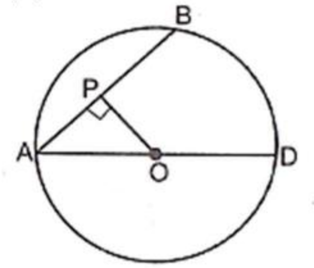
Circles
<u>Exercise 10.1</u>

1. AD is a diameter of a circle and AB is a chord. If AD = 34 cm, AB = 30 cm, the distance of AB from the centre of the circle is:

(A) 17 cm
(B) 15 cm
(C) 4 cm
(D) 8 cm



Sol. Draw $OP \perp AB.$

As perpendicular from the centre to a chord bisect the chord, so

$$AP = \frac{1}{2} \times AB = \frac{1}{2} \times 30 = 15cm$$

Radius $OA = \frac{1}{2} \times 32 = 17cm$

In right $\Delta OPA,$ we have

$$OP = \sqrt{OA^2 - AP^2} = \sqrt{(17)^2 - (15)^2}$$
$$= \sqrt{289 - 225} = \sqrt{64} = 8cm$$

Hence, (d) is the correct answer.

2. In Fig. 10.3, if OA = 5 cm, AB = 8 cm and OD is perpendicular to AB, then CD is equal to:

(A) 2 cm
(B) 3 cm
(C) 4 cm
(D) 5 cm

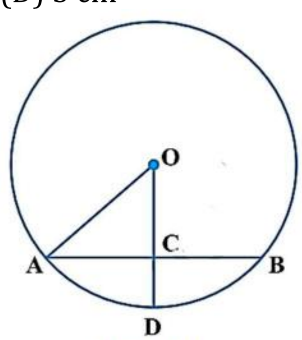


Fig. 10.3

Sol. As perpendicular from the centre to a chord bisect the chord,

$$AC = \frac{1}{2} \times AB = \frac{1}{2} \times 8 = 4cm$$

$$OC = \sqrt{OA^2 - AC^2} = \sqrt{25-16} = \sqrt{9}$$

OC = 3 cm

Now, CD = OD – OC

= 5 cm – 3 cm = 2 cm

Hence, (c) is the correct answer.

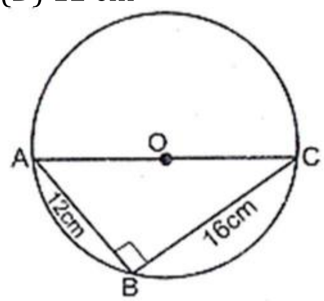
3. If AB = 12 cm, BC = 16 cm and AB is perpendicular to BC, then the radius of the circle passing through the points A, B and C is:

(A) 6 cm

(B) 8 cm

(C) 10 cm

(D) 12 cm



Sol. AB is perpendicular to BC, therefore ABC is a right triangle.

In right ΔABC, we have

$$AC = \sqrt{AB^2 + BC^2}$$

$$= \sqrt{(12)^2 + (16)^2}$$

$$= \sqrt{144 + 256}$$

$$= \sqrt{400}$$

= 20 cm

$\therefore$ Radius $= \frac{1}{2} \times diameter = \frac{1}{2} \times 20cm = 10cm$

Hence, (c) is the correct answer.

4. In Fig.10.4, if ∠ABC = 20º, then ∠AOC is equal to:

(A) 20º

(B) 40º

(C) 60º

(D) 10º

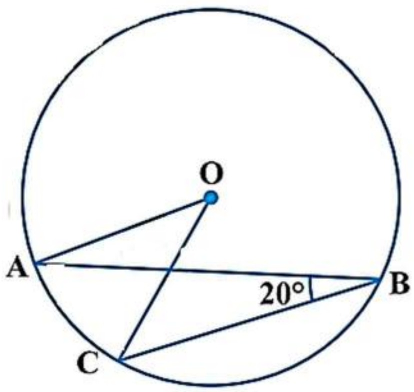


Fig. 10.4

Sol. Arc AC of a circle subtends AOC at the centre O and ABC at a point B on the remaining part of the circle,

$\therefore \quad \angle AOC = 2\angle ABC$

$= 2\times 20^0 \times 40^0$

Hence, (b) is the correct answer.

5. In Fig.10.5, if AOB is a diameter of the circle and AC = BC, then ∠CAB is equal to:

(A) 30º

(B) 60º

(C) 90º

(D) 45º

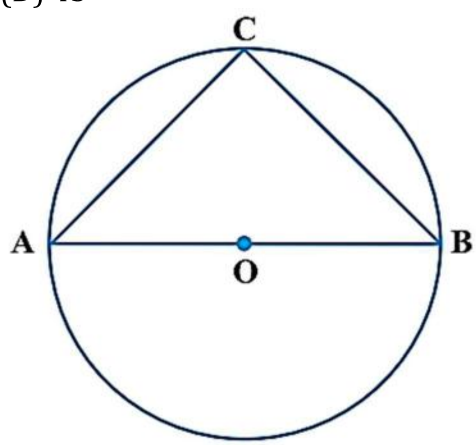


Fig. 10.5

Sol. As AOB is a diameter of the circle,

$\angle C = 90^0$

[∵ Angles in a semi-circle is 90º]

Now, AC = BC

$\angle A = \angle B$

[∵ Angles opposite to equal sides of triangle are equal]

$\angle A + \angle B + \angle C = 180^0 \Rightarrow 2\angle A + 90^0 = 180^0$

$\Rightarrow \quad 2\angle A = 90^0 \Rightarrow \angle A = 90^0 \div 2 = 45^0$

Hence, (d) is the correct answer.

6. In Fig. 10.6, if ∠OAB = 40º, then ∠ACB is equal to:

(A) 50º
(B) 40º
(C) 60º
(D) 70°

Sol. In ΔOAB,

OA = OB [Radii of circle]

$\therefore \quad \angle OAB = \angle OBA = 40^0$

[∵ Angles opposite to equal sides are equal]

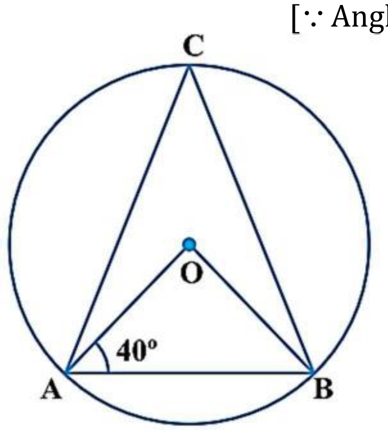


Fig. 10.6

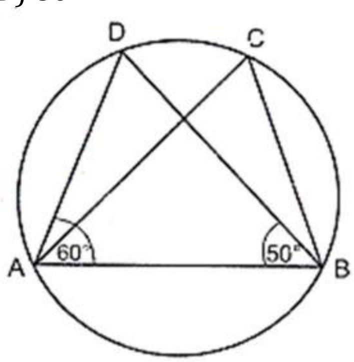
So, $\angle AOB = 180^0 - (40^0 + 40^0) = 100^0$

$\therefore \angle ACB = \frac{1}{2}\angle AOB = \frac{1}{2}\times 100^0 = 50^0$

Hence, (a) is the correct answer.

7. In Fig. 10.7, if ∠DAB = 60º, ∠ABD = 50º, then ∠ACB is equal to:

(A) 60º
(B) 50º
(C) 70º
(D) 80º



Sol. In $\Delta ADB,$ we have

$\angle A+\angle B+\angle D=180^0$

$\Rightarrow$ $60^0+50^0+\angle D=180^0$

$\Rightarrow$ $\angle D=180^0-110^0=70^0$

i.e., $\angle ABD=70^0$

Now, $\angle ACB=\angle ADB=70^0$

[∵ Angles in the same segment of a circle are equal]

Hence, (c) is the correct answer.

