Unit 11(Exponents & Powers)

Multiple Choice Questions (MCQs)

Question 1:

$[(-3)^2]^3$ is equal to

(a) $(-3)^8$ (b) $(-3)^6$

(c) $(-3)^5$ (d) $(-3)^{23}$

Solution:

(b) We know that, if 'a' is a rational number, m and n are natural numbers, then $(a^m)^n = a^{m \times n}$

So, $[(-3)^2]^3 = (-3)^{2 \times 3} = (-3)^6$

Question 2:

For a non-zero rational number x, $x^8 \div x^2$ is equal to

(a) x^4 (b) x^6 (c) x^{10} (d) x^{16}

Solution:

(b) We know that, if 'a' is a rational number, m and n are natural numbers such that m> n, then

$a^m \div a^n = a^{m-n}$

So, $x^8 \div x^2 = x^8/x^2 = x^{8-2} = x^6$

Question 3:

x is a non-zero rational number. Product of the square of x with the cube of x is equal to the

(a) second power of x (b) third power of x

(c) fifth power of x (d) sixth power of x

Solution:

(c) Square of x = x^2

Cube of x = x^3

Product of square with the cube of x = $x^2 \times x^3 = x^{2+3}$ [$\because a^m \times a^n = a^{m+n}$]

i.e. fifth power of x.

Question 4:

For any two non-zero rational numbers x and y, $x^5 \div y^5$ is equal to

(a) $(x \div y)^1$ (b) $(x \div y)^0$

(c) $(x \div y)^5$ (d) $(x \div y)^{10}$

Solution:

(c) Given, $x^5 \div y^5 = \frac{x^5}{y^5}$

As we know, $\frac{p^n}{q^n} = \left(\frac{p}{q}\right)^n$

Thus, $\frac{x^5}{y^5} = \left(\frac{x}{y}\right)^5 = (x + y)^5$

Question 5:

a^m x a^n is equal to

(a) $(a^2)^{mn}$ (b) a^{m-n}

(c) a^{m+n} (d) a^{mn}

Solution:

(c) We know that, if 'a' is a rational number, m and n are natural numbers, then

a^m x a^n =a^{m+n}

Question 6:

(1° + 2° + 3°) is equal to

(a) 0 (b) 1 (c) 3 (d) 6

Solution:

(c) As we know, a0 = 1

10 + 2° + 3° =1 + 1 + 1 = 3.

Question 7:

The value of $\frac{10^{22} + 10^{20}}{10^{20}}$ is

(a) 10 (b) 10^{42} (c) 101 (d) 10^{22}

Solution:

(c) We can write the given expression as

$$\frac{10^{22}}{10^{20}} + \frac{10^{20}}{10^{20}} = 10^{22-20} + 1 \qquad \left[\because \frac{a^m}{a^n} = a^{m-n}, m > n\right]$$

$$= 10^2 + 1 = 10 \times 10 + 1 = 100 + 1 = 101$$

Question 8:

The standard form of the number 12345 is

(a) 1234.5 x 10^1 (b) 123.45 x 10^2

(c) 12.345 x 10^3 (d) 1.2345 x 10^4

Solution:

(d) A number in standard form is written as a x 10^k, where a is a terminating decimal such that 1 ≤ a ≤ 10 and k is any integer.

So, 12345 = 1.2345 x 10^4

Question 9:

If $2^{1998} - 2^{1997} - 2^{1996} + 2^{1995} = k \cdot 2^{1995}$, then the value of k is

(a) 1 (b) 2

(c) 3 (d) 4

Solution:

(c) Given,

$$2^{1998} - 2^{1997} - 2^{1996} + 2^{1995} = k \cdot 2^{1995}$$

$$\Rightarrow \quad 2^{1995+3} - 2^{1995+2} - 2^{1995+1} + 2^{1995} \times 1 = k \cdot 2^{1995}$$

$$\Rightarrow \quad 2^{1995}[2^3 - 2^2 - 2^1 + 1] = k \cdot 2^{1995} \qquad [\because a^{m+n} = a^m \times a^n]$$

$$\Rightarrow \quad 2^{1995}[8 - 4 - 2 + 1] = k \cdot 2^{1995}$$

$$\Rightarrow \quad 3 = \frac{k \cdot 2^{1995}}{2^{1995}}$$

$$\Rightarrow \quad 3 = k \text{ or } k = 3$$

So, the value of k is 3.

Question 10:

Which of the following is equal to 1?

(a) 2° + 3° + 4° (b) 2° x 3° x 4°

(c) (3° -2°) x 4° (d) (3° – 2°) x (3° + 2°)

Solution:

(b) Let us solve all the expressions,

For option (a),

2° + 3° + 4° = 1 + 1 + 1 [∴ a° = 1]

= 3

For option (b),

2° x 3° x 4° = 1 x 1 x 1 [∴ a° = 1]

= 1

Hence, option (b) is the answer.

Question 11:

In standard form, the number 72105.4 is written as 7.21054 x 10^n, where n is equal to

(a) 2 (b) 3 (c) 4 (d) 5

Solution:

(c) We know that, if the given number is greater than or equal to 10, then the power of 10 (i.e. n) is a positive integer equal to the number of places the decimal point has been shifted.

Hence, 72105.4 = 7.21054 x 10^4

Question 12:

Square of [-2/3] is

(a) $\frac{-2}{3}$ (b) $\frac{2}{3}$ (c) $\frac{-4}{9}$ (d) $\frac{4}{9}$

Solution:

(d) Square of $\frac{-2}{3}$ is $\left(\frac{-2}{3}\right)^2$.

So, $\left(\frac{-2}{3}\right)^2 = \left(\frac{-2}{3}\right) \times \left(\frac{-2}{3}\right) = \frac{4}{9}$

[∵ multiplication of two rational numbers with same sign is always positive]

Question 13:

The cube [-1/4] is

(a) $\frac{-1}{12}$ (b) $\frac{1}{16}$ (c) $\frac{-1}{64}$ (d) $\frac{1}{64}$

Solution:

(c) Cube of $\left(\frac{-1}{4}\right)$ is $\left(\frac{-1}{4}\right)^3$.

So, $\left(\frac{-1}{4}\right)^3 = \left(\frac{-1}{4}\right) \times \left(\frac{-1}{4}\right) \times \left(\frac{-1}{4}\right) = \frac{(-1) \times (-1) \times (-1)}{4 \times 4 \times 4} = \frac{-1}{64}$

Question 14:

Which of the following is not equal to $\left(\frac{-5}{4}\right)^4$?

(a) $\frac{(-5)^4}{4^4}$ (b) $\frac{5^4}{(-4)^4}$

(c) $-\frac{5^4}{4^4}$ (d) $\left(-\frac{5}{4}\right)\times\left(-\frac{5}{4}\right)\times\left(-\frac{5}{4}\right)\times\left(-\frac{5}{4}\right)$

Solution:

(c) We know that, $\left(\frac{p}{q}\right)^m = \frac{p^m}{q^m}$

So, $\left(\frac{-5}{4}\right)^4 = \frac{(-5)^4}{(4)^4}$ or $\left(\frac{-5}{4}\right)^4 = \frac{(5)^4}{(-4)^4}$

or $\left(\frac{-5}{4}\right)^4 = \left(\frac{-5}{4}\right)\times\left(\frac{-5}{4}\right)\times\left(\frac{-5}{4}\right)\times\left(\frac{-5}{4}\right)$

Hence, option (c) is not equal to 1.

Question 15:

Which of the following is not equal to 1?

(a) $\frac{2^3\times3^2}{4\times18}$ (b) $[(-2)^3\times(-2)^4]\div(-2)^7$

(c) $\frac{3^0\times5^3}{5\times25}$ (d) $\frac{2^4}{(7^0+3^0)^3}$

Solution:

(d) Let us solve the expressions.

For option (a), $\frac{2^3\times3^2}{4\times18} = \frac{2\times2\times2\times3\times3}{4\times18} = \frac{4\times18}{4\times18} = 1$

For option (b), $[(-2)^3\times(-2)^4]\div(-2)^7 = \frac{(-2)^3\times(-2)^4}{(-2)^7}$

$= \frac{(-2)^{3+4}}{(-2)^7}$ $[\because a^m\times a^n = a^{m+n}]$

$= \frac{(-2)^7}{(-2)^7} = 1$ $\left[\because \frac{a^m}{a^n} = a^{m-n}\right]$

For option (c), $\frac{3^0\times5^3}{5\times25} = \frac{1\times5\times5\times5}{5\times25}$

$= \frac{5\times25}{5\times25}$ $[\because a^0 = 1]$

$= 1$

For option (d), $\frac{2^4}{(7^0+3^0)^3} = \frac{2^4}{(1+1)^3}$ $[\because a^0 = 1]$

$= \frac{2^4}{2^3} = 2^{4-3}$ $\left[\because \frac{a^m}{a^n} = a^{m-n}, m>n\right]$

$= -2^1 = 2$

Hence, option (d) is not equal to 1.

