Chapter-10

Vector Algebra

- Position vector of a point P (x, y) is given as $\overline{OP}(=\overline{r}) = x\hat{i} + y\hat{j} + z\hat{k}$ and its magnitude by $\sqrt{x^2 + y^2 + z^2}$
- The scalar components of a vector are its direction ratios, and represent its projections along the respective axes.
- The magnitude (r), direction ratios (a, b, c) and direction cosines (l, m, n) of any vector are related as: $l = \frac{a}{r}, m = \frac{b}{r}, n = \frac{c}{r}$
- The vector sum of the three sides of a triangle taken in order is $\overline{O}$
- The vector sum of two conidial vectors is given by the diagonal of the parallelogram whose adjacent sides are the given vectors.
- The multiplication of a given vector by a scalar λ, changes the magnitude of the vector by the multiple |λ|, and keeps the direction same (or makes it opposite) according as the value of λ is positive (or negative).
- For a given vector $\overline{a}$, the vector $\hat{a} = \frac{\overline{a}}{|\overline{a}|}$ gives the unit vector in the direction of $\overline{a}$
- The position vector of a point R dividing a line segment joining the points P and Q whose position vectors are $\overline{a}$ *and* $\overline{b}$ respectively, in the ratio $m : n$

  (i) internally, is given by $\frac{n\overline{a} + m\overline{b}}{m + n}$

  (ii) externally, is given by $\frac{m\overline{b} - n\overline{a}}{m - n}$
- The scalar product of two given vectors $\overline{a}$ and $\overline{b}$ having angle θ between them is defined as

  $\overline{a} \cdot \overline{b} = |\overline{a}||\overline{b}| \cos\theta$

Also, when $\overline{a}\,.\,\overline{b}$ is given, the angle 'θ' between the vectors $\overline{a}$ *and* $\overline{b}$ may be determined by

$$cos\theta = \frac{\overline{a}\,.\,\overline{b}}{|\overline{a}||\overline{b}|}$$

- If θ is the angle between two vector $\overline{a}$ and $\overline{b}$, then their cross product is given as

  $\overline{a}\times\overline{b} = |\overline{a}||\overline{b}|\sin\theta\hat{n}$ where $\hat{n}$ is a unit vector perpendicular to the plane containing $\overline{a}$ *and* $\overline{b}$. Such that $\overline{a}$, $\overline{b}$, $\hat{n}$ form right handed system of coordinate axes.

- If we have two vectors $\overline{a}$ *and* $\overline{b}$ given in component form as $\overline{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\overline{b} = b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ and λ any scalar, then, $a+b = (a_1+b_1)\hat{i}+(a_2+b_2)\hat{j}+(a_3+b_3)\hat{k}$

  $\lambda\overline{a} = (\lambda a_1)\hat{i}+(\lambda a_2)\hat{j}+(\lambda a_3)\hat{k};$

  $\overline{a}.\overline{b} = a_1b_1+a_2b_2+a_3b_3$

  and $\overline{a}\times\overline{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$