8. ABCD is a cyclic quadrilateral such that AB is a diameter of the circle circumscribing it and ∠ADC = 140º, then ∠BAC is equal to:

(A) 80º

(B) 50º

(C) 40º

(D) 30º

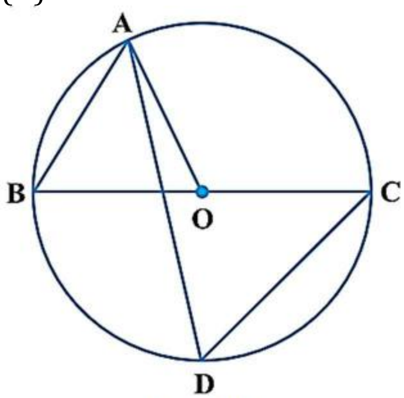


Fig. 10.8

Sol. $\angle ADC+\angle ABC=180^0$

$\Rightarrow$ $140^0+\angle ABC=180^0$

$\therefore$ $\angle ABC=180^0-140^0=40^0$

ABCD is a cyclic quadrilateral such that AB is the diameter of the circle circumscribing it.

Now, Join AC. $\angle C=90^0$

[∵ Angle in a semi-circle is a right angle]

In $\Delta ABC,$ we have

$\angle BAC=180^0(90^0+40^0)$

$=50^0$

Hence, (b) is the correct answer.

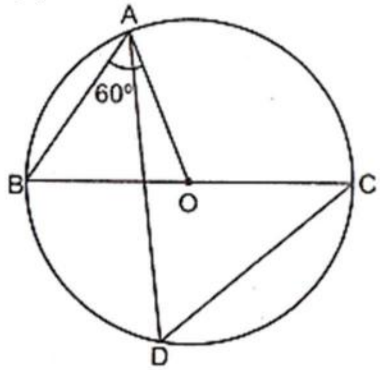
9. In Fig. 10.8, BC is a diameter of the circle and ∠BAO = 60º. Then ∠ADC is equal to:

(A) 30º

(B) 45º

(C) 60º

(D) 120º



Sol. In ΔOAB, we have

OA = OB [Radii of the same circle]

$\therefore$ $\angle ABO = \angle BAO$ [Angles opp. To equal sides are equal]

$\therefore$ $\angle ABO = \angle BAO = 60^0$ [Given]

Now, $\angle ADC = \angle ABC = 60^0$

[$\because$ $\angle ABC$ and $\angle ADC$ are angles in the same segment of a circle, are equal]

Hence, $\angle ADC = 60^0$

So, (c) is the correct answer.

10. In Fig. 10.9, ∠AOB = 90º and ∠ABC = 30º, then ∠CAO is equal to:

(A) 30º
(B) 45º
(C) 90º
(D) 60º

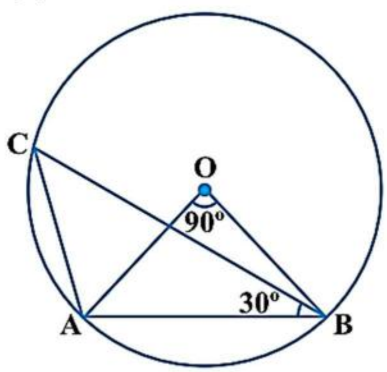


Fig. 10.9

Sol. In ΔOAB, we have

OA = OB

[Radii of the same circle]

$\therefore$ $\angle OAB = \angle OBA$

$\therefore$ $2\angle OAB = (180^0 - \angle AOB)$

$= (180^0 - 90^0)$ [$\because$ Sum of angles of Δ is 180°]

$\Rightarrow \quad \angle OAB = \frac{1}{2} \times 90^0 = 45^0$

Also, $\angle ACB = \frac{1}{2} \angle AOB = \frac{1}{2} \times 90^0 = 45^0$

Now, in ΔCAB, we have

$\angle CAB = 180^0 - (\angle ABC + \angle ACB)$

$= 180^0 - (30^0 + 45^0) = 105^0$

Now, $\angle CAO = \angle CAB - \angle OAB$

$\Rightarrow \quad \angle CAO = 105^0 - 45^0 = 60^0$

Hence, (d) is the correct answer.

Circles

<u>Exercise 10.2</u>

Write True or False and justify your answer in each of the following:

1. Two chords AB and CD of a circle are each at distances 4 cm from the centre. Then AB = CD.

Sol. We know that chords equidistant from the centre of a circle are equal.
Here we are given that two chords AB and CD of a circle are each at distance 4 cm (equidistance) from the centre of a circle. So, chords are equal, i.e., AB = CD.
Hence, the given statement is true.

2. Two chords AB and AC of a circle with centre O are on the opposite sides of OA. Then ∠OAB = ∠OAC.

Sol. The given statement is false, because the angles will be equal if AB = AC.

3. Two congruent circles with centres O and O′ intersect at two points A and B. Then ∠AOB = ∠AO′ B.

Sol. The given statement is true because equal chords of congruent circles subtend equal angles at the respective centre.

4. Through three collinear points a circle can be drawn.

Sol. The given statement is false because a circle through two points cannot pass through a point which is collinear to these two points.

5. A circle of radius 3 cm can be drawn through two points A, B such that AB = 6 cm.

Sol. Radius of circle = 3 cm,
∴ Diameter of circle = 2 × r = 2 × 3 cm = 6 cm
Now, AB = 6 cm, so the given statement is true because AB will be the diameter.

6. If AOB is a diameter of a circle and C is a point on the circle, then $AC^2 + BC^2 = AB^2$.

Sol. AOB is a diameter of a circle and C is a point on the circle.
∴ $\angle ACB = 90^0$ [∵ Angle in a semicircle is a right angle]
In right ΔABC,
$AC^2 + BC^2 = AB^2$ [By Pythagoras theorem]
Hence, the given statement is true.

7. ABCD is a cyclic quadrilateral such that ∠A = 90°, ∠B = 70°, ∠C = 95° and ∠D = 105°.

Sol. We know that opposite angles of a cyclic quadrilateral are supplementary.
Here, sum of opposite angles is not. 180^o
$\angle A + \angle C = 90^0 + 95^0 = 185^0$
Hence, ABCD is not a cyclic quadrilateral. The given statement is false.

8. If A, B, C, D are four points such that ∠BAC = 30° and ∠BDC = 60°, then D is the centre of the circle through A, B and C.

Sol. The given statement is false because there can be many points D such that ∠BDC = 60° and each such point cannot be centre of the circle through A, B, C.

9. If A, B, C and D are four points such that ∠BAC = 45° and ∠BDC = 45°, then A, B, C, D are concyclic.

Sol. The given statement is true, because the two angles ∠BAC = 45° and ∠BDC = 45° are in the same segment of a circle.

10. In Fig. 10.10, if AOB is a diameter and ∠ADC = 120°, then ∠CAB = 30°.

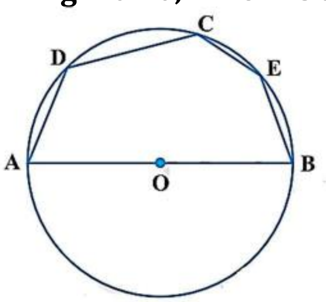


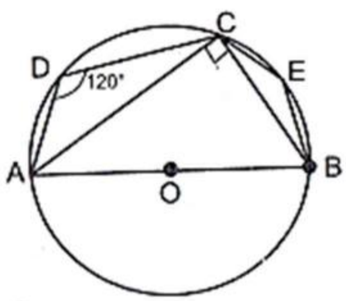
Fig. 10.10

Sol. AOB is a diameter of circle with centre O.

$\angle ADC + \angle ABC = 180^0$

[∵ ABCD is a cyclic quadrilateral]

$\Rightarrow \quad 120^0 + \angle ABC = 180^0$



$\Rightarrow \quad \angle ABC = 180^0 - 120^0 = 60^0$

In ΔABC, we have

$\angle ACB = 90^0$

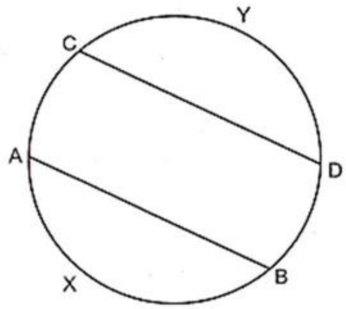
[∵ Angle in a semicircle and $\angle ABC = 60^0$ (Proved above)]

$\therefore \angle CAB = 180^0 - (90^0 + 60^0) = 30^0$

Hence, the given statement is true.