Question 16:

$\left(\frac{2}{3}\right)^3\times\left(\frac{5}{7}\right)^3$ is equal to

(a) $\left(\frac{2}{3}\times\frac{5}{7}\right)^9$ (b) $\left(\frac{2}{3}\times\frac{5}{7}\right)^6$ (c) $\left(\frac{2}{3}\times\frac{5}{7}\right)^3$ (d) $\left(\frac{2}{3}\times\frac{5}{7}\right)^0$

Solution:

(c) We know that, if *a*, *b* and *m* are rational numbers, then

$$a^m\times b^m = (ab)^m$$

So, $\left(\frac{2}{3}\right)^3\times\left(\frac{5}{7}\right)^3 = \left(\frac{2}{3}\times\frac{5}{7}\right)^3$

Question 17:

In standard form, the number 829030000 is written as K x 10^8, where K is equal to

(a) 82903 (b) 829.03 (c) 82.903 (d) 8.2903

Solution:

(d) We have,

A number in a standard form is written as K x 10^8, then K will be a terminating decimal such that $1 \le K \le 10$.

So, there is only one option, where K = 8.2903 < 10.

Question 18:

Which of the following has the largest value?

(a) 0.0001 (b) $\frac{1}{10000}$

(c) $\frac{1}{10^6}$ (d) $\frac{1}{10^6} \div 0.1$

Solution:

(a,b) For option (a), $0.0001 = \frac{1}{10000}$

For option (b), $\frac{1}{10000}$

For option (c), $\frac{1}{10^6} = \frac{1}{10 \times 10 \times 10 \times 10 \times 10 \times 10} = \frac{1}{1000000}$

For option (d), $\frac{1}{10^6} \div 0.1 = \frac{1}{10^6} \times \frac{1}{0.1} = \frac{1}{10^6} \times \frac{10}{1} = \frac{10}{10^6} = \frac{1}{10^5}$

$= \frac{1}{10 \times 10 \times 10 \times 10 \times 10} = \frac{1}{100000}$

The fraction whose denominator is smallest will be largest.

Hence, (a) and (b) are the largest.

Question 19:

In standard form 72 crore is written as

(a) 72 x 10^7 (b) 72 x 10^8 (c)7.2 x 10^8 (d)7.2 x 10^7

Solution:

(c) We know that,

A number in standard form is written as a x 10^k, where a is the terminating decimal such that $1 \le a \le 10$ and k is any integer.

So, 72 crore = 720000000 =7.2 x10^8

Note Here, power of 10 (i.e. k) is a positive integer equal to the number of places the decimal point has been shifted.

Question 20:

For non-zero numbers a and b, $\left(\frac{a}{b}\right)^m \div \left(\frac{a}{b}\right)^n$, where $m > n$, is equal to

(a) $\left(\frac{a}{b}\right)^{mn}$ (b) $\left(\frac{a}{b}\right)^{m+n}$ (c) $\left(\frac{a}{b}\right)^{m-n}$ (d) $\left(\left(\frac{a}{b}\right)^m\right)^n$

Solution:

(c) We know that,

$$a^m \div a^n = a^{m-n}, (m > n)$$

So, $\left(\frac{a}{b}\right)^m \div \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^{m-n}$

Question 21:

Which of the following is not true?

(a) $3^2 > 2^3$ (b) $4^3 = 2^6$ (c) $3^3 = 9$ (d) $2^5 > 5^2$

Solution:

(c) For option (a), $3^2 > 2^3$

$3 \times 3 > 2 \times 2 \times 2$

$9 > 8$ (true)

For option (b), $4^3 = 2^6$

$(2^2)^3 = 2^6$

$2^6 = 2^6$ (true) $\quad [\because (a^m)^n = a^{m \times n}]$

For option (c), $3^3 = 9$

$3 \times 3 \times 3 = 9$

$27 \neq 9$ (not true)

For option (d), $2^5 > 5^2$

$2 \times 2 \times 2 \times 2 \times 2 > 5 \times 5$

$32 > 25$ (true)

Hence, option (c) is not true.

Question 22:

Which power of 8 is equal to 2^6?

(a) 3 (b) 2 (c) 1 (d) 4

Solution:

(b) Let power of 8 be x.

According to the question,

$8^x = 2^6$

$\Rightarrow (2^3)^x = 2^6 \quad [\because 8 = 2 \times 2 \times 2 = 2^3]$

$\Rightarrow 2^{3x} = 2^6 \quad [\because (a^m)^n = a^{m \times n}]$

Since, base are equal, then by equating their exponents, we get

$3x = 6$

$\Rightarrow \frac{3x}{3} = \frac{6}{3}$ [dividing both sides by 3]

$\Rightarrow x = 2$

Hence, the power of 8 is 2, which is equal to 2^6.

Fill in the Blank

In questions 23 to 39, fill in the blanks to make the statements True .

Question 23:

$(-2)^{31} \times (-2)^{13} = (-2)^{\_}$

Solution:

Here, $(-2)^{31} \times (-2)^{13} = (-2)^{31+13} \quad [\because a^m \times a^n = a^{m+n}]$

$= (-2)^{44}$

$\therefore (-2)^{31} \times (-2)^{13} = (-2)^{44}$

Question 24:

$(-3)^8 \div (-3)^5 = (-3)^{\_}$

Solution:

Here, $(-3)^8 \div (-3)^5 = (-3)^{8-5} = (-3)^3 \quad [\because a^m \div a^n = a^{m-n}, m > n]$

$\therefore (-3)^8 \div (-3)^5 = (-3)^3$

Question 25:

$\left(\frac{11}{15}\right)^4 \times (\_\_\_)^5 = \left(\frac{11}{15}\right)^9$

Solution:

Let $\left(\frac{11}{15}\right)^4 \times (x)^5 = \left(\frac{11}{15}\right)^9$

$\Rightarrow \quad (x)^5 = \dfrac{\left(\frac{11}{15}\right)^9}{\left(\frac{11}{15}\right)^4}$

$\Rightarrow \quad (x)^5 = \left(\frac{11}{15}\right)^9 \left(\frac{11}{15}\right)^{-4}$

$\Rightarrow \quad (x)^5 = \left(\frac{11}{15}\right)^{9-4} \quad [\because a^m \times a^{-n} = a^{m+(-n)} = a^{m-n}]$

$\Rightarrow \quad (x)^5 = \left(\frac{11}{15}\right)^5$

Since, the powers are same.

$\therefore \quad x = \frac{11}{15}$

Hence, $\left(\frac{11}{15}\right)^4 \times \left(\frac{\mathbf{11}}{\mathbf{15}}\right)^5 = \left(\frac{11}{15}\right)^9$

Question 26:

$\left(\frac{-1}{4}\right)^3 \times \left(\frac{-1}{4}\right)^{—} = \left(\frac{-1}{4}\right)^{11}$

Solution:

Let $\left(\frac{-1}{4}\right)^3 \times \left(\frac{-1}{4}\right)^x = \left(\frac{-1}{4}\right)^{11}$

$\Rightarrow \quad \left(\frac{-1}{4}\right)^x = \dfrac{\left(\frac{-1}{4}\right)^{11}}{\left(\frac{-1}{4}\right)^3}$

$\Rightarrow \quad \left(\frac{-1}{4}\right)^x = \left(\frac{-1}{4}\right)^{11-3} \quad [\because a^m \div a^n = a^{m-n}, m > n]$

$\Rightarrow \quad \left(\frac{-1}{4}\right)^x = \left(\frac{-1}{4}\right)^8$

Since, base are equal. So, by equating the powers, we get

$x = 8$

$\therefore \quad \left(-\frac{1}{4}\right)^3 \times \left(-\frac{1}{4}\right)^{\mathbf{8}} = \left(-\frac{1}{4}\right)^{11}$

Question 27:

$\left[\left(\frac{7}{11}\right)^3\right]^4 = \left(\frac{7}{11}\right)^{—}$

Solution:

Here, $\left[\left(\frac{7}{11}\right)^3\right]^4 = \left(\frac{7}{11}\right)^{3\times 4} = \left(\frac{7}{11}\right)^{12} \quad [\because (a^m)^n = a^{mn}]$

$\therefore \quad \left[\left(\frac{7}{11}\right)^3\right]^4 = \left(\frac{7}{11}\right)^{\mathbf{12}}$

Question 28:

$\left(\frac{6}{13}\right)^{10} \div \left[\left(\frac{6}{13}\right)^5\right]^2 = \left(\frac{6}{13}\right)^{—}$

Solution:

$$\text{Here, } \left(\frac{6}{13}\right)^{10} \div \left[\left(\frac{6}{13}\right)^{5}\right]^{2} = \left(\frac{6}{13}\right)^{10} \div \left(\frac{6}{13}\right)^{5\times 2} \qquad [\because (a^m)^n = a^{mn}]$$

$$= \left(\frac{6}{13}\right)^{10} \div \left(\frac{6}{13}\right)^{10} = \left(\frac{6}{13}\right)^{10-10} \qquad [\because a^m \div a^n = a^{m-n}, (m > n)]$$

$$= \left(\frac{6}{13}\right)^{0}$$

$$\therefore \quad \left(\frac{6}{13}\right)^{10} \div \left[\left(\frac{6}{13}\right)^{5}\right]^{2} = \left(\frac{6}{13}\right)^{0}$$

Question 29:

$$\left[\left(\frac{-1}{4}\right)^{16}\right]^{2} = \left(\frac{-1}{4}\right)^{\_\_}$$

