CHAPTER 7

# TRIANGLES

## (A) Main Concepts and Results

Triangles and their parts, Congruence of triangles, Congruence and correspondence of vertices, Criteria for Congruence of triangles: (i) SAS (ii) ASA (iii) SSS (iv) RHS

AAS criterion for congruence of triangles as a particular case of ASA criterion.

- Angles opposite to equal sides of a triangle are equal,
- Sides opposite to equal angles of a triangle are equal,
- A point equidistant from two given points lies on the perpendicular bisector of the line-segment joining the two points and its converse,
- A point equidistant from two intersecting lines lies on the bisectors of the angles formed by the two lines,
- In a triangle
  - (i) side opposite to the greater angle is longer
  - (ii) angle opposite the longer side is greater
  - (iii) the sum of any two sides is greater than the third side.

## (B) Multiple Choice Questions

Write the correct answer :

**Sample Question 1 :** If $\Delta$ ABC $\cong$ $\Delta$ PQR and $\Delta$ ABC is not congruent to $\Delta$ RPQ, then which of the following is not true:

(A) BC = PQ (B) AC = PR (C) QR = BC (D) AB = PQ

**Solution :** Answer (A)

## EXERCISE 7.1

In each of the following, write the correct answer:

**1.** Which of the following is not a criterion for congruence of triangles?

(A) SAS (B) ASA (C) SSA (D) SSS

**2.** If AB = QR, BC = PR and CA = PQ, then

(A) $\Delta$ ABC $\cong$ $\Delta$ PQR (B) $\Delta$ CBA $\cong$ $\Delta$ PRQ

(C) $\Delta$ BAC $\cong$ $\Delta$ RPQ (D) $\Delta$ PQR $\cong$ $\Delta$ BCA

**3.** In $\Delta$ ABC, AB = AC and $\angle$B = 50°. Then $\angle$C is equal to

(A) 40° (B) 50° (C) 80° (D) 130°

**4.** In $\Delta$ ABC, BC = AB and $\angle$B = 80°. Then $\angle$A is equal to

(A) 80° (B) 40° (C) 50° (D) 100°

**5.** In $\Delta$ PQR, $\angle$R = $\angle$P and QR = 4 cm and PR = 5 cm. Then the length of PQ is

(A) 4 cm (B) 5 cm (C) 2 cm (D) 2.5 cm

**6.** D is a point on the side BC of a $\Delta$ ABC such that AD bisects $\angle$BAC. Then

(A) BD = CD (B) BA > BD (C) BD > BA (D) CD > CA

**7.** It is given that $\Delta$ ABC $\cong$ $\Delta$ FDE and AB = 5 cm, $\angle$B = 40° and $\angle$A = 80°. Then which of the following is true?

(A) DF = 5 cm, $\angle$F = 60° (B) DF = 5 cm, $\angle$E = 60°

(C) DE = 5 cm, $\angle$E = 60° (D) DE = 5 cm, $\angle$D = 40°

**8.** Two sides of a triangle are of lengths 5 cm and 1.5 cm. The length of the third side of the triangle cannot be

(A) 3.6 cm (B) 4.1 cm (C) 3.8 cm (D) 3.4 cm

**9.** In $\Delta$ PQR, if $\angle$R > $\angle$Q, then

(A) QR > PR (B) PQ > PR (C) PQ < PR (D) QR < PR

**10.** In triangles ABC and PQR, AB = AC, $\angle$C = $\angle$P and $\angle$B = $\angle$Q. The two triangles are

(A) isosceles but not congruent (B) isosceles and congruent

(C) congruent but not isosceles (D) neither congruent nor isosceles

**11.** In triangles ABC and DEF, AB = FD and $\angle$A = $\angle$D. The two triangles will be congruent by SAS axiom if

(A) BC = EF (B) AC = DE (C) AC = EF (D) BC = DE

## (C) Short Answer Questions with Reasoning

**Sample Question 1:** In the two triangles ABC and DEF, AB = DE and AC = EF. Name two angles from the two triangles that must be equal so that the two triangles are congruent. Give reason for your answer.