Circles
Exercise 10.3

1. If arcs AXB and CYD of a circle are congruent, find the ratio of AB and chord CD.

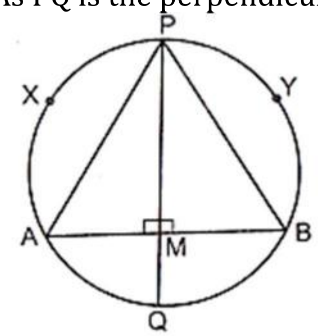


Sol. We have $\overset{\frown}{AXB} \cong \overset{\frown}{CYD}$

Since if two arcs of a circle are congruent, then their corresponding arcs are equal, so we have chord AB = chord CD Hence, AB : CD = 1 : 1.

2. If the perpendicular bisector of a chord AB of a circle PXAQBY intersects the circle at P and Q, prove that arc PXA ≅ Arc PYB.

Sol. As PQ is the perpendicular bisect of AB,



So, AM = BM

In ΔAPM and ΔBPM, we have

AM = BM [Proved above]

$\angle AMP = \angle BMP$ [Each = 90^o]

PM = PM [Common side]

$\therefore$ $\Delta APM \cong \Delta BPM$ [By SAS congruence rule]

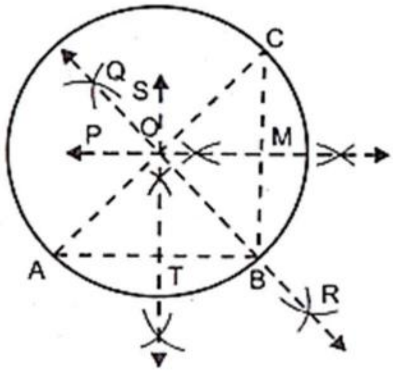
So, AP = BP [CPCT]

Hence, arc PXA ≅ Arc PYB

[If two chords of a circle are equal, then their corresponding arcs are congruent]

3. A, B and C are three points on a circle. Prove that the perpendicular bisectors of AB, BC and CA are concurrent.

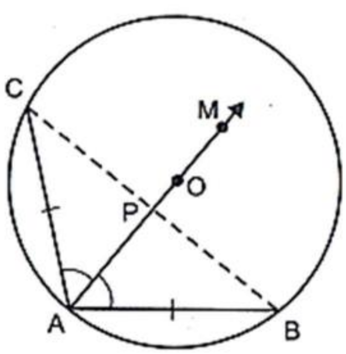
Sol. Given: Three non-collinear points A, B and C are on a circle.



To prove: Perpendicular bisectors of AB, BC and CA are concurrent.
Construction: Join AB, BC and CA.
Draw perpendicular bisectors ST of AB, PM of BC and QR of CA are respectively. As point A, B and C are not collinear, so ST, PM and QR are not parallel and will intersect.
Proof: $\because$ lies on ST, the $\perp$ bisector of AB
$\therefore$ OA = OB ...(1)
Similarly, O lies on PM, the $\perp$ bisector of BC
$\therefore$ OB = OC ...(2)
And, O lies on QR, the $\perp$ bisector of CA
$\therefore$ OC = OA ...(3)
From (1), (2) and (3), OA = OB = OC = r(say)
With O as a centre and r as the radius, draw circle C(O, r) which will pass through A, B and C.
This prove that there is a circle passing through the points A, B and C. Since ST, PM or QR can cut each other at one and only one point O.
$\therefore$ O is the only point equidistance from A, B and C.
Hence, the perpendicular bisectors of AB, BC and CA re concurrent.

4. AB and AC are two equal chords of a circle. Prove that the bisector of the angle BAC passes through the centre of the circle.

Sol. Given: AB and AC are two chords which are equal with centre O. AM is the bisector of $\angle BAC$.



To prove: AM passes through O.
Construction: Join BC. Let AM intersect BC at P. Proof: In ΔBAP and ΔCAP
AB = AC [Given]

$\angle BAP = \angle CAP$ [Given]

And AP = BP [Common side]

$\therefore$ $\Delta BAP \cong \Delta CAP$ [By SAS]

$\therefore$ $\angle BPA = \angle CPA$ [CPCT]

And CP = PB

But $\angle BPA + \angle CPA = 180^0$ [Linear pair $\angle s$]

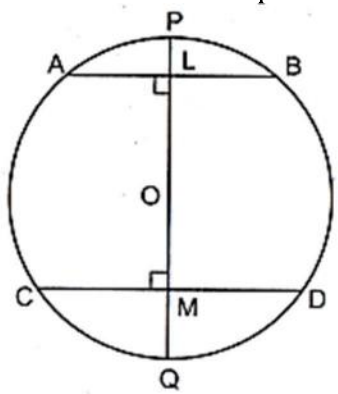
$\therefore$ $\angle BPA = \angle CPA = 90^0$

$\therefore$ AP is perpendicular bisector of the chord BC, which will pass through the centre O on being produced.

Hence, AM passes through O.

5. If a line segment joining mid-points of two chords of a circle passes through the centre of the circle, prove that the two chords are parallel.

Sol. Given: AB and CD are two chords of a circle whose centre of O. The mid-points of AB and CD are L and M respectively.



To prove: AB||CD

Proof: $\because$ L is the mid-point of chord AB $\therefore OL \perp AB$, or $\angle ALO = 90^0$

[$\because$ The line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord]

Similarly, $\angle CMO = 90^0$

$\therefore$ $\angle ALO = \angle CMO$

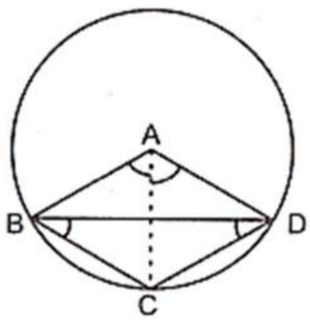
But, these are corresponding angles.

So, AB||CD.

Hence, proved

6. ABCD is such a quadrilateral that A is the centre of the circle passing through B, C and D. Prove that $\angle CBD + \angle CDB = \frac{1}{2}\angle BAD$

Sol. ABCD is such a quadrilateral that A is the centre of the circle passing through B, C and D. We have to prove that



$$\angle CBD + \angle CDB = \frac{1}{2}\angle BAD$$

Join AC.

Since angle subtended by an arc at the centre is double the angle subtended by it at point on the remaining part of the circle.

Therefore, $\angle CAD = 2\angle CBD$ ...(1)

And $\angle BAC = 2\angle CDB$ ...(2)

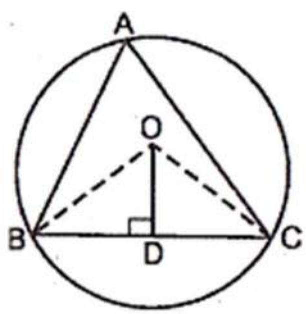
Adding (1) and (2), we get

$$\angle CAD + \angle BAC = 2(\angle CBD + \angle CDB)$$

$\Rightarrow$ $\angle BAD = 2(\angle CBD + \angle CDB)$

Hence, $\angle CBD + \angle CBD = \frac{1}{2}\angle BAD$

7. O is the circumcentre of the triangle ABC and D is the mid-point of the base BC. Prove that ∠BOD = ∠A.



Sol. Given: O is the circumcentre of ΔABC and $OD \perp BC$.

To prove: $\angle BOD = \angle A$.

Construction: Join OB and OC.

Proof: In ΔOBD and ΔOCD, we have

OB = OC [Each equal to radius of the circumcircle]

$\angle ODB = \angle ODC$ [Each of 90°]

OD = OD [Common]

$\therefore$ $\angle OBD \cong \angle OCD$

$\Rightarrow$ $\angle BOC = 2\angle BOD = 2\angle COD$

Now, are BC subtends $\angle BOC$ at the centre and $\angle BAC = \angle A$ at a point in the remaining part of the circle.