Solution:

$$\text{Here, } \left[\left(\frac{-1}{4}\right)^{16}\right]^{2} = \left(\frac{-1}{4}\right)^{16\times 2} \qquad [\because (a^m)^n = a^{mn}]$$

$$= \left(\frac{-1}{4}\right)^{32}$$

$$\therefore \quad \left[\left(-\frac{1}{4}\right)^{16}\right]^{2} = \left(-\frac{1}{4}\right)^{32}$$

Question 30:

$$\left(\frac{13}{14}\right)^{5} \div (\_\_)^{2} = \left(\frac{13}{14}\right)^{3}$$

Solution:

$$\text{Let } \left(\frac{13}{14}\right)^{5} \div (x)^{2} = \left(\frac{13}{14}\right)^{3}$$

$$\Rightarrow \quad (x)^2 = \frac{\left(\frac{13}{14}\right)^{5}}{\left(\frac{13}{14}\right)^{3}}$$

$$\Rightarrow \quad (x)^2 = \left(\frac{13}{14}\right)^{5-3} \qquad \left[\because \frac{a^m}{a^n} = a^{m-n}\right]$$

$$\Rightarrow \quad (x)^2 = \left(\frac{13}{14}\right)^{2}$$

Since, powers are same.

$$\therefore \quad x = \frac{13}{14}$$

$$\text{Hence, } \quad \left(\frac{13}{14}\right)^{5} \div \left(\frac{13}{14}\right)^{2} = \left(\frac{13}{14}\right)^{3}$$

Question 31:

$a^6 \times a^5 \times a^0 = a^{\_\_}$

Solution:

Since, $a^6 \times a^5 \times a^0 = a^{6+5+0} = a^{11}$

$\therefore a^6 \times a^5 \times a^0 = a^{11}$

Question 32:

1 lakh = $10^{\_\_}$

Solution:

Here, 1 lakh = 100000 = 10^5.

$\therefore$ 1 lakh = 10^5

Question 33:

1 million = $10^{\_\_\_}$

Solution:

Here, 1 million = 1000000 = 10^6

∴ 1 million = 10^6

Question 34:

729 = $3^{\_\_\_}$

Solution:

Here, firstly we find out the factors of given expression.

3	729
3	243
3	81
3	27
3	9
3	3
	1

So, 729 = 3 x 3 x 3 x 3 x 3 x 3=3^6

∴ 729 = 3^6

Question 35:

432 = 2^4 x $3^{\_\_\_}$

Solution:

Firstly, we find out the factors of given expression.

2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

So, 432 = 2 x 2 x 2 x 2 x 3 x 3 x 3= 2^4 x 3^3

∴ 432 =2^4 x 3^3

Question 36:

53700000 = ___ x 10^7

Solution:

Given, 53700000

For standard form, 53700000 = 537 x 10^5 Also, 537 = 5.37 x 10^2

So, 5.37 x 10^2 x 10^5 = 5.37 x 10^7

53700000 = 5.37 x 10^7

Question 37:

88880000000 = ___ x 10^{10} .

Solution:

Given, 88880000000

For standard form, 88880000000 = 8888 x 10^7 Also, 8888 = 8.888 x 10^3

So, 8.888 x 10^3 x 10^7 = 8.888 x 10^{10}

88880000000 = 8.888 x 10^{10}

Question 38:

27500000 = 2.75 x 10^{--}

Solution:

Given, 27500000

For standard form, 27500000 = 275 x 10^5 Also, 275 = 2.75 x 10^2

So, 2.75 x 10^2 x 10^5= 2.75 x 10^7 [$\because a^m \times a^n = a^{m+n}$]

27500000 = 2.75 x 10^7

Question 39:

340900000 = 3.409 x 10^{--}

Solution:

For standard form, 340900000 = 3409 x 10^5

Also, 3409= 3.409 x 10^3

So, 3.409 x 10^3 x 10^5 = 3.409 x 10^8 [$\because a^m \times a^n = a^{m+n}$]

340900000 = 3.409 x 10^8

Question 40:

(a) 3^2 ______ 15 (b) 2^3 ______ 3^2

(c) 7^4 ______ 5^4 (d) 10000 ______ 10^5

(e) 6^3 ______ 4^4

Solution:

(a) 3^2 ______ 15

$\because$ $3^2 = 3 \times 3 = 9$

So, $9 < 15$

$\therefore$ $3^2 < 15$

(b) 2^3 ______ 3^2

$\because$ $2^3 = 2 \times 2 \times 2 = 8$

and $3^2 = 3 \times 3 = 9$

So, $8 < 9$

$\therefore$ $2^3 < 3^2$

(c) 7^4 ______ 5^4

Obviously, $7^4 > 5^4$ [$\because$ powers are same, so if base is greater, then the number is greater]

(d) 10000 ______ 10^5

$\because 10000 = 10^4$

So, $10^4 < 10^5$ [$\because$ base are same, so if power is greater, then the number is greater]

$\therefore$ $10000 < 10^5$

(e) 6^3 ______ 4^4

$\because$ $6^3 = 6 \times 6 \times 6 = 216$

and $4^4 = 4 \times 4 \times 4 \times 4 = 256$

So, $216 < 256$

$\therefore$ $6^3 < 4^4$

True/ False

In questions 41 to 65, state whether the given statements are True or False.

Question 41:

One million = 10^7

Solution:

False

One million = 10 lakhs = 1000000 = 10^6

Hence, $10^6 \neq 10^7$

Question 42:

One hour = 60^2 seconds

Solution:

True

1 h = 60 min = 60 x 60 s = 60^2 s

Question 43:

$1° \times 0^1 = 1$

Solution:

False

$\therefore 1° \times 0^1 = 1 \times 0 = 0 \neq 1$.

Question 44:

$(-3)^4 = -12$

Solution:

False

$(-3)^4 = (-3) \times (-3) \times (-3) \times (-3) = 81 \neq -12$

Question 45:

$3^4 > 4^3$

Solution:

True

$\therefore 3^4 = 3 \times 3 \times 3 \times 3 = 81$

and $4^3 = 4 \times 4 \times 4 = 64$

81 > 64

Hence, $3^4 > 4^3$

Question 46:

$$\left(\frac{-3}{5}\right)^{100} = \frac{-3^{100}}{-5^{100}}$$

Solution:

True

Taking LHS, we have

$$\left(\frac{-3}{5}\right)^{100} = \left(\frac{-1 \times 3}{5}\right)^{100} \quad [\because -3 = -1 \times 3]$$

$$= \frac{(-1)^{100} \times 3^{100}}{5^{100}} \quad [\because (a \times b)^m = a^m \times b^m]$$

$$= \frac{1 \times 3^{100}}{5^{100}} \quad [\because (-1)^n = 1, \text{ if } n \text{ is even}]$$

$$= \frac{3^{100}}{5^{100}}$$

Now, taking RHS, we have $\frac{-3^{100}}{-5^{100}} = \frac{3^{100}}{5^{100}}$ [$\because$ if both numerator and denominator have negative sign, then it is cancelled out]

$\therefore$ LHS = RHS

Hence, $\frac{-3^{100}}{5} = \frac{-3^{100}}{-5^{100}}$

Question 47:

$(10 + 10)^{10} = 10^{10} + 10^{10}$

Solution:

False

We know that, $(a \times b)^m = a^m \times b^m$

So, $(10 \times 10)^{10} = 10^{10} \times 10^{10}$

Question 48:

$x^\circ \times x^\circ = x^\circ + x^\circ$ is true for all non-zero values of x.

Solution:

True

$\therefore x^\circ \times x^\circ = 1 \times 1 = 1$ [$\because a^\circ = 1$]

and $x^\circ + x^\circ = 1+1 = 1$ [$\because a^\circ=1$]

Hence, $x^\circ x\, x^\circ = x^\circ + x^\circ$

Question 49:

In the standard form, a large number can be expressed as a decimal number between 0 and 1, multiplied by a power of 10.

Solution:

False

A number in standard form is written as $a \times 10^k$, where $1 \le a \le 10$ and k is any integer.

Question 50:

4^2 is greater than 2^4.

Solution:

False

$4^2 = 4 \times 4 = 16$ [$\because a^m = a \times a \times a \times \ldots \times a$ (m times)]

and $2^4 = 2 \times 2 \times 2 \times 2 = 16$

So, $4^2 = 2^4$

Question 51:

$x^m + x^m = x^{2m}$, where x is a non-zero rational number and m is a positive integer.

Solution:

False

$a^m \times a^n = a^{m+n}$

$x^m \times x^m = x^{m+m} = x^{2m}$

Also, $a^k + a^k = 2a^k$

So, $x^m + x^m = 2x^m$

Question 52:

$x^m \times y^m = (x \times y)^{2m}$, where x and y are non-zero rational numbers and m is a positive integer.

Solution:

False

If a and b are rational numbers, then

$x^m \times y^m = (ab)^m$

$x^m \times y^m = (xy)^m = (x \times y)^m$

Hence, $x^m \times y^m \neq (x \times y)^{2m}$

Question 53:

$x^m \div y^m = (x \div y)^m$, where x and y are non-zero rational numbers and m is a positive integer.