**Solution:** The required two angles are $\angle A$ and $\angle E$. When $\angle A = \angle E$, $\Delta\, ABC \cong \Delta\, EDF$ by SAS criterion.

**Sample Question 2:** In triangles ABC and DEF, $\angle A = \angle D$, $\angle B = \angle E$ and AB = EF. Will the two triangles be congruent? Give reasons for your answer.

**Solution:** Two triangles need not be congruent, because AB and EF are not corresponding sides in the two triangles.

## EXERCISE 7.2

1. In triangles ABC and PQR, $\angle A = \angle Q$ and $\angle B = \angle R$. Which side of $\Delta$ PQR should be equal to side AB of $\Delta$ ABC so that the two triangles are congruent? Give reason for your answer.
2. In triangles ABC and PQR, $\angle A = \angle Q$ and $\angle B = \angle R$. Which side of $\Delta$ PQR should be equal to side BC of $\Delta$ ABC so that the two triangles are congruent? Give reason for your answer.
3. "If two sides and an angle of one triangle are equal to two sides and an angle of another triangle, then the two triangles must be congruent." Is the statement true? Why?
4. "If two angles and a side of one triangle are equal to two angles and a side of another triangle, then the two triangles must be congruent." Is the statement true? Why?
5. Is it possible to construct a triangle with lengths of its sides as 4 cm, 3 cm and 7 cm? Give reason for your answer.
6. It is given that $\Delta\, ABC \cong \Delta\, RPQ$. Is it true to say that BC = QR? Why?
7. If $\Delta$ PQR $\cong$ $\Delta$ EDF, then is it true to say that PR = EF? Give reason for your answer.
8. In $\Delta$ PQR, $\angle P = 70°$ and $\angle R = 30°$. Which side of this triangle is the longest? Give reason for your answer.
9. AD is a median of the triangle ABC. Is it true that AB + BC + CA > 2 AD? Give reason for your answer.
10. M is a point on side BC of a triangle ABC such that AM is the bisector of $\angle BAC$. Is it true to say that perimeter of the triangle is greater than 2 AM? Give reason for your answer.

**11.** Is it possible to construct a triangle with lengths of its sides as 9 cm, 7 cm and 17 cm? Give reason for your answer.

**12.** Is it possible to construct a triangle with lengths of its sides as 8 cm, 7 cm and 4 cm? Give reason for your answer.

## (D) Short Answer Questions

**Sample Question 1 :** In Fig 7.1, PQ = PR and $\angle Q = \angle R$. Prove that $\Delta$ PQS $\cong$ $\Delta$ PRT.

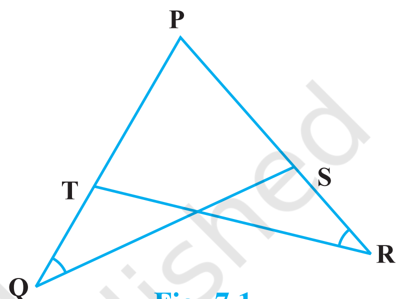

Fig. 7.1

**Solution :** In $\Delta$ PQS and $\Delta$ PRT,

PQ = PR (Given)

$\angle Q = \angle R$ (Given)

and $\angle QPS = \angle RPT$ (Same angle)

Therefore, $\Delta$ PQS $\cong$ $\Delta$ PRT (ASA)

**Sample Question 2 :** In Fig.7.2, two lines AB and CD intersect each other at the point O such that BC || DA and BC = DA. Show that O is the mid-point of both the line-segments AB and CD.

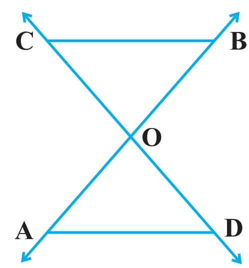

Fig. 7.2

**Solution :** BC || AD (Given)

Therefore, $\angle CBO = \angle DAO$ (Alternate interior angles)

and $\angle BCO = \angle ADO$ (Alternate interior angles)

Also, BC = DA (Given)

So, $\Delta$ BOC $\cong$ $\Delta$ AOD (ASA)

Therefore, OB = OA and OC = OD, i.e., O is the mid-point of both AB and CD.