$\therefore \quad \angle BOC = 2\angle A$

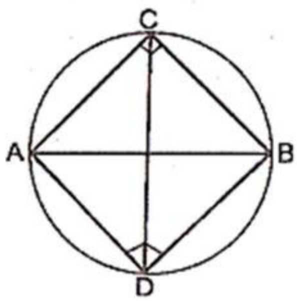
$\Rightarrow \quad 2\angle BOD = 2\angle A \qquad [\because \angle BOC = 2\angle BOD]$

$\Rightarrow \quad \angle BOD = \angle A$

Hence, proved.

8. On a common hypotenuse AB, two right triangles ACB and ADB are situated on opposite sides. Prove that ∠BAC = ∠BDC.

Sol. In right triangle ACB and ADB, we have



$\angle ACB = 90^0$ and $\angle ADB = 90^0$

$\therefore \quad \angle ACB + \angle ADB = 90^0 + 90^0 = 180^0$

If the sum of any pair of opposite angles of a quadrilateral is 180º, then the quadrilateral is cyclic. So, ADBC is a cyclic quadrilateral.

Join CD. Angles $\angle BAC$ and $\angle BDC$ are made by $\overset{\frown}{BC}$ in the same segment BDAC.

Hence, $\angle BAC = \angle BDC$.

[∵ Angles in the same segment of a circle are equal]

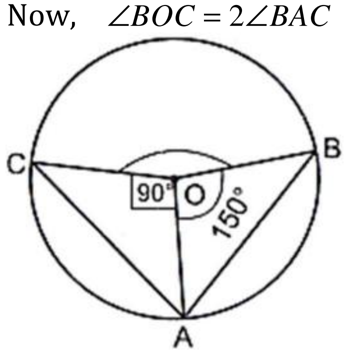
9. Two chords AB and AC of a circle subtends angles equal to 90º and 150º, respectively at the centre. Find ∠BAC, if AB and AC lie on the opposite sides of the centre.

Sol. We have

Reflex $\angle BOC = 90^0 + 150^0 = 240^0$

$\therefore \quad \angle BOC = 360^0 - 240^0 = 120^0$

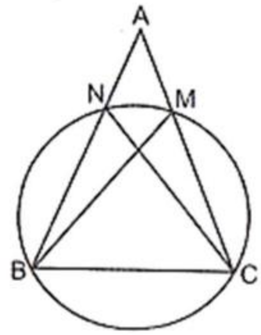
Now, $\angle BOC = 2\angle BAC$



Hence, $\angle BAC = \frac{1}{2}\angle BOC = \frac{1}{2}\times 120^0 - 60^0$

10. If BM and CN are the perpendiculars drawn on the sides AC and AB of the triangle ABC, prove that the points B, C, M and N are concyclic.

Sol. As BM and CN are the perpendiculars drawn on the sides AC and AB of the triangle ABC



$\therefore \quad \angle BMC = \angle BNC = 90^0$

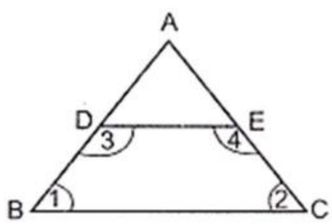
Since, if a line segment (here BC) joining two points (here B and C) subtends equal angles (here $\angle BMC$ and $\angle BNC$) at M and N on the same side of the line (here BC) containing the segment, the four points (here B, C, M and N) are concyclic.

Hence, B, C, M and N are concyclic.

11. If a line is drawn parallel to the base of an isosceles triangle to intersect its equal sides, prove that the quadrilateral so formed is cyclic.

Sol. ΔABC is an isosceles triangle in which AB = AC. DE is drawn parallel to BC. We have to prove that quadrilateral BCED is a cyclic quadrilateral i.e., point B, C, E and D lie on a circle.

In ΔABC, we have



AB = AC [Given]

$\therefore \quad \angle 1 = \angle 2$ [$\because$ Angles opp. To equal sides are equal]

Now, DE||BC and AB cuts them,

$\therefore \quad \angle 1 + \angle 3 = 180^0$

[$\because$ Sum of int. $\angle s$ on the same side of the transversal]

$\Rightarrow \quad \angle 2 + \angle 3 = 180^0$ [$\because \angle 1 = \angle 2$]

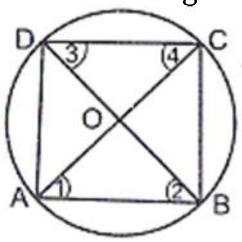
Similarly, we can show that $\angle 1 + \angle 4 = 180^0$

Since if pair of opposite angles of a quadrilateral is supplementary, then the quadrilateral is cyclic.

Hence, BCED is a quadrilateral.

12. If a pair of opposite sides of a cyclic quadrilateral are equal, prove that its diagonals are also equal.

Sol. ABCD is cyclic quadrilateral in which one pair of opposite sides AB = DC. We have to prove that diagonal AC = diagonal BD.



In ΔAOB and ΔDOC we have

$\angle 1 = \angle 3$

[Angles in the same segment of the circle are equal]

AB = DC [Given]

Also, $\angle 2 = \angle 4$ [Same reason as in step – 1]

$\therefore$ $\Delta AOB \cong \Delta DOC$ [By ASA congruence rule]

$\therefore$ AO = OD [CPCT] ...(1)

And OC = BO ...(2)

Now, adding (1) and (2),we get

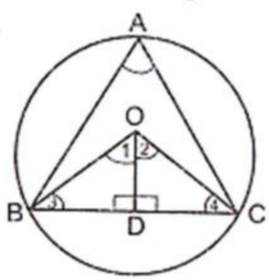
AO + OC = BO + OD

$\Rightarrow$ AC = BD

Hence, proved.

13. The circumcentre of the triangle ABC is O. Prove that ∠OBC + ∠BAC = 90º.

Sol. ABC is a triangle and O is the circumcentre.



Draw $OD \perp BC$. Join OB and OC.

In right ΔOBD and right ΔOCD, we have

hyp. OB = hyp. OC

[Radii of the same circle]

OD = OD

[Common side]

$\therefore$ $\Delta OBD \cong \Delta OCD$ [By RHs cong. Rule]

$\therefore$ $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ [CPCT]

Now, $\angle BOC = 2\angle 1$ and $\angle BOC = 2\angle A$

$\therefore$ $2\angle 1 = 2\angle A \Rightarrow \angle 1 = \angle A$

$\therefore$ $\angle A = \angle 2$ ...(1) $[\because \angle 1 = \angle 2]$

$\Rightarrow$ $\angle A + \angle 4 = \angle 2 + \angle 4$ [Adding $\angle 4$ to both sides]

$\Rightarrow$ $\angle A + \angle 3 = 90^0$ $[\because \angle 2 + \angle 4 = 90^0$ and $\angle 4 = \angle 3]$

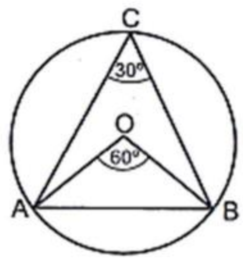
$\Rightarrow \quad \angle BOC = \angle A = 90^0$

Hence, proved.

14. A chord of a circle is equal to its radius. Find the angle subtended by this chord at a point in major segment.

Sol. Since chord of a circle is equal radius, so we have AB = OA = OB.

Therefore, ABC is an equilateral triangle.



Since each angle of an equilateral triangle is 60°, so we have $\angle AOB = 60^0$ Since angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle, so we have

$$\angle AOB = 2\angle ACB$$

Hence, $\angle ACB = \frac{1}{2}\angle AOB = \frac{1}{2} \times 60^0 = 30^0$

15. In Fig.10.13, ∠ADC = 130° and chord BC = chord BE. Find ∠CBE.

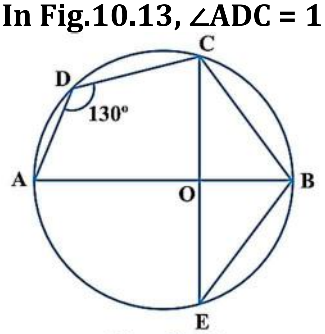


Fig. 10.13

Sol. In the given figure, we have $\angle ADC = 130^0$ and chord BC = chord BE. We have to find $\angle CBE.$ Since ABCD is a cyclic quadrilateral and the opposite angles of a cyclic quadrilateral are supplementary.