Solution:

True

If x and y are rational numbers, then $(x/y)^m = (x^m/y^m)$ or $x^m \div y^m = (x \div y)^m$

Question 54:

$a^m \times a^n = a^{m+n}$, where x is a non-zero rational number and m, n are positive integers.

Solution:

True

If x is a rational number and m and n are positive integers, then
$a^m \times a^n = a^{m+n}$

Question 55:

4^9 is greater than 16^3.

Solution:

True

$\therefore 16^3 = (4^2)^3$ [$\because 16 = 4 \times 4 = 4^2$]

$= 4^6$

Now, in 4^9 and 4^6, $4^9 > 4^6$ as powers $9 > 6$

Question 56:

$$\left(\frac{2}{5}\right)^3 \div \left(\frac{5}{2}\right)^3 = 1$$

Solution:

False

Here, $\left(\frac{2}{5}\right)^3 \div \left(\frac{5}{2}\right)^3 = \left(\frac{2}{5}\right)^3 \times \left(\frac{2}{5}\right)^3$ $\left[\because \left(\frac{a}{b}\right) \div \left(\frac{c}{d}\right) = \frac{a}{b} \times \frac{d}{c}\right]$

$= \left(\frac{2}{5}\right)^{3+3} = \left(\frac{2}{5}\right)^6$ $[\because a^m \times a^n = a^{m+n}]$

Hence, $\left(\frac{2}{5}\right)^3 \div \left(\frac{5}{2}\right)^3 \neq 1$

Question 57:

$$\left(\frac{4}{3}\right)^5 \times \left(\frac{5}{7}\right)^5 = \left(\frac{4}{3} \div \frac{5}{7}\right)^5$$

Solution:

False

Here, $\left(\frac{4}{3}\right)^5 \times \left(\frac{5}{7}\right)^5 = \left(\frac{4}{3} \times \frac{5}{7}\right)^5$ $[\because a^m \times b^m = (ab)^m]$

and $\left[\left(\frac{4}{3}\right) \div \left(\frac{5}{7}\right)\right]^5 = \left(\frac{4}{3} \times \frac{7}{5}\right)^5$ $\left[\because \left(\frac{a}{b}\right) \div \left(\frac{c}{d}\right) = \frac{a}{b} \times \frac{d}{c}\right]$

Hence, $\left(\frac{4}{3} \times \frac{5}{7}\right)^5 \neq \left(\frac{4}{3} \times \frac{7}{5}\right)^5$

Question 58:

$$\left(\frac{5}{8}\right)^9 \div \left(\frac{5}{8}\right)^4 = \left(\frac{5}{8}\right)^4$$

Solution:

False

Here, $\left(\frac{5}{8}\right)^9 \div \left(\frac{5}{8}\right)^4 = \left(\frac{5}{8}\right)^{9-4} = \left(\frac{5}{8}\right)^5$ $[\because a^m \div a^n = a^{m-n}]$

Hence, $\left(\frac{5}{8}\right)^9 \div \left(\frac{5}{8}\right)^4 \neq \left(\frac{5}{8}\right)^4$

Question 59:

$$\left(\frac{7}{3}\right)^2 \times \left(\frac{7}{3}\right)^5 = \left(\frac{7}{3}\right)^{10}$$

Solution:

False

$$\text{Here, } \left(\frac{7}{3}\right)^2 \times \left(\frac{7}{3}\right)^5 = \left(\frac{7}{3}\right)^{2+5} \quad [\because a^m \times a^n = a^{m+n}]$$

$$= \left(\frac{7}{3}\right)^7$$

$$\text{Hence, } \left(\frac{7}{3}\right)^2 \times \left(\frac{7}{5}\right)^5 \neq \left(\frac{7}{3}\right)^{10}$$

Question 60:

5° x 25° x 125° =(50)6

Solution:

True

Here,5° x 25° x 125°=5° x(5 x 5)° x (5 x 5 x 5)° [∴ 25 = 5 x 5 and 125 = 5 x 5 x 5]

= 5° x 5° x 5° x 5° x 5° x 5° [∴ a^m x b^m = $a^m b^m$]

= $(5°)^6$

Hence, 5° x 25° x 125° =$(5^0)^6$

Question 61:

876543 = 8 x 10^5 + 7 x 10^4 + 6 x 10^3 +5 x 10^2 + 4 x 10^1 + 3 x 10°

Solution:

True

Take RHS = 8 x 10^5 + 7 x 10^4 + 6 x 10^3 +5 x 10^2 + 4 x 10^1 + 3 x 10°

= 8 x 100000 + 7 x 10000 + 6 x 1000 + 5 x 100 + 4 x 10 + 3 x 1 [∴ a° = 1]

= 800000 + 70000 + 6000 + 500+ 40+3 = 876543 = LHS

Hence, RHS = LHS

Question 62:

600060 = 6 x 10^5 + 6 x 10^2

Solution:

False

Take RHS =6 x 10^5 + 6 x 10^2 = 6 x 100000 + 6 x 100= 600000 + 600 = 600600 ≠ LHS

Hence, RHS ≠ LHS

Question 63:

4 x 10^5+ 3 x 10^4 + 2 x 10^3 + 1 x 10° = 432010

Solution:

False

Take LHS

= 4 x 10^5+ 3 x 10^4 + 2 x 10^3 + 1 x 10°

= 4 x 100000+ 3 x 10000 + 2 x 1000+ 1 x 1 [∴ a° = 1]

= 400000 + 30000 + 2000 + 1 = 432001 ≠ RHS

Hence, LHS ≠ RHS

Question 64:

8 x 10^6 + 2 x 10^4 + 5 x 10^2 + 9 x 10° = 8020509

Solution:

True

Take LHS

= 8 x 10^6 + 2 x 10^4 + 5 x 10^2 + 9 x 10°

= 8 x 1000000 + 2 x 10000 + 5 x 100 + 9 x 1 [∴ a° = 1]

= 8000000 + 2000 + 500+9 = 8020509 = RHS

Hence, LHS = RHS

Question 65:

$4° + 5° + 6° = (4 + 5 + 6)°$

Solution:

False

Here, $4° + 5° + 6° = 1 + 1 + 1 = 3$ [$\therefore a° = 1$]

and $(4 + 5 + 6)° = (15)° = 1$

Hence, $4° + 5° + 6° \neq (4 + 5 + 6)°$

Question 66:

Arrange in ascending order.

$2^5, 3^3, 2^3 \times 2, (3^3)^2, 3^5, 4°, 2^3 \times 3^1$

Solution:

In ascending order, the numbers are arranged from smallest to largest.

We have, $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$.

$3^3 = 3 \times 3 \times 3 = 27$.

$2^3 \times 2 = 2 \times 2 \times 2 \times 2 = 16$.

$(3^3)^2 = 3^{3 \times 2}$ [$\therefore (a^m)^n = a^{mn}$]

$3^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$.

$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$.

$4° = 1$ [$\therefore a^0 = 1$]

and $2^3 \times 3^1 = 2 \times 2 \times 2 \times 3 = 24$ Thus, the required ascending order will be

$4° < 2^3 \times 2$ < 23 x 31 < 33 < 25 < 35 < (33)2.

Question 67:

Arrange the following exponents in descending order. 2^{2+3}, $(2^2)^3$, (2×2^2), $3^5/3^2$, $(3^2 \times 3^0)$, $(2^2 \times 5^2)$.

Solution:

In descending order, the numbers are arranged from largest to smallest. We have, 22 + 3 = 25=2 x 2 x 2 x 2 x 2 = 32.

$(2^2)^3 \Rightarrow 2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$.

$2 \times 2^2 = 2^{1+2} \Rightarrow 2^3 = 8$.

$3^5/3^2 = 3^{5-2} \Rightarrow 3^3 = 27$.

$3^2 \times 3^0 = 3^{2+0} \Rightarrow 3^2 = 9$.

$2^3 \times 5^2 = 2 \times 2 \times 2 \times 5 \times 5 \Rightarrow 8 \times 25 = 200$.

Thus, the required descending order will be

$(2^2 \times 5^2) > (2^2)^3 > 2^{2+3} > 3^5/3^2 > (3^2 \times 3^0) > (2 \times 2^2)$.

Question 68:

By what number should $(-4)^5$ be divided so that the quotient may be equal to $(-4)^3$?

Solution:

In order to find the number, which should divide $(-4)^5$ to get the quotient $(-4)^3$, we will divide $(-4)^5$ by $(-4)^3$.

Hence, required number = = $(-4)^{5-3} = (-4)^2$

Question 69:

Find m, so that $\left(\frac{2}{9}\right)^3 \times \left(\frac{2}{9}\right)^6 = \left(\frac{2}{9}\right)^{2m-1}$.

Solution:

We have, $\left(\frac{2}{9}\right)^3 \times \left(\frac{2}{9}\right)^6 = \left(\frac{2}{9}\right)^{2m-1}$

$\Rightarrow \left(\frac{2}{9}\right)^{3+6} = \left(\frac{2}{9}\right)^{2m-1}$ $[\because a^m \times a^n = a^{m+n}]$

$\Rightarrow \left(\frac{2}{9}\right)^{9} = \left(\frac{2}{9}\right)^{2m-1}$

$\Rightarrow 9 = 2m - 1$ $[\because a^m = a^n \Rightarrow m = n]$

$\Rightarrow 9 + 1 = 2m$ [transposing (– 1) to LHS]

$\Rightarrow 10 = 2m$

$\Rightarrow \frac{10}{2} = \frac{2m}{2}$ [dividing both sides by 2]

$\Rightarrow 5 = m$

Hence, $m = 5$.

