFFERENTIATION

Type I

Find the derivatives of the following functions from first principle.

1) x

2) $\frac{1}{x}$

3) $-x$

4) $(-x)^{-1}$

5) x^n

6) NCERT Ex 13.2 Q4

** 7) sin x

** 8) cos x

** 9) tan x

** 10) sec x

** 11) cosec x

** 12) sin 2x

** 13) cos 3x

14) x sin x

15) $\sin^2 x$

16) sin(x+1)

17) Misc Example 19

Type II

Find the derivatives of the following

* 1) $3cot\ x + 5\ cosec\ x$

2) $5\sin x - 6\cos x + 7$

3) $2\tan x - 7\ cosec\ x$

4) $x^3 - 27$

5) $2x - \frac{3}{4}$

* 6) $x^2 + \sin x + \frac{1}{x^2}$

Type III

Ex 13.2 Q1

Ex 13.2 Q2

Ex 13.2 Q3

Ex 13.2 Q5

Type IV

$$\frac{d}{dx}(uv) = u.\frac{dv}{dx} + v.\frac{du}{dx}$$

NCERT Ex 13.2

* 1) Q7 (1),(2)

* 2) Q9 (2) ,(3), (4),(5)

* 3) Q11 (1)

* 4) Example 18

** 5) Misc Ex Q 15, Q22, Q25, Q29.

Type V

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Misc Exercise Q 5,6,7,8,9,16,17,18,20,26.

Additional questions and HOT Questions

1. Find the derivatives of the following functions

   a) $\dfrac{\sec x + \tan x}{\sec x - \tan x}$

   ** b) $(x^2 - 3x + 2)(x + 2)$

   ** c) $\dfrac{\sin x - x\cos x}{x\sin x + \cos x}$

2. If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$, show that $\frac{dy}{dx} = y$.