$\therefore \quad \angle D + \angle ABC = 180^0$

$\Rightarrow \quad 130^0 + \angle ABC = 180^0$

$\Rightarrow \quad \angle ABC = 180^0 - 130^0 = 50^0$

$\Rightarrow \quad \angle OBC = 50^0$ ...(1)

In ΔOBC and ΔOBE, we have

BC = BE [Given]

OC = OE [Radii of same circle]

OB = OB [Common side]

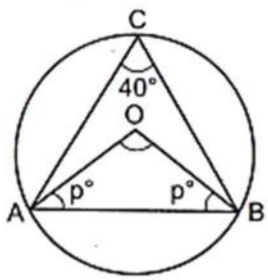
$\therefore \quad \Delta OBC \cong \Delta OBE$ [By SSS cong. Rule]

$\therefore \quad \angle OBC = \angle OBE = 50^0$ [CPCT and by (1) $\angle OBC = 50^0$]

$\therefore \angle OBC + \angle OBE = 50^0 + 50^0 = 100^0$

Hence, $\angle CBE = 100^0$

16. In Fig.10.14, ∠ACB = 40º. Find ∠OAB.



Sol. Since the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle, so we have

$$\angle AOB = 2\angle ACB = 2\times 40^0 = 80^0$$

So, in ΔOAB we have

$$p^0 + p^0 + \angle AOB = 180^0$$

$\Rightarrow \quad 2p^0 + 80^0 = 180^0 \Rightarrow 2p^0 = 180^0 - 80^0$

$\Rightarrow \quad 2p^0 = 100^0 \Rightarrow p^0 = 100^0 \div 2 = 50^0$

Hence, $\angle OAB = 50^0$

17. A quadrilateral ABCD is inscribed in a circle such that AB is a diameter and ∠ADC = 130º. Find ∠BAC.

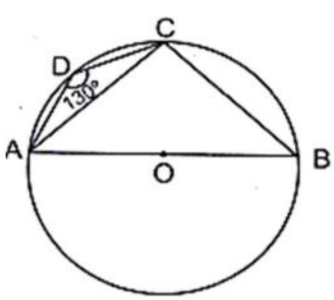
Sol. Since the opposite angles of a cyclic quadrilateral are supplementary.

$\therefore \quad \angle B + \angle D = 180^0$

$\Rightarrow \quad \angle B + \angle 130^0 = 180^0$

$\Rightarrow \quad \angle B = 180^0 - \angle 130^0 = 50^0$

Now, in ΔABC, $\angle ACB = 90^0$ [∵ Angle in a semi-circle = 90º]

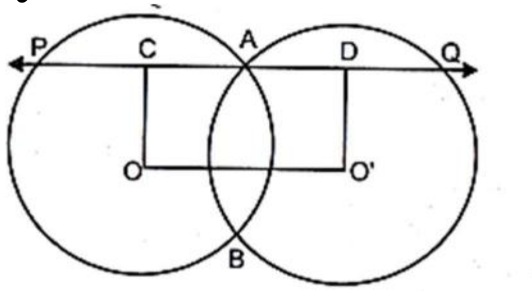


And $\angle ABC = 50^0$

$\therefore \quad \angle BAC = 180^0 - (90^0 + 50^0)$

$= 180^0 - 140^0 = 40^0$

18. Two circles with centres O and O′ intersect at two points A and B. A line PQ is drawn parallel to OO′ through A(or B) intersecting the circles at P and Q. Prove that PQ = 2 OO′.



Sol. Two circles with centre O and O' intersect at two points A and B. A line PQ is drawn parallel to OO' through A (or B) intersecting the circles at P and Q.

Draw $OC \perp PA$ and $O'D \perp AQ$.

We have to prove that PQ = 2 OO'.

Since perpendicular from the centre to a chord bisect the chord, so

PA = 2CA ...(1)

And AQ = 2 AD ...(2)

Adding (1) and (2), we get

PA + AQ = 2CA + 2AD

$\Rightarrow$ PQ = 2(CA + AD) = 2CD

Hence, PQ = 2 OO' [$\because$ CD and OO' are opposite sides of a rectangle]

19. In Fig.10.15, AOB is a diameter of the circle and C, D, E are any three points on the semi-circle. Find the value of ∠ACD + ∠BED.

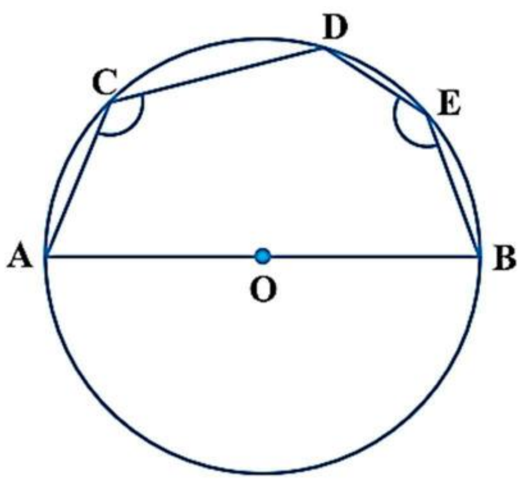


Fig. 10.15

Sol. Join BC.

Since angle in a semicircle is 90^o, we have

$\angle ACB = 90^o$

As ABCD is a cyclic quadrilateral and opposite angles of a cyclic quadrilateral are supplementary

$\therefore \quad \angle BCD + \angle BED = 180^0$

Now, adding $\angle ACB$ to both sides, we get

$(\angle BCD + \angle ACB) + \angle BED = 180^0 + \angle ACB$

Hence, $\angle ACD + \angle BED = 180^0 + 90^0 = 270^0$

20. In Fig. 10.16, ∠OAB = 30º and ∠OCB = 57º. Find ∠BOC and ∠AOC.

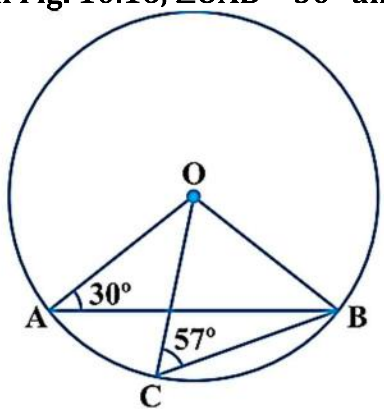


Fig. 10.16

Sol. In ΔOBC, we have

OB = OC [Radii of the same circle]

$\therefore \quad \angle OCB = \angle OBC = 57^0 \quad [\because \angle OCB = 57^0 \text{ (Given)}]$

Now, in ΔBOC, we have

$\angle OCB + \angle OBC + \angle BOC = 180^0$

$\Rightarrow \quad 57^0 + 57^0 + \angle BOC = 180^0$

$\Rightarrow \quad 114^0 + \angle BOC = 180^0$

$\Rightarrow \quad \angle BOC = 180^0 - 114^0 = 66^0$

Again, in ΔAOB, we have

$\angle OAB + \angle OBA + \angle AOB = 180^0$

$\Rightarrow 30^0 + 30^0 + (\angle AOC + \angle BOC) = 180^0$

$\Rightarrow \quad 60^0 + \angle AOC + 66^0 = 180^0$

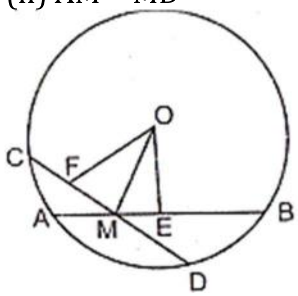
$\Rightarrow \quad \angle AOC = 180^0 - 126^0 = 54^0$

Hence, $\angle BOC = 66^0$ and $\angle AOC = 54^0$

Circles
Exercise 10.4

1. If two equal chords of a circle intersect, prove that the parts of one chord are separately equal to the parts of the other chord.