Question 70:

If $\frac{p}{q} = \left(\frac{3}{2}\right)^2 \div \left(\frac{9}{4}\right)^0$, find the value of $\left(\frac{p}{q}\right)^3$.

Solution:

We have, $\left(\frac{p}{q}\right) = \left(\frac{3}{2}\right)^2 \div \left(\frac{9}{4}\right)^0$

$\Rightarrow \frac{p}{q} = \left(\frac{3}{2}\right)^2 \div \frac{1}{1}$ $[\because a^0 = 1]$

$\Rightarrow \frac{p}{q} = \left(\frac{3}{2}\right)^2$ $\left[\because \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}\right]$

$\Rightarrow \frac{p}{q} = \frac{3^2}{2^2}$ $\left[\because \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\right]$

$\Rightarrow \frac{p}{q} = \frac{9}{4}$

On taking cube both sides, we get

$$\left(\frac{p}{q}\right)^3 = \left(\frac{9}{4}\right)^3$$

$$\therefore \quad \left(\frac{p}{q}\right)^3 = \frac{9 \times 9 \times 9}{4 \times 4 \times 4} = \frac{729}{64}$$

Question 71:

Find the reciprocal of the rational number $\left(\frac{1}{2}\right)^2 \div \left(\frac{2}{3}\right)^3$.

Solution:

Given, $\left(\frac{1}{2}\right)^2 \div \left(\frac{2}{3}\right)^3 = \dfrac{\left(\frac{1}{2}\right)^2}{\left(\frac{2}{3}\right)^3}$ $\left[\because a \div b = \frac{a}{b}\right]$

$= \dfrac{\frac{(1)^2}{(2)^2}}{\frac{(2)^3}{(3)^3}} = \dfrac{\left(\frac{1}{4}\right)}{\left(\frac{8}{27}\right)}$ $\left[\because \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\right]$

$[\because 1^2 = 1, 2^2 = 4, 2^3 = 8 \text{ and } 3^3 = 27]$

$= \frac{1}{4} \times \frac{27}{8} = \frac{27}{4 \times 8} = \frac{27}{32}$ $\left[\because \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}\right]$

We know that, reciprocal of a rational number is obtained by interchanging numerator and denominator.

$\therefore$ Reciprocal of given number $= \frac{32}{27}$

Question 72:

Find the value of

(a) 7^0 (b) $7^7 \div 7^7$

(c) $(-7)^{2\times7-6-8}$ (d) $(2^0+3^0+4^0)(4^0-3^0-2^0)$

(e) $2\times3\times4\div2^0\times3^0\times4^0$ (f) $(8^0-2^0)\times(8^0+2^0)$

Solution:

(a) $7^0 = 1$ $[\because a^0 = 1]$

(b) $7^7 \div 7^7 = \dfrac{7^7}{7^7} = 7^{7-7}$ $\left[\because \dfrac{a^m}{a^n} = a^{m-n}\right]$

$= 7^0 = 1$ $[\because a^0 = 1]$

(c) $(-7)^{2\times7-6-8} = (-7)^{14-14} = (-7)^0 = 1$ $[\because a^0 = 1]$

(d) $(2^0+3^0+4^0)(4^0-3^0-2^0) = (1+1+1)(1-1-1)$ $[\because a^0 = 1]$

$= (3)(-1) = -3$

(e) $2\times3\times4\div2^0\times3^0\times4^0 = 2\times3\times4\div1\times1\times1$ $[\because a^0 = 1]$

$= \dfrac{2\times3\times4}{1\times1\times1} = 2\times3\times4 = 24$

(f) $(8^0-2^0)\times(8^0+2^0) = (1-1)\times(1+1) = 0\times2 = 0$ $[\because a^0 = 1]$

Question 73:

Find the value of n, where n is an integer and $2^{n-5}\times6^{2n-4} = \dfrac{1}{12^4\times2}$.

Solution:

We have, $2^{n-5}\times6^{2n-4} = \dfrac{1}{12^4\times2}$

$\Rightarrow \dfrac{2^n}{2^5}\times\dfrac{6^{2n}}{6^4} = \dfrac{1}{12^4\times2}$ $\left[\because a^{m-n} = \dfrac{a^m}{a^n}\right]$

$\Rightarrow \dfrac{2^n\times6^{2n}}{2^5\times6^4} = \dfrac{1}{(2\times6)^4\times2}$ $[\because 12 = 6\times2]$

$\Rightarrow 2^n\times(6^2)^n = \dfrac{2^5\times6^4}{2^4\times6^4\times2}$ [by cross-multiplication]

$[\because a^{mn} = (a^m)^n$ and $(a\times b)^m = a^m\times b^m]$

$\Rightarrow 2^n\times36^n = \dfrac{2^5\times6^4}{2^5\times6^4}$ $[\because a^m\times a^n = a^{m+n}]$

$\Rightarrow 2^n\times36^n = 1$

$\Rightarrow (2\times36)^n = 1$ $[\because a^m\times b^m = (ab)^m]$

$\Rightarrow (72)^n = (72)^0$ $[\because a^0 = 1]$

$\therefore n = 0$ $[\because$ if $a^m = a^n \Rightarrow m = n]$

Question 74:

Express the following in usual form.

(a) 8.01×10^7 (b) 1.75×10^{-3}

Solution:

(a) Here, $8.01\times10^7 = \dfrac{801}{100}\times10000000 = 80100000$

(b) Here, $1.75\times10^{-3} = \dfrac{175}{100}\times\dfrac{1}{10^3} = \dfrac{175}{100000} = 0.00175$ $\left[\because a^{-m} = \dfrac{1}{a^m}\right]$

Question 75:

Find the value of

(a) 2^5 (b) (-3^5) (c) $-(-4)^4$

Solution:

We know that, $a^n = a\times a\times a\times\ldots\times a$ (n times)

(a) $2^5 = 2\times2\times2\times2\times2 = 32$

(b) $(-3^5) = (-1)^5\times3^5 = -1\times3\times3\times3\times3\times3 = -243$ $[\because (-1)^n = -1$, if n is odd$]$

(c) $-(-4)^4 = -[(-4)\times(-4)\times(-4)\times(-4)] = -[(-1)^4(4\times4\times4\times4)] = -(256) = -256$

$[\because (-1)^n = 1$, if n is even$]$

Question 76:

Express the following in exponential form.

(a)3 x 3 x 3 x a x a x a x a (b) a x a x b x b x b x c x c x c x c (c) s x s x t x t x s x s x t

Solution:

We know that,

a x a x … x a (n times) = a^n

(a) 3 x 3 x 3 x a x a x a x a=3^3 x a^4 = 27 a^4

(b) a x a x b x b x b x c x c x c x c = a^2 x b^3 x c^4

(c) s x s x t x t x s x s x t = s x s x s x s x t x t x t = s^4 x t^3

Question 77:

How many times of 30 must be added together to get a sum equal to 30^7.

Solution:

Let n be the number of times that 30 must be added together to get a sum equal to 30^7.
Therefore, we can write that

$$\underbrace{30 + 30 + \dots + 30}_{n \text{ times}} = 30^7$$

$\Rightarrow \quad 30 \times n = 30^7 \quad [\because \underbrace{a + a + \dots + a}_{n \text{ times}} = a \times n]$

$\Rightarrow \quad \dfrac{30 \times n}{30} = \dfrac{30^7}{30}$ [dividing both sides by 30]

$\Rightarrow \quad n = 30^{7-1} \quad \left[\because \dfrac{a^m}{a^n} = a^{m-n}\right]$

$\therefore \quad n = 30^6$

Hence, if 30 is added 30^6 times, then we get 30^7.

Question 78:

Express each of the following numbers using exponential notations,

(a) 1024 (b) 1029 (c) $\dfrac{144}{875}$

Using prime factorization of 1024, we have

Solution:

For solving these type of questions, we use here prime factorisation method.

(a) Given, 1024

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Using prime factorisation of 1024, we have

$$1024 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^{10}$$

(b) Given, 1029

3	1029
7	343
7	49
7	7
	1

Using prime factorisation of 1029, we have

$$1029 = 3 \times 7 \times 7 \times 7 = 3 \times 7^3$$

(c) Given, $\frac{144}{875}$

2	144
2	72
2	36
2	18
3	9
3	3
	1

5	875
5	175
5	35
7	7
	1

Using prime factorisation of 144 and 875, we have

$$\frac{144}{875} = \frac{2 \times 2 \times 2 \times 2 \times 3 \times 3}{5 \times 5 \times 5 \times 7} = \frac{2^4 \times 3^2}{5^3 \times 7^1}$$

Question 79:

Identify the greater number, in each of the following.