**Sample Question 3 :** In Fig.7.3, PQ > PR and QS and RS are the bisectors of $\angle Q$ and $\angle R$, respectively. Show that SQ > SR.

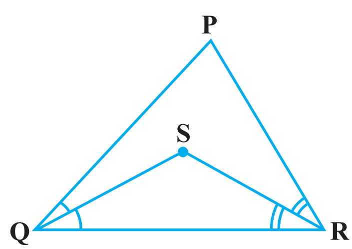

Fig. 7.3

**Solution :** PQ > PR (Given)

Therefore, $\angle R > \angle Q$(Angles opposite the longer side is greater)

So, $\angle SRQ > \angle SQR$ (Half of each angle)

Therefore, SQ > SR(Side opposite the greater angle will be longer)

## EXERCISE 7.3

1. ABC is an isosceles triangle with AB = AC and BD and CE are its two medians. Show that BD = CE.
2. In Fig.7.4, D and E are points on side BC of a Δ ABC such that BD = CE and AD = AE. Show that Δ ABD ≅ Δ ACE.
3. CDE is an equilateral triangle formed on a side CD of a square ABCD (Fig.7.5). Show that Δ ADE ≅ Δ BCE.

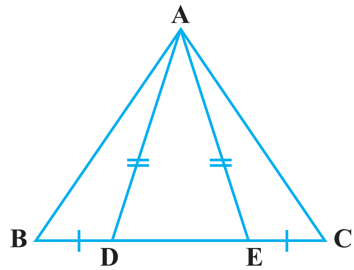

Fig. 7.4

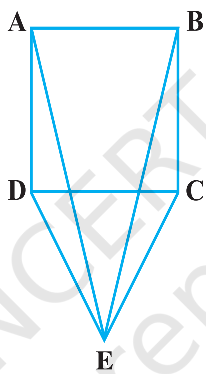

Fig. 7.5

4. In Fig.7.6, BA ⊥ AC, DE ⊥ DF such that BA = DE and BF = EC. Show that Δ ABC ≅ Δ DEF.
5. Q is a point on the side SR of a Δ PSR such that PQ = PR. Prove that PS > PQ.
6. S is any point on side QR of a Δ PQR. Show that: PQ + QR + RP > 2 PS.
7. D is any point on side AC of a Δ ABC with AB = AC. Show that CD < BD.
8. In Fig. 7.7, $l \parallel m$ and M is the mid-point of a line segment AB. Show that M is also the mid-point of any line segment CD, having its end points on $l$ and $m$, respectively.
9. Bisectors of the angles B and C of an isosceles triangle with AB = AC intersect each other at O. BO is produced to a point M. Prove that ∠MOC = ∠ABC.

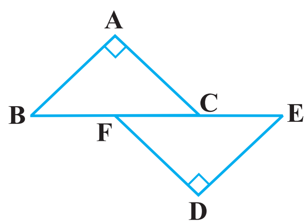

Fig. 7.6

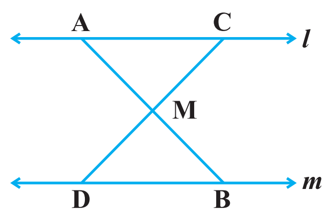

Fig. 7.7

**10.** Bisectors of the angles B and C of an isosceles triangle ABC with AB = AC intersect each other at O. Show that external angle adjacent to $\angle$ABC is equal to $\angle$BOC.

**11.** In Fig. 7.8, AD is the bisector of $\angle$BAC. Prove that AB > BD.

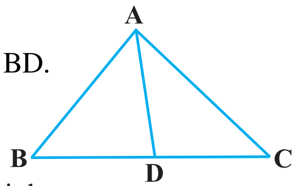


Fig. 7.8

## (E) Long Answer Questions

**Sample Question 1:** In Fig. 7.9, ABC is a right triangle and right angled at B such that $\angle$BCA = 2 $\angle$BAC. Show that hypotenuse AC = 2 BC.