Sol. AB and CD are two equal chords of a circle with centre O, intersect each other at M.
We have to prove that,
(i) MB = MC and
(ii) AM = MD



AB is a chord and $OE \perp$ to it from the centre O,

$\therefore \quad AE = \frac{1}{2} AB$ [∵ Perpendicular from the centre to a chord bisect the chord]

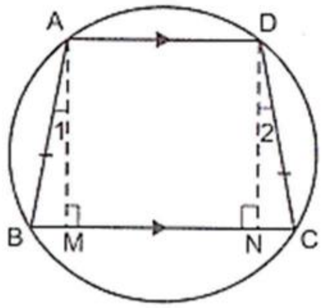
Similarly, $FD = \frac{1}{2} CD$

As $AB = CD \Rightarrow \frac{1}{2} AB = \frac{1}{2} CD$ [Given]

So, AE = FD ...(1)
Since equal chords are equidistance from the centre,
So, OE = OF [∵ AB = CD]
Now, in right ΔMOE and MOF, [Proved above]
hyp. OE = hyp. OF [Common side]
OM = OM
$\therefore \quad \Delta MOE \cong \Delta MOF$
$\therefore$ ME = MF ...(2)
Subtracting (2) from (1), we get
AE – ME = FD – MF
$\Rightarrow$ AM = MD [Proved part (ii)]
Again, AB = CD [Given]
And AM = MD [Proved]
$\therefore$ AB – AM = CD – MD [Equals subtracted from equal]
Hence, MB = MC [Proved part (i)]

2. If non-parallel sides of a trapezium are equal, prove that it is cyclic.

Sol. Given: ABCD is a trapezium in which AD||BC and its non-parallel sides.
AB and DC are equal i.e.,



AB = DC.

To prove: Trapezium ABCD is cyclic.

Construction: Draw AM and DN $\perp s$ on BC.

Proof: In right Δs AMB and DNC,

$\angle AMB = \angle DNC$ [Each 90°]

AB = DC [Given]

AM = DN

[$\perp$ Distance between two ||lines are same]

$\therefore$ $\Delta AMB \cong \Delta DNC$ [By RHS congruence rule]

$\therefore$ $\angle B = \angle C$ [CPCT]

And $\angle 1 = \angle 2$

$\therefore$ $\angle BAD = \angle 1 + 90^0$

$= \angle 2 + 90^0$ [$\because \angle 1 = \angle 2$ (proved above)]

$= \angle CDA$

Now, in quadrilateral ABCD

$\angle B + \angle C + \angle CDA + \angle BAD = 360^0$

$\Rightarrow \angle B + \angle B + \angle CDA + \angle CDA = 360^0$ [$\because \angle B = \angle C$ and $\angle CDA = \angle BAD$ (Proved above)]

$\Rightarrow 2(\angle B + \angle CDA) = 360^0$

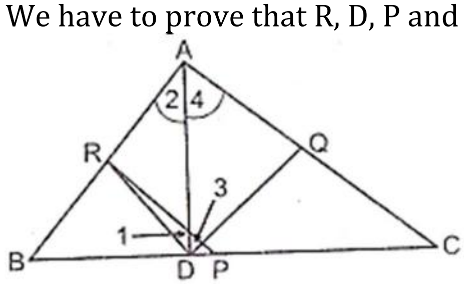
$\Rightarrow$ $\angle B + \angle CDA = 180^0$

We know that if the same of any pair of opposite angles of a quadrilateral is 180°, then the quadrilateral is cyclic.

Hence, the trapezium ABCD is cyclic.

3. If P, Q and R are the mid-points of the sides BC, CA and AB of a triangle and AD is the perpendicular from A on BC, prove that P, Q, R and D are concyclic.

Sol. We have to prove that R, D, P and Q are concyclic.

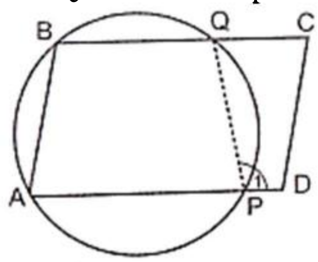


Join RD, QD, PR and PQ.

$\because$ RP joints R and P, the mid-point of AB and BC.
$\therefore$ RP||AC [Mid-point theorem]
Similarly, PQ||AB.
$\therefore$ ARPQ is a ||gm
So, $\angle RAQ = \angle RPQ$ [Opposite angles of a ||gm]...(1)
$\because$ ABD is a rt. $\angle D\Delta$ and DR is a median,
$\therefore$ RA = DR and $\angle 1 = \angle 2$ ...(2)
Similarly $\angle 3 = \angle 4$ ...(3)
Adding (2) and (3), we get
$\angle 1 + \angle 3 = \angle 2 + \angle 4$
$\Rightarrow$ $\angle RDQ = \angle RAQ$
$\angle RPQ$ [Proved above]
Hence, R, D, P and Q are concyclic.
[$\because \angle D$ and $\angle P$ are subtended by RQ on the same side of it.]

4. ABCD is a parallelogram. A circle through A, B is so drawn that it intersects AD at P and BC at Q. Prove that P, Q, C and D are concyclic.

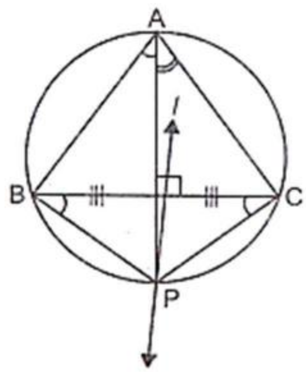
Sol. ABCD is a parallelogram. A circle through A, B is so drawn that it intersects AD at P and BC at Q. We have to prove that P, Q, C and D are concyclic. Join PQ.



Now, side AP of the cyclic quadrilateral APQB is produced to D.
$\therefore$ $Ext.\angle 1 = \text{int}.opp.\angle B$
$\because$ BA||CD and BC cuts them
$\therefore$ $\angle B + \angle C = 180^0$
[$\because$ Sum of int. $\angle s$ on the same side of the transversal is 180°]
Or $\angle 1 + \angle C = 180^0$ [$\therefore \angle 1 = \angle B$ (proved)]
$\therefore$ PDCQ is cyclic quadrilateral.
Hence, the points P, Q, C and D are concyclic.

5. Prove that angle bisector of any angle of a triangle and perpendicular bisector of the opposite side if intersect, they will intersect on the circumcircle of the triangle.

Sol. Given: A ΔABC and l is perpendicular bisector of BC.



To prove: Angles bisector of $\angle A$ and perpendicular bisector of BC intersect on the circumcircle of ΔABC.

Proof: Let the angle bisector of $\angle A$ intersect circumcircle of ΔABC at D. Join BP and CP.

$\Rightarrow \quad \angle BAP = \angle BCP$

[Angles in the same segment are equal]

$\Rightarrow \quad \angle BAP = \angle BCP = \frac{1}{2}\angle A$ ...(1) [AP is bisector of $\angle A$]

Similarly, we have

$\angle PAC = \angle PBC = \frac{1}{2}\angle A$ ...(2)

From equation (1) and (2), we have

$\angle BCP = \angle PBC$

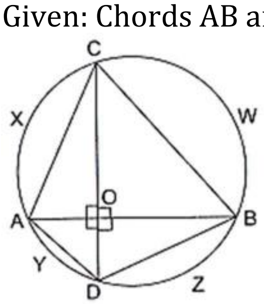
$\Rightarrow$ BP = CP [$\because$ If the angles subtended by two Chords of a circle at the centre are equal, the chords are equal, the chords are equal]

$\Rightarrow$ P is on perpendicular bisector of BC.

Hence, angle bisector of $\angle A$ and perpendicular bisector of BC intersect on the circumcircle of ΔABC.

6. If two chords AB and CD of a circle AYDZBWCX intersect at right angles (see Fig.10.18), prove that arc CXA + arc DZB = arc AYD + arc BWC = semicircle.

Sol. Given: Chords AB and CD of circle AYDZBWCX intersect at right angles.



To prove: arc CXA + are DZM = are AYD + arc BWC = semicircle.

Construction: Join AC, AD, BD and BC.