(a) 2^6 or 6^2 (b) 2^9 or 9^2 (c) 7.9×10^4 or 5.28×10^5

Solution:

(a) We have, $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$ and $6^2 = 6 \times 6 = 36$ So, $2^6 > 6^2$

(b) We have, $2^9 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 512$ and $9^2 = 9 \times 9 = 81$

So, $2^9 > 9^2$

(c) We have, $7.9 \times 10^4 = 7.9 \times 10000 = 79000$ and $5.28 \times 10^5 = 5.28 \times 100000 = 528000$ So, $5.28 \times 10^5 > 7.9 \times 10^4$

Question 80:

Express each of following as a product of powers of their prime factors,

(a) 9000 (b) 2025 (c) 800

Solution:

(a)

2	9000
2	4500
2	2250
3	1125
3	375
5	125
5	25
5	5
	1

Using prime factorisation of 9000, we have

$$9000 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5$$

$\therefore$ $9000 = 2^2 \times 3^2 \times 5^3$

(b)

3	2025
3	675
3	225
3	75
5	25
5	5
	1

Using prime factorisation of 2025, we have

$$2025 = 3 \times 3 \times 3 \times 3 \times 5 \times 5$$

$\therefore$ $2025 = 3^4 \times 5^2$

(c)

2	800
2	400
2	200
2	100
2	50
5	25
5	5
	1

Using prime factorisation of 800, we have

$$800 = 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$$

$\therefore$ $800 = 2^5 \times 5^2$

Question 81:

Express each of the following in single exponential form,

(a) $2^3 \times 3^3$

(b) $2^4 \times 4^2$

(c) $5^2 \times 7^2$

(d) $(-5)^5 \times (-5)$

(e) $(-3)^3 \times (-10)^3$

(f) $(-11)^2 \times (-2)^2$

Solution:

(a) We have, $2^3 \times 3^3 = (2 \times 3)^3$ $[\because a^m \times b^m = (a \times b)^m]$

$= 6^3$

(b) We have, $2^4 \times 4^2 = 2^4 \times (2^2)^2$ $[\because 4 = 2^2]$

$= 2^4 \times 2^4$ $[\because (a^m)^n = a^{mn}]$

$= 2^{4+4}$ $[\because a^m \times a^n = a^{m+n}]$

$= 2^8$

(c) We have, $5^2 \times 7^2 = (5 \times 7)^2$ $[\because a^m \times b^m = (a \times b)^m]$

$= 35^2$

(d) We have, $(-5)^5 \times (-5) = (-5)^{5+1} = (-5)^6$ $[\because a^m \times a^n = a^{m+n}]$

$= (-1 \times 5)^6 = (-1)^6 \times (5)^6$ $[\because (a \times b)^m = a^m \times b^m]$

$= 1 \times 5^6$ $[\because (-1)^n = 1, \text{ if } n \text{ is even}]$

$= 5^6$

(e) We have, $(-3)^3 \times (-10)^3 = [(-3) \times (-10)]^3$ $[\because a^m \times b^m = (a \times b)^m]$

$= (30)^3$ $[\because (-3) \times (-10) = 30]$

(f) We have, $(-11)^2 \times (-2)^2 = [(-11) \times (-2)]^2$ $[\because a^m \times b^m = (a \times b)^m]$

$= 22^2$ $[\because (-11) \times (-2) = 22]$

Question 82:

Express the following numbers in standard form.

(a) 76,47,000 (b) 8,19,00,000

(c) 5,83,00,00,00,000 (d) 24 billion

Solution:

(a) We have, 76,47,000 = 7647000,00

A number in standard form is written as $a \times 10^k$, where a is the terminating decimal such that $1 < a < 10$ and k is any integer.

So,7647000 = 7647×10^3

= $7.647 \times 10^3 \times 10^3 = 7.647 \times 10^6$

Similarly,

(b) 8,19,00,000 = 81900000.00= 819 x 10 s = $8.19 \times 10^2 \times 10^5 = 8.19 \times 10^7$

(c) 5,83,00,00,00,000 = 583000000000.00= $583 \times 10^9 = 5.83 \times 10^2 \times 10^9 = 5.83 \times 10^{11}$

(d) 24 billion = 24,00,00,00,000 = $24 \times 10^9 = 2.4 \times 10^1 \times 10^9 = 2.4 \times 10^{10}$

Question 83:

The speed of light in vacuum is 3×10^8 m/s. Sunlight takes about 8 minutes to reach the Earth. Express distance of Sun from Earth in standard form.

Solution:

It is given that,

Speed of light = 3×10^8 m/s

Time taken by light to reach the Earth = 8 min= 8 x 60 s= 480 s [∴ 1 min = 60s]

We know that,

Distance = Speed x Time=$3 \times 10^8 \times 480 = 1440 \times 10^8 = 1.440 \times 10^3 \times 10^8$

= 1.44×10^{11} [∴ $10^3 \times 10^8 = 10^{11}$]

Hence, the distance of Sun from the Earth is 1.44×10^{11} m.

Question 84:

Simplify and express each of the following in exponential form.

(a) $\left[\left(\frac{3}{7}\right)^4 \times \left(\frac{3}{7}\right)^5\right] \div \left(\frac{3}{7}\right)^7$ (b) $\left[\left(\frac{7}{11}\right)^5 \div \left(\frac{7}{11}\right)^2\right] \times \left(\frac{7}{11}\right)^2$

(c) $(3^7 \div 3^5)^4$ (d) $\left(\frac{a^6}{a^4}\right) \times a^5 \times a^0$

(e) $\left[\left(\frac{3}{5}\right)^3 \times \left(\frac{3}{5}\right)^8\right] \div \left[\left(\frac{3}{5}\right)^2 \times \left(\frac{3}{5}\right)^4\right]$

(f) $(5^{15} \div 5^{10}) \times 5^5$

Solution:

(a) We have, $\left[\left(\frac{3}{7}\right)^4 \times \left(\frac{3}{7}\right)^5\right] \div \left(\frac{3}{7}\right)^7$

$$= \left(\frac{3}{7}\right)^{4+5} \div \left(\frac{3}{7}\right)^7 \qquad [\because a^m \times a^n = a^{m+n}]$$

$$= \left(\frac{3}{7}\right)^9 \div \left(\frac{3}{7}\right)^7 = \frac{\left(\frac{3}{7}\right)^9}{\left(\frac{3}{7}\right)^7} = \left(\frac{3}{7}\right)^{9-7} \qquad \left[\because \frac{a^m}{a^n} = a^{m-n}\right]$$

$$= \left(\frac{3}{7}\right)^2$$

(b) We have, $\left[\left(\frac{7}{11}\right)^5 \div \left(\frac{7}{11}\right)^2\right] \times \left(\frac{7}{11}\right)^2$

$$= \left[\frac{\left(\frac{7}{11}\right)^5}{\left(\frac{7}{11}\right)^2}\right] \times \left(\frac{7}{11}\right)^2 = \left(\frac{7}{11}\right)^{5-2} \times \left(\frac{7}{11}\right)^2 \qquad \left[\because \frac{a^m}{a^n} = a^{m-n}\right]$$

$$= \left(\frac{7}{11}\right)^3 \times \left(\frac{7}{11}\right)^2 = \left(\frac{7}{11}\right)^{3+2} \qquad [\because a^m \times a^n = a^{m+n}]$$

$$= \left(\frac{7}{11}\right)^5$$

(c) We have, $(3^7 \div 3^5)^4 = \left(\frac{3^7}{3^5}\right)^4 = (3^{7-5})^4 \qquad \left[\because \frac{a^m}{a^n} = a^{m-n}\right]$

$$= (3^2)^4 = 3^{2 \times 4} \qquad [\because (a^m)^n = a^{mn}]$$

$$= 3^8$$

(d) We have, $\left(\frac{a^6}{a^4}\right) \times a^5 \times a^0 = (a^{6-4} \times a^5 \times 1) \qquad \left[\because \frac{a^m}{a^n} = a^{m-n} \text{ and } a^0 = 1\right]$

$$= a^2 \times a^5 = a^{2+5} \qquad [\because a^m \times a^n = a^{m+n}]$$

$$= a^7$$

(e) We have, $\left[\left(\frac{3}{5}\right)^3 \times \left(\frac{3}{5}\right)^8\right] \div \left[\left(\frac{3}{5}\right)^2 \times \left(\frac{3}{5}\right)^4\right]$

$$= \left(\frac{3}{5}\right)^{3+8} \div \left(\frac{3}{5}\right)^{2+4} \qquad [\because a^m \times a^n = a^{m+n}]$$

$$= \left(\frac{3}{5}\right)^{11} \div \left(\frac{3}{5}\right)^6 = \frac{\left(\frac{3}{5}\right)^{11}}{\left(\frac{3}{5}\right)^6} = \left(\frac{3}{5}\right)^{11-6} \qquad \left[\because \frac{a^m}{a^n} = a^{m-n}\right]$$

$$= \left(\frac{3}{5}\right)^5$$

(f) We have, $(5^{15} \div 5^{10}) \times 5^5 = \left(\frac{5^{15}}{5^{10}}\right) \times 5^5 = 5^{15-10} \times 5^5 \qquad \left[\because \frac{a^m}{a^n} = a^{m-n}\right]$

$$= 5^5 \times 5^5 = 5^{5+5} = 5^{10} \qquad [\because a^m \times a^n = a^{m+n}]$$

Question 85:

