Chapter-13

Probability

The salient features of the chapter are –

- The conditional probability of an event E, given the occurrence of the event F is given by

  $$P(E|F) = \frac{P(E \cap F)}{P(F)}, P(F) \neq 0$$

  $$0 \le P(E|F) \le 1,$$
  $$P(E'|F) = 1 - P(E|F)$$
  $$P((E \cup F)|G) = P(E|G) + P(F|G) - P((E \cap F)|G)$$

- $$P(E \cap F) = P(E)P(E|F), P(E) \neq 0$$
  $$P(E \cap F) = P(F)P(E|F), P(F) \neq 0$$

- $$P(E \cap F) = P(E)\,P(F)$$
  $$P(E|F) = P(E), P(F) \neq 0$$
  $$P(F|E) = P(F), P(E) \neq 0$$

- **Theorem of total probability:**

  Let $\{E_1, E_2, \ldots, E_n)$ be a partition of a sample space and suppose that each of $E_1, E_2, \ldots, E_n$ has non zero probability. Let A be any event associated with S, then

  $$P(A) = P(E_1)P(A|E_1) + P(E_2) + P(A|E_2) + \ldots\ldots + P(E_n)P(A|E_n)$$

- **Bayes' theorem:** If $E_1, E_2, \ldots, E_n$ are events which constitute a partition of sample space S, i.e. $E_1, E_2, \ldots, E_n$ are pairwise disjoint and $E_1 4, E_2 4, \ldots, 4E_n = S$ *and* A be any event with non-zero probability, then, $P(E_i|A) = \dfrac{P(E_i)\,P(A|E_i)}{\sum_{j\ 1}^{n} P(E_i)\,P(A|E_i)}$

- A random variable is a real valued function whose domain is the sample space of a random experiment.

- The probability distribution of a random variable X is the system of numbers

X	:	x_1	x_2	.....	x_n
P(X)	:	p_1	p_2	......	p_n

Where, $p_i > o, \sum_{i=1}^{n} p_i = 1, \; i = 1, 2, ..., n$

- Let X be a random variable whose possible values $x_{1,} x_{2,} \; x_3 .... \; ..., \; x_n$ occur with probabilities $p_{1,} p_{2,} \; p_3 .... \; ..., \; p_n$ respectively. The mean of X, denoted by μ is the number $\sum_{i=1}^{n} x_i p_i$. The mean of a random variable X is also called the expectation of X, denoted by E (X).
- Let X be a random variable whose possible values $x_{1,} x_{2,} \; x_3 .... \; ..., \; x_n$ occur with probabilities $p(x_1), p(x_2), ...p(x_n)$ respectively. Let $\mu = E(X)$ be the mean of X. The variance of X, denoted by Var (X) or σ_x^2 is defined as $x^2 \; Var\,(X) = \sum_{i=1}^{n} (x_i \; \mu)^2 \;\; p(x_i)$ or equivalently $\sigma_x^2 = E\,(X - \mu)^2$. The non-negative number, $_x \sqrt{Var\,(X)} = \sqrt{\sum_{1=i}^{n} (x_i \; \mu)^2 \, p(x_i)}$ is called the standard deviation of the random variable X.

$$Var\,(X) = E\left(X^2\right) - \left[E(X)\right]^2$$

- Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions:

(i) There should be a finite number of trials.

(ii) The trials should be independent.

(iii) Each trial has exactly two outcomes: success or failure.

(iv) The probability of success remains the same in each trial.

For Binomial distribution $B(n, p), \; P(X{=}x) = {}^n C_x q^{n-x} P^x, \; x = 0, 1, ..., n (q = 1 - p)$