Proof: O is any point inside the circle. Now, consider the chord CA.

The angle subtended by the chord AC at the circumference is $\angle CBA$.

Similarly, the angle subtended by the chord BD at the circumference is $\angle BCD$.

Now, consider the right triangle BOC.

Thus, by angle sum property, we have:

$$\angle COB + \angle CBA + \angle BCD = 180^0$$

$\Rightarrow$ $90^0 + \angle CBA + \angle BCD = 180^0$

$\Rightarrow$ $\angle CBA + \angle BCD = 180^0 - 90^0$

$\Rightarrow$ $\angle CBA + \angle BCD = 90^0$

That is the sum of angle subtended by the arc CXA and the angles subtended by the arc BZD = 90o.

$$arc\widehat{CXA} + arc\widehat{BZD} = 90^0 \qquad ...(1)$$

Now, consider the chord BC.

The angles subtended by the chord BC at the centre is $\angle BAC$.

Similarly, the angle subtended by the Chord AD at the centre is $\angle ACD$.

Now, consider the right triangle AOC.

Thus, by angle sum property, we have:

$$\angle COA + \angle BAC + \angle ACD = 180^0$$

$\Rightarrow$ $90^0 + \angle BAC + \angle ACD = 180^0$

$\Rightarrow$ $\angle BAC + \angle ACD = 180^0 - 90^0$

$\Rightarrow$ $\angle BAC + \angle ACD = 90^0$

That is the sum of angle subtended by the are CWB and the angle subtended by the are AYD = 90o.

$$arc\widehat{CWB} + arc\widehat{AYD} = 90^0 \qquad ..(2)$$

From equations (1) and (2), we have

$$arc\widehat{CXA} + arc\widehat{BYD} = arc\widehat{CWB} + arc\widehat{AYD} = 90^0$$

We know that the are of a circle subtending a right angle at any point of the circle in its alternate segment is a semi-circle.

Thus, we have

$$arc\widehat{CXA} + arc\widehat{BZD} = arc\widehat{CXA} + arc\widehat{BZD} = \text{semicircle}$$

Hence proved.

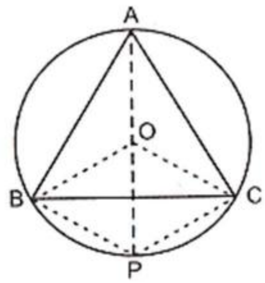
7. If ABC is an equilateral triangle inscribed in a circle and P be any point on a minor arc BC which does not coincide with B or C, prove that PA is angle bisector of ∠BPC.

Sol. Since equal chords of a circle subtends equal angles at the centre, so we have

Chord AB = chord AC [Given]

So $\angle AOB = \angle AOC$ ...(1)

Since the angles subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle,



$\therefore \quad \angle APC = \frac{1}{2}\angle AOC$ ...(2)

And $\quad \angle APB = \frac{1}{2}\angle AOB$ ...(3)

$\therefore \quad \angle APC = \angle APB$ [From (1), (2) and (3)]

Hence, PA is the bisector of $\angle BPC$.

8. In the given fig., AB and CD are two chords of a circle intersecting each other at point E. Prove that $\angle AEC = \frac{1}{2}$ (Angles subtended by an arc CXA at the centre + angle subtended by arc DYB at the centre.)

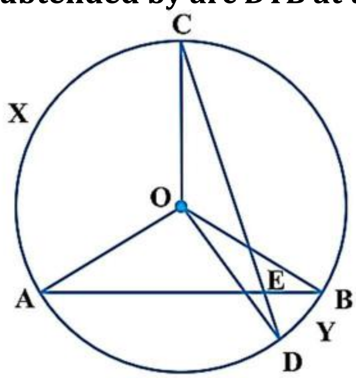


Fig. 10.19

Sol. AB and CD are two chords of a circle intersecting each other at point E.

We have to prove that $\angle AEC = \frac{1}{2}$ (Angles subtended by an arc CXA at the centre + angle subtended by arc DYB at the centre.)

Join AC, BC and BD.

Since, the angles subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle, now arc CXA subtends $\angle AOC$ at the centre and $\angle ABC$ at the remaining part of the circle, so

$\angle AOC = 2\angle ABC$ ...(1)

Similarly, $\quad \angle BOD = 2\angle BCD$ ...(2)

Now, adding (1) and (2), we get

$\angle AOC + \angle BOD = 2(\angle ABC + \angle BCD)$ ...(3)

Since exterior angle of a triangle is equal to the sum of interior opposite angles, so in ΔCEB, we have

$\therefore \quad \angle AEC = \angle ABC + \angle BCD \quad ...(4)$

From (3) and (4), we get

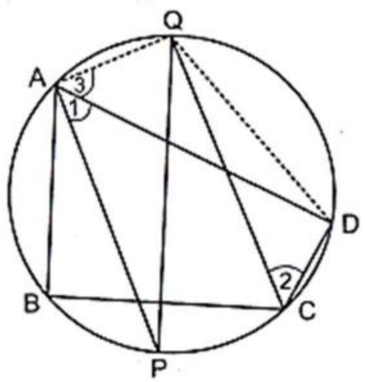
$$\angle AOC + \angle BOD = 2\angle AEC$$

Or $\quad \angle AEC = \frac{1}{2}(\angle AOC + \angle BOD)$

Hence, $\angle AEC = \frac{1}{2}$ (angles subtended by an arc CXA at the centre + angle subtended by an arc DYB at the centre).

9. If bisectors of opposite angles of a cyclic quadrilateral ABCD intersect the circle, circumscribing it at the points P and Q, prove that PQ is a diameter of the circle.

Sol. The bisectors of opposite angles $\angle A$ and $\angle C$ of a cyclic quadrilateral ABCD intersect the circle at the point P and Q, respectively.



We have to prove that PQ is a diameter of the circle.

Join AQ and DQ.

Since opposite angles of a cyclic quadrilateral are supplementary, so in cyclic quadrilateral ABCD, we have

$$\angle DAB + \angle DCB = 180^0$$

So, $\quad \frac{1}{2}\angle DAB + \frac{1}{2}\angle DCB = \frac{1}{2}\left(180^0\right)$

$\Rightarrow \quad \angle 1 + \angle 2 = 90^0$

[$\because$ AP and CQ are the bisectors of $\angle A$ and $\angle C$ respectively]

$\therefore \quad \angle 1 + \angle 3 = 90^0 \quad [\because \angle 2 = \angle 3]$

[$\because \angle 2$ and $\angle 3$ are the angles in the same segment of a circle with chord QD]

$\Rightarrow \quad \angle PAQ = 90^0$

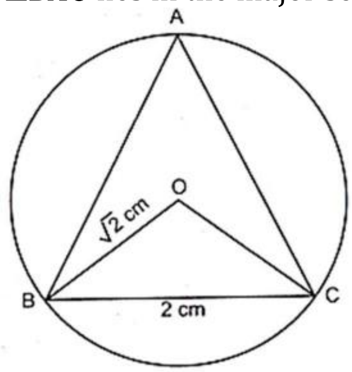
$\therefore \quad \angle PAQ$ is in a semi-circle

Hence, PQ is a diameter of circle.

10. A circle has radius $\sqrt{2}cm$. It is divided into two segments by a chord of length 2 cm. Prove that the angle subtended by the chord at a point in major segment is 45º.

Sol. A circle with centre O and radius $\sqrt{2}cm$. Chord BC, 2 c long divides the circle into two segment.

$\angle BAC$ lies in the major segment.



We have to prove that $\angle BAC = 45^0$

Join OB and OC.

$$BC^2 = (2)^2 = 4 = 2 + 2 = \left(\sqrt{2}\right)^2 + \left(\sqrt{2}\right)^2$$

$\Rightarrow$ $BC^2 = OB^2 + OC^2$

In ΔBOC, we have

$$BC^2 = OB^2 + OC^2$$

$\therefore$ $\angle BOC = 90^0$ [By converse of Pythagoras theorem]

Now, $\overset{\frown}{BC}$ subtends $\angle BOC$ at the centre O and $\angle BAC$ at the remaining part of the circle.