Evaluate

(a) $\frac{7^8 \times a^{10} b^7 c^{12}}{7^6 \times a^8 b^4 c^{12}}$

(b) $\frac{5^4 \times 7^4 \times 2^7}{8 \times 49 \times 5^3}$

(c) $\frac{125 \times 5^2 \times a^7}{10^3 \times a^4}$

(d) $\frac{3^4 \times 12^3 \times 36}{2^5 \times 6^3}$

(e) $\left(\frac{6 \times 10}{2^2 \times 5^3}\right)^2 \times \frac{25}{27}$

(f) $\frac{15^4 \times 18^3}{3^3 \times 5^2 \times 12^2}$

(g) $\frac{6^4 \times 9^2 \times 25^3}{3^2 \times 4^2 \times 15^6}$

Solution:

(a) We have, $\frac{7^8 \times a^{10} b^7 c^{12}}{7^6 \times a^8 b^4 c^{12}} = \left(\frac{7^8}{7^6}\right) \times \left(\frac{a^{10}}{a^8}\right) \times \left(\frac{b^7}{b^4}\right) \times \left(\frac{c^{12}}{c^{12}}\right)$

$= 7^{8-6} \times a^{10-8} \times b^{7-4} \times c^{12-12}$ $\left[\therefore \frac{a^m}{a^n} = a^{m-n}\right]$

$= 7^2 \times a^2 \times b^3 \times c^0 = 49 a^2 b^3$ $[\because c^0 = 1]$

(b) We have, $\frac{5^4 \times 7^4 \times 2^7}{8 \times 49 \times 5^3} = \frac{5^4 \times 7^4 \times 2^7}{2^3 \times 7^2 \times 5^3}$ $[\because 8 = 2^3 \text{ and } 49 = 7^2]$

$= \left(\frac{5^4}{5^3}\right) \times \left(\frac{2^7}{2^3}\right) \times \left(\frac{7^4}{7^2}\right) = 5^{4-3} \times 2^{7-3} \times 7^{4-2}$ $\left[\because \frac{a^m}{a^n} = a^{m-n}\right]$

$= 5 \times 2^4 \times 7^2 = 5 \times 16 \times 49$

$= 3920$

(c) We have, $\frac{125 \times 5^2 \times a^7}{10^3 \times a^4} = \frac{5^3 \times 5^2 \times a^7}{(2 \times 5)^3 \times a^4}$ $[\because 125 = 5^3]$

$= \frac{5^{3+2} \times a^7}{2^3 \times 5^3 \times a^4}$ $[\because a^m \times a^n = a^{m+n} \text{ and } (a \times b)^m = a^m \times b^m]$

$= \frac{5^5 \times a^7}{2^3 \times 5^3 \times a^4} = \left(\frac{5^5}{5^3}\right) \times \left(\frac{a^7}{a^4}\right) \times \left(\frac{1}{2^3}\right)$

$= \frac{5^{5-3} \times a^{7-4}}{2^3}$ $\left[\because \frac{a^m}{a^n} = a^{m-n}\right]$

$= \frac{5^2 \times a^3}{2^3} = \frac{25a^3}{8}$

(d) We have, $\frac{3^4 \times 12^3 \times 36}{2^5 \times 6^3}$

$= \frac{3^4 \times (2^2 \times 3)^3 \times (2^2 \times 3^2)}{2^5 \times (2 \times 3)^3}$

$[\because 12 = 2 \times 2 \times 3 \text{ and } 36 = 2 \times 2 \times 3 \times 3]$

$= \frac{3^4 \times 2^6 \times 3^3 \times 2^2 \times 3^2}{2^5 \times 2^3 \times 3^3}$ $[\because (a \times b)^m = a^m \times b^m]$

$= \frac{(3^4 \times 3^2 \times 3^3) \times (2^6 \times 2^2)}{(2^5 \times 2^3) \times 3^3}$

$= \frac{3^{4+2+3} \times 2^{6+2}}{2^{5+3} \times 3^3}$ $[\because a^m \times a^n = a^{m+n}]$

$= \frac{3^9 \times 2^8}{3^3 \times 2^8} = 3^{9-3} \times 2^{8-8}$ $\left[\because \frac{a^m}{a^n} = a^{m-n}\right]$

$= 3^6 \times 2^0 = 3^6 \times 1 = 729$ $[\because a^0 = 1]$

(e) We have, $\left(\frac{6 \times 10}{2^2 \times 5^3}\right)^2 \times \frac{25}{27} = \left(\frac{2 \times 3 \times 2 \times 5}{2^2 \times 5^3}\right)^2 \times \frac{5^2}{3^3}$ $[\because 6 = 2 \times 3 \text{ and } 10 = 2 \times 5]$

$= \left(\frac{2^2 \times 3 \times 5}{2^2 \times 5^3}\right)^2 \times \frac{5^2}{3^3} = \left(\frac{3}{5^2}\right)^2 \times \frac{5^2}{3^3}$ $[\because (a \times b)^m = a^m \times b^m]$

$= \frac{3^2}{5^4} \times \frac{5^2}{3^3} = \frac{1}{5^2 \times 3} = \frac{1}{25 \times 3} = \frac{1}{75}$ $[\because (a^m)^n = a^{mn}]$

(f) We have, $\frac{15^4 \times 18^3}{3^3 \times 5^2 \times 12^2} = \frac{(3 \times 5)^4 \times (2 \times 3^2)^3}{3^3 \times 5^2 \times (2^2 \times 3)^2}$ $[\because 18 = 2 \times 3 \times 3 \text{ and } 12 = 2 \times 2 \times 3]$

$= \frac{3^4 \times 5^4 \times 2^3 \times 3^6}{3^3 \times 5^2 \times 2^4 \times 3^2}$ $[\because (a \times b)^m = a^m \times b^m]$

$= \frac{3^{4+6-3-2} \times 5^{4-2}}{2^{4-3}}$ $\left[\because \frac{a^m}{a^n} = a^{m-n} \text{ and } a^m \times a^n = a^{m+n}\right]$

$= \frac{3^5 \times 5^2}{2} = \frac{243 \times 25}{2} = \frac{6075}{2}$

(g) We have, $\frac{6^4 \times 9^2 \times 25^3}{3^2 \times 4^2 \times 15^6} = \frac{(2 \times 3)^4 \times (3^2)^2 \times (5^2)^3}{3^2 \times (2^2)^2 \times (3 \times 5)^6}$

$= \frac{2^4 \times 3^4 \times 3^4 \times 5^6}{3^2 \times 2^4 \times 3^6 \times 5^6}$ $[\because (a \times b)^m = a^m \times b^m]$

$= 2^{4-4} \times 3^{4+4-2-6} \times 5^{6-6}$ $\left[\because \frac{a^m}{a^n} = a^{m-n}\right]$

$= 2^0 \times 3^0 \times 5^0$

$= 1 \times 1 \times 1 = 1$

Question 86:

Express the given information in Scientific notation (standard form) and then arrange them in ascending order of their size.

S. N.	Deserts of the World	Area (in sq km)
1.	Kalahari, South Africa	932,400
2.	Thar, India	199,430
3.	Gibson, Australia	155,400
4.	Great Victoria, Australia	647,500
5	Sahara, North Africa	8,598,800

Solution:

1. Area of Kalahari, South Africa = 932,400 = 932400.00 [$\therefore$ standard form $a \times 10^k$]
$= 9324 \times 10^2 = 9.324 \times 10^3 \times 10^2 = 9.324 \times 10^6$
2. Area of Thar, India = 199,430 = 199430.00 = $19943 \times 10^1 = 1.9943 \times 10^4 \times 10^1 = 1.9943 \times 10^5$
3. Area of Gibson, Australia = 155,400= 155400.00= $1554 \times 10^2 = 1.554 \times 10^3 \times 10^2 = 1.554 \times 10^5$
4. Area of Great Victoria, Australia = 647,500= 647500.00
$= 6475 \times 10^2 = 6.475 \times 10^3 \times 10^5 = 6475 \times 10^5$
5. Area of Sahara, North-Africa = 8,598,800= 8598800.00
$= 85988 \times 10^2 = 8.5988 \times 10^4 \times 10^2 = 85988 \times 10^6$

Two numbers written in scientific notation can be compared. The number with the larger power of 10 is greater than the number with the smaller power of 10. If the powers of ten are the same, then the number with larger factor is the larger number.

Hence, required ascending order of the size will be

Gibson, Australia < Thar, India < Great Victoria, Australia < Kalahari, South-Africa < Sahara, North-Africa.

Question 87:

Express the given information in scientific notation and then arrange them in descending order of their size.

S. N.	Name of the planet	Mass (in kg)
1.	Mercury	330000000000000000000000
2.	Venus	4870000000000000000000000
3.	Earth	5980000000000000000000000
4.	Mars	642000000000000000000000
5.	Jupiter	1900000000000000000000000000
6.	Saturn	569000000000000000000000000
7.	Uranus	86900000000000000000000000
8.	Neptune	102000000000000000000000000
9.	Pluto	13100000000000000000000

Solution:

A number is written in standard form as $a \times 10^k$, where a is terminating decimal and k is an integer.