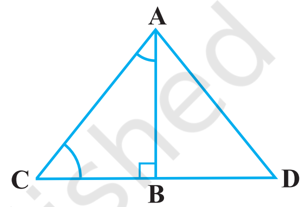


Fig. 7.9

**Solution:** Produce CB to a point D such that BC = BD and join AD.

In $\Delta$ ABC and $\Delta$ ABD, we have

BC = BD (By construction)

AB = AB (Same side)

$\angle$ABC = $\angle$ABD (Each of 90°)

Therefore, $\Delta$ ABC $\cong$ $\Delta$ ABD (SAS)

So, $\angle$CAB = $\angle$DAB (CPCT) (1)

and AC = AD (CPCT) (2)

Thus, $\angle$CAD = $\angle$CAB + $\angle$BAD = $x + x = 2x$ [From (1)] (3)

and $\angle$ACD = $\angle$ADB = $2x$ [From (2), AC = AD] (4)

That is, $\Delta$ACD is an equilateral triangle. [From (3) and (4)]

or AC = CD, i.e., AC = 2 BC (Since BC = BD)

**Sample Question 2 :** Prove that if in two triangles two angles and the included side of one triangle are equal to two angles and the included side of the other triangle, then the two triangles are congruent.

**Solution:** See proof of Theorem 7.1 of Class IX Mathematics Textbook.

**Sample Question 3 :** If the bisector of an angle of a triangle also bisects the opposite side, prove that the triangle is isosceles.

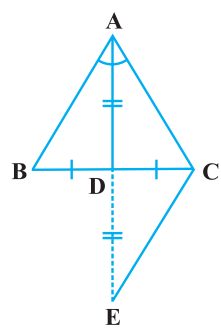


Fig. 7.10

**Solution :** We are given a point D on side BC of a $\Delta$ ABC such that $\angle$BAD = $\angle$CAD and BD = CD (see Fig. 7.10). We are to prove that AB = AC.

Produce AD to a point E such that AD = DE and then join CE.

Now, in $\Delta$ ABD and $\Delta$ ECD, we have

$BD = CD$ (Given)

$AD = ED$ (By construction)

and $\angle ADB = \angle EDC$ (Vertically opposite angles)

Therefore, $\Delta\, ABD \cong \Delta\, ECD$ (SAS)

So, $AB = EC$ (CPCT) (1)

and $\angle BAD = \angle CED$ (CPCT) (2)

Also, $\angle BAD = \angle CAD$ (Given)

Therefore, $\angle CAD = \angle CED$ [From (2)]

So, $AC = EC$ [Sides opposite the equal angles] (3)

Therefore, $AB = AC$ [From (1) and (3)]

**Sample Question 4 :** S is any point in the interior of $\Delta$ PQR. Show that SQ + SR < PQ + PR.

**Solution :** Produce QS to intersect PR at T (See Fig. 7.11).

From $\Delta$ PQT, we have

PQ + PT > QT (Sum of any two sides is greater than the third side)

i.e., $PQ + PT > SQ + ST$ (1)

From $\Delta$ TSR, we have

$ST + TR > SR$ (2)

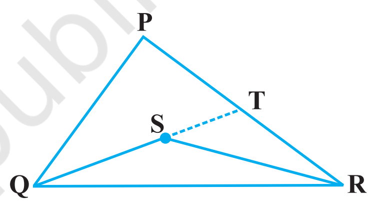

Fig. 7.11

Adding (1) and (2), we get

$PQ + PT + ST + TR > SQ + ST + SR$

i.e., $PQ + PT + TR > SQ + SR$

i.e., $PQ + PR > SQ + SR$

or $SQ + SR < PQ + PR$

## EXERCISE 7.4

1. Find all the angles of an equilateral triangle.
2. The image of an object placed at a point A before a plane mirror LM is seen at the point B by an observer at D as shown in Fig. 7.12. Prove that the image is as far behind the mirror as the object is in front of the mirror.