$\therefore$ $\angle BAC = \frac{1}{2}\angle BOC = \frac{1}{2}\times 90^0 = 45^0$

Hence, proved.

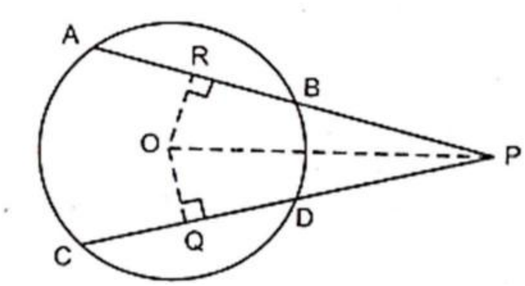
11. Two equal chords AB and CD of a circle when produced intersect at a point P. Prove that PB = PD.

Sol. Given: AB and CD two equal chords of a circle with centre O when produced intersect at P.

To prove: PB = PD.

Construction: Draw $OR \perp AB$ and $OQ \perp CD$. Joint OP.

Proof: $\because OR \perp AB$ and $OQ \perp CD$ from the centre O of circle



$\therefore$ R is mid-point of AB and Q is the mid-point of CD.

[$\because$ $\perp$ from the centre to a chord bisects the chord]

$\because$ AB = CD [Given]

$\therefore \quad \frac{1}{2}AB = \frac{1}{2}CD$

$\therefore$ AR = CQ and RB = QD ...(1)

$\because$ AB = CD, $\therefore$ OR = OQ ...(2)

[$\because$ Equal chords are equidistance from the centre]

Now, in right-angled Δs ORP and OQP, we have

$\angle ORP = \angle OQP$ [Each 90°]

hyp. OP = hyp. OP [Common side]

OR = OQ [From (2)]

$\therefore \quad ORP \cong OQP$ [By R.H.S axiom]

$\therefore$ RP = QP [CPCT]...(3)

Now, subtracting (1) from (3)

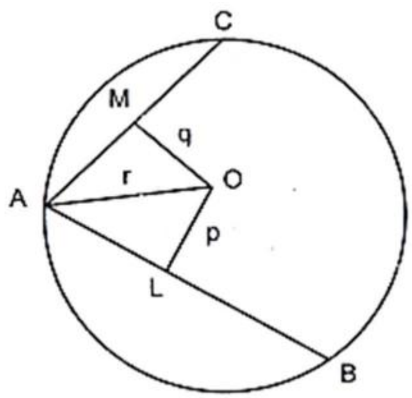
RP – RB = QP – QD

$\Rightarrow$ PB = PD

Hence, proved

12. AB and AC are two chords of a circle of radius r such that AB = 2AC. If p and q are the distances of AB and AC from the centre, prove that $4q^2 = p^2 + 3r^2$.

Sol. A circle with centre O and radius r in which there are two chords such that AB = 2AC. $OL \perp AB$ and $OM \perp AC$. OL = p and OM = q.



We have to prove that $4q^2 = p^2 + 3r^2$ since perpendicular from the centre to a bisects the chord

In right ΔAOL, we have

$$r^2 = AL^2 + p^2$$

$\Rightarrow \quad AL^2 = r^2 - p^2$

$\therefore \quad \left(\frac{1}{2}AB\right)^2 = r^2 - p^2 \Rightarrow \frac{1}{4}AB^2 = r^2 - p^2$

$\Rightarrow \quad AB^2 = 4(r^2 - p^2)$

$\Rightarrow \quad (2AC)^2 = 4(r^2 - p^2)$ [$\because$ AB = 2AC]

$\Rightarrow \quad 4AC^2 = 4(r^2 - p^2)$ ...(1)

Again, in right ΔAOM, we have

$$r^2 = AM^2 + q^2 \Rightarrow AM^2 = r^2 - q^2$$

Since, $\perp$ from the centre to a chord bisects the chord

$\therefore \quad \left(\frac{1}{2}AC\right)^2 = r^2 - q^2 \Rightarrow \frac{1}{4}AC^2 = r^2 - q^2$

$\Rightarrow \quad AC^2 = 4(r^2 - q^2)$ ...(2)

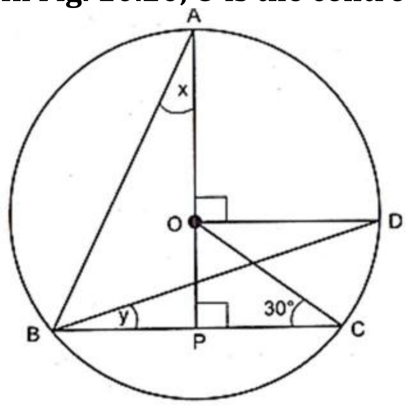
From (1) and (2), we get

$$4\{4(r^2 - q^2)\} = 4(r^2 - p^2)$$

$\Rightarrow \quad 4r^2 - 4q^2 = r^2 - p^2 \Rightarrow 4q^2 = 3r^2 + p^2$

Hence, $4q^2 = p^2 + 3r^2$

13. In Fig. 10.20, O is the centre of the circle, ∠BCO = 30°. Find x and y.



Sol. O is the centre of the circle and $\angle BCO = 30^0$. We have to find the values of x and y.

In right ΔOCP, we have

$$\angle POC = 180^0 - (\angle OPC + \angle PCO)$$

$\Rightarrow \quad \angle POC = 180^0 - (90^0 + 30^0) = 60^0$

And $\quad \angle AOD = 90^0$ [Given]

$\angle AOD + \angle DOP = 180^0$ [Angles of a linear pair]

$\therefore \quad \angle DOP = 180^0 - \angle AOD$

$= 180^0 - 90^0 = 90^0$

Now, $\quad \angle COD = 90^0 - \angle POC = 90^0 - 60^0 = 30^0$

Since the angles subtended by an arc at the centre is double the angle subtended by it any point on the remaining part of the circle,

$\therefore \quad \angle CBD = \frac{1}{2}\angle COD \Rightarrow y = \frac{1}{2} \times 30^0 = 15^0$

Also, $\quad \angle ABD = \frac{1}{2}\angle AOD = \frac{1}{2} \times 90^0 = 45^0$

Now, in ΔABP, we have $x + (45^0 + y) + 90^0 = 180^0$

$\Rightarrow \quad x + 45^0 + 15^0 + 90^0 = 180^0$

$\Rightarrow \quad x = 180^0 - 150^0 = 30^0$

Hence, $x = 30^0$ and $y = 15^0$.

14. In Fig. 10.21, O is the centre of the circle, BD = OD and CD ⊥ AB. Find ∠CAB.

Sol. In ΔABP, we have

BD = OD [Given]

$\therefore$ $\angle DOB = \angle DBO$ [∵ Angles opp. To equal sides of triangle are equal]

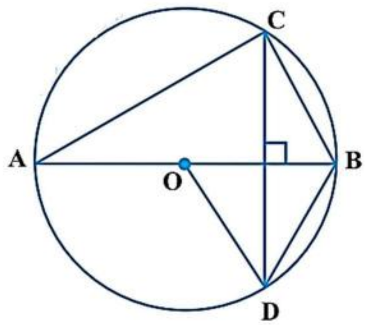


Fig. 10.21

In ΔODP and ΔBDP, we have

$\angle DOP = \angle DBP$ [∵ $\angle DOB = \angle DBO$ (proved above)]

$\angle DPO = \angle DPB$ [Each = 90°]

OD = BD [Given]

$\therefore$ $\Delta ODP \cong \Delta BDP$ [By AAS congruence rule]

$\therefore$ $\angle ODP = \angle DBP$ ...(1) [CPCT]

Now, OD = OB [Radii of the same circle]

And OD = BD [Given]

$\therefore$ OB = OD = BD, so ΔOBD is equilateral.

$\therefore$ $\angle ODP = 60^0$ [∵ Each angle of equilateral triangle is 60°]

Now, $\angle BDP = \frac{1}{2} \times 60^0 = 30^0$ or $\angle CDB = 30^0$

Since, angles in the same segment of a circle are equal, so we have

So, $\angle CAB = \angle CDB = 30^0$