S. N.	Name of the planet	Mass (in standard form)
1.	Mercury	3.3×10^{23}
2.	Venus	4.87×10^{24}
3.	Earth	5.98×10^{24}
4.	Mars	6.42×10^{23}
5.	Jupiter	1.9×10^{27}
6.	Saturn	5.69×10^{26}
7.	Uranus	8.69×10^{25}
8.	Neptune	1.02×10^{26}
9.	Pluto	1.31×10^{22}

Two numbers written in scientific notation can be compared. The number with the larger power of 10 is greater than the number with the smaller power of 10. If the powers of ten are the same, then the number with larger factor is the larger number.

Hence, the required descending order of the size will be

Jupiter > Saturn > Neptune > Uranus > Earth > Venus > Mars > Mercury > Pluto

Question 88:

Write the number of seconds in scientific notation.

S. N.	Unit	Value in seconds
1.	1 minute	60
2.	1 hour	3,600
3.	1 day	86,400
4.	1 month	2,600,000
5.	1 year	32,000,000
6.	10 years	3,20,000,000

Solution:

1. 1 min = 60 s = 60 x 10^1 s = 6 x 101 s
2. 1 h = 3,600 s = 36 x 10^2 s = 3.6 x 10 x 10^2 s = 3.6 x 10^3 s
3. 1 day = 86,400 s = 8.64 x 10^2 s = 8.64 x 10^2 x 10^2 s = 8.64 x 10^4 s = 8.6 x 10^4 s
4. 1 month = 2,600,000 s=26 x 10^5 s =2.6 x 10 x 10^5 s = 2.6 x 10^6 s
5. 1 yr = 32,000,000 s = 32 x 10^6 s = 32 x 10^6 s = 3.2 x 10 x 10^6 s = 32 x 10^7 s
6. 10 yr = 320,000,000 s = 32 x 10^7 S = 32 x 10 x 10^7 s = 3.2 x 10^8 s

Question 89:

In our own planet Earth, 361,419,000 square kilometre of area is covered with water and 148,647,000 square kilometre of area is covered by land. Find the approximate ratio of area covered with water to area covered by land converting these numbers into scientific notation.

Solution:

Given,
Area covered by water = 361419000 km^2
Area covered by land = 148647000 km^2
Convertion of area into scientific notation,

$$361419000 = 361419 \times 10^3$$

Also, $361419 = 3.61419 \times 10^5$

So, $3.61419 \times 10^5 \times 10^3 = 3.61419 \times 10^8$

∴ Area covered by water $= 3.61419 \times 10^8 \text{ km}^2$

Similarly, $148647000 = 148647 \times 10^3$

Also, $148647 = 1.48647 \times 10^5$

So, $1.48647 \times 10^5 \times 10^3 = 1.48647 \times 10^8$

∴ Area covered by land $= 1.48647 \times 10^8 \text{ km}^2$

Let $3.61419 \times 10^8 \approx 3.6 \times 10^8$

and $1.48647 \times 10^8 \approx 1.5 \times 10^8$

∴ Ratio of water to land $= \frac{3.6}{1.5} = 12 : 5$

Question 90:

If $2^{n+2} + 2^{n+1} + 2n = c \times 2^n$, then find c.

Solution:

We have, $2^{n+2} + 2^{n+1} + 2n = c \times 2^n$

=> $2^n\, 2^2 + 2^n\, 2^1 + 2^n = c \times 2^n$ [∴ $a^{m+n} = a^m \times a^n$]

=> $2^n\, [2^2 – 2^1 + 1] = c \times 2^n$ [taking common 2^n in LHS]

=> $2^n[4-2 + 1]=c \times 2^n$

=> $3 \times 2^n = c \times 2^n$

$3 \times 2^n \times 2^{-n} = c \times 3^n \times c^{-n}$ [multiplying both sides by 2^{-n}]

=> $3 \times 2^{n-1} = c \times 2^{n-n}$ [∴ $a^{m+n} = a^m \times a^n$]

=> $3 \times 2^{\circ} = c \times 2^{\circ}$

=> $3 \times 1 = c \times 1$ [∴ $a^{\circ}=1$]

∴ 3 = c

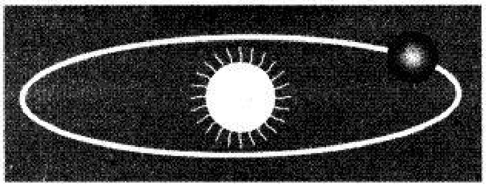
Question 91:

A light year is the distance that light can travel in one year.

1 light year = 9,460,000,000,000 km.

(a) Express one light year in scientific notation.

(b) The average distance between Earth and Sun is 1.496×10^8 km. Is the distance between Earth and the Sun greater than, less than or equal to one light year?



Solution:

(a) Given, 1 light year = 9,460,000,000, 000 km

For standard form $= 946 \times 10^{10} \text{ km} = \frac{946}{100} \times 10^{10} \times 100 \text{ km}$

$= 9.46 \times 10^{12} \text{ km}$

(b) The average distance between Earth and Sun $= 1.496 \times 10^8$ km

∴ Distance between Earth and Sun $= \frac{1.496}{10000} \times 10^8 \times 10^4 \text{ km} = 0.0001496 \times 10^{12} \text{ km}$

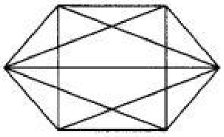
Since, $9.46 > 0.0001496$

So, the distance between Earth and Sun less than one light year.

Question 92:

Geometry Application

The number of diagonals of an n-sided figure is ½(n^2 – 3n). Use the formula to find the number of diagonals for a 6-sided figure (hexagon).



Solution:

Given, a polygon has n sides, then number of diagonals is $\frac{1}{2}(n^2 - 3n)$.

In hexagon, there are six sides.

Therefore for calculating number of diagonals in hexagon, put $n = 6$ in the above formula.

$\therefore$ Number of diagonals $= \frac{1}{2}[n^2 - 3n] = \frac{1}{2}(6^2 - 3 \times 6)$

$= \frac{1}{2}(6 \times 6 - 3 \times 6) = \frac{1}{2}(36 - 18) = \frac{1}{2}(18) = 9$

Hence, a hexagon has 9 diagonals.

Question 93:

Life Science

Bacteria can divide in every 20 minutes. So, 1 bacterium can multiply to 2 in 20 minutes, 4 in 40 minutes, and so on. How many bacteria will there be in 6 hours? Write your answer using exponents, then evaluate.

Most Bacteria reproduce by a type of simple cell division known as binary fission.

Each species reproduce best at a specific temperature and moisture level.

Solution:

We know that, 1 h = 60 min

6 h = 60 x 6 min = 360 min Given, a bacteria doubles itself in every 20 min.

Number of times it will double itself = 360 min/20 min = 18

∴ Bacteria will there in 6 h = 2 x 2 x 2 x … x 2 (18 times) =2^{18}

Question 94:

Blubber makes up 27 per cent of a blue whale’s body weight. Deepak found the average weight of blue whales and used it to calculate the average weight of their blubber. He wrote the amount as 22 x 32 x 5 x 17 kg. Evaluate this amount.

Solution:

Weight calculated by Deepak = 22 x 32 x 5 x 17 kg

= 2 x 2 x 3 x 3 x 5 x 17 = 4 x 9 x 5 x 17

= 36 x 5 x 17= 180 x 17 = 3060 kg Hence, weight calculated by Deepak was 3060 kg.

Question 95:

Life Science Application

The major components of human blood are red blood cells, white blood cells, platelets and plasma. A typical red blood cell has a diameter of approx 7 x 10^{-6} metre. A typical platelet has a diameter of approximately 2.33 x 10^{-6} metre.

Which has a greater diameter, a red blood cell or a platelet?

Solution:

Given, diameter of red blood cell = 7 x 10^{-6} m

and diameter of platelet = 2.33 x 10^{-6} m

We know that, two numbers written in scientific notation can be compared. The number with the larger power of 10 is greater than the number with the smaller power of 10. If the powers of ten are the same, then the number with the larger factor is the larger number. Therefore, red blood cell has a greater diameter than a platelet.

Question 96:

A googol is the number 1 followed by 100 zeroes.

(a) How is a googol written as a power?

(b) How is a googol times a googol written as a power?

Solution:

(a) $1 \text{ googol} = \underbrace{1000\_\_\_0}_{100 \text{times}} = 1 \times 10^{100}$ [as there are 100 zeroes after 1]

(b) Googol times googol means multiply googol by googol.

$\therefore$ Required number $= \text{googol} \times \text{googol} = 10^{100} \times 10^{100}$ $[\because 1 \text{ googol} = 10^{100}]$

$= 10^{100+100}$ $[\because a^m \times a^n = a^{m+n}]$

$= 100^{200}$

Question 97:

What's the Error?

A student said that $3^5/5^5$ is the same as 1/3. What mistake has the student made?

Solution:

We have, $\frac{3^5}{9^5} = \frac{3^5}{(3^2)^5}$ $[\because 9 = 3 \times 3 = 3^2]$

$= \frac{3^5}{3^{10}} = \frac{1}{3^{10-5}} = \frac{1}{3^5}$ $\left[\because \frac{a^m}{a^n} = a^{m-n}\right]$

So, $\frac{1}{3}$ is not same as $\frac{1}{3^5}$.

Student has multiplied the base by its exponent.

This is the error.