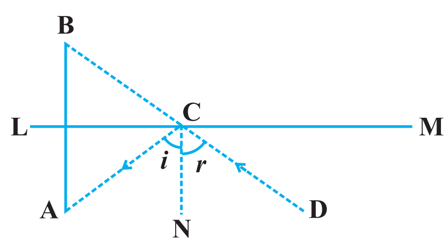

Fig. 7.12

[**Hint:** CN is normal to the mirror. Also, angle of incidence = angle of reflection].

**3.** ABC is an isosceles triangle with AB = AC and D is a point on BC such that AD $\perp$ BC (Fig. 7.13). To prove that $\angle$BAD = $\angle$CAD, a student proceeded as follows:

In $\Delta$ ABD and $\Delta$ ACD,

| | | | |
|---|---|---|---|
| | AB = | AC | (Given) |
| | $\angle$B = | $\angle$C | (because AB = AC) |
| and | $\angle$ADB = | $\angle$ADC | |
| Therefore, | $\Delta$ ABD $\cong$ | $\Delta$ ACD | (AAS) |
| So, | $\angle$BAD = | $\angle$CAD | (CPCT) |

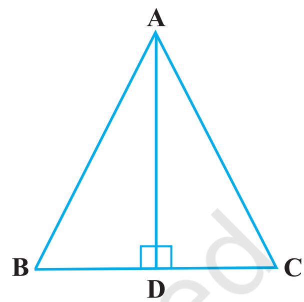

Fig. 7.13

What is the defect in the above arguments?

[**Hint:** Recall how $\angle$B = $\angle$C is proved when AB = AC].

**4.** P is a point on the bisector of $\angle$ABC. If the line through P, parallel to BA meet BC at Q, prove that BPQ is an isosceles triangle.

**5.** ABCD is a quadrilateral in which AB = BC and AD = CD. Show that BD bisects both the angles ABC and ADC.

**6.** ABC is a right triangle with AB = AC. Bisector of $\angle$A meets BC at D. Prove that BC = 2 AD.

**7.** O is a point in the interior of a square ABCD such that OAB is an equilateral triangle. Show that $\Delta$ OCD is an isosceles triangle.

**8.** ABC and DBC are two triangles on the same base BC such that A and D lie on the opposite sides of BC, AB = AC and DB = DC. Show that AD is the perpendicular bisector of BC.

**9.** ABC is an isosceles triangle in which AC = BC. AD and BE are respectively two altitudes to sides BC and AC. Prove that AE = BD.

**10.** Prove that sum of any two sides of a triangle is greater than twice the median with respect to the third side.

**11.** Show that in a quadrilateral ABCD, AB + BC + CD + DA < 2 (BD + AC)

**12.** Show that in a quadrilateral ABCD,

AB + BC + CD + DA > AC + BD

**13.** In a triangle ABC, D is the mid-point of side AC such that BD = $\frac{1}{2}$ AC. Show that $\angle$ABC is a right angle.

**14.** In a right triangle, prove that the line-segment joining the mid-point of the hypotenuse to the opposite vertex is half the hypotenuse.

**15.** Two lines $l$ and $m$ intersect at the point O and P is a point on a line $n$ passing through the point O such that P is equidistant from $l$ and $m$. Prove that $n$ is the bisector of the angle formed by $l$ and $m$.

**16.** Line segment joining the mid-points M and N of parallel sides AB and DC, respectively of a trapezium ABCD is perpendicular to both the sides AB and DC. Prove that AD = BC.

**17.** ABCD is a quadrilateral such that diagonal AC bisects the angles A and C. Prove that AB = AD and CB = CD.

**18.** ABC is a right triangle such that AB = AC and bisector of angle C intersects the side AB at D. Prove that AC + AD = BC.

**19.** AB and CD are the smallest and largest sides of a quadrilateral ABCD. Out of $\angle$B and $\angle$D decide which is greater.

**20.** Prove that in a triangle, other than an equilateral triangle, angle opposite the longest side is greater than $\frac{2}{3}$ of a right angle.

**21.** ABCD is quadrilateral such that AB = AD and CB = CD. Prove that AC is the perpendicular bisector of BD.