UNIT 8

RATIONAL NUMBERS

(A) Main Concepts and Results

- A number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called a rational number.
- All integers and fractions are rational numbers.
- If the numerator and denominator of a rational number are multiplied or divided by a non-zero integer, we get a rational number which is said to be equivalent to the given rational number.
- Rational numbers are classified as positive, zero or negative rational numbers. When the numerator and denominator both are positive integers or both are negative integers, it is a positive rational number. When either the numerator or the denominator is a negative integer, it is a negative rational number.
- The number 0 is neither a positive nor a negative rational number.
- There are unlimited number of rational numbers between two rational numbers.
- A rational number is said to be in the standard form, if its denominator is a positive integer and the numerator and denominator have no common factor other than 1.
- Two rational numbers with the same denominator can be added by adding their numerators, keeping with the same denominator.
- Two rational numbers with different denominators are added by first taking the LCM of the two denominators and then converting both the rational numbers to their equivalent forms having the LCM as the denominator and adding them as above.

- While subtracting two rational numbers, we add the additive inverse of the rational number to be subtracted to the other rational number.
- Product of rational numbers = $\dfrac{\text{Product of numerators}}{\text{Product of denominators}}$
- The reciprocal of a non-zero rational number $\dfrac{p}{q}$ is $\dfrac{q}{p}$.
- To divide one rational number by the other non-zero rational number, we multiply the first rational number by the reciprocal of the other.

(B) Solved Examples

In Examples 1 to 4, there are four options, out of which one is correct. Choose the correct one.

Example 1: Which of the following rational numbers is equivalent to $\dfrac{2}{3}$?

(a) $\dfrac{3}{2}$ (b) $\dfrac{4}{9}$ (c) $\dfrac{4}{6}$ (d) $\dfrac{9}{4}$

Solution: Correct answer is (c).

Example 2: Which of the following rational numbers is in standard form?

(a) $\dfrac{20}{30}$ (b) $\dfrac{10}{4}$ (c) $\dfrac{1}{2}$ (d) $\dfrac{1}{-3}$

Solution: Correct answer is (c).

Example 3: The sum of $\dfrac{-3}{2}$ and $\dfrac{1}{2}$ is

(a) –1 (b) –2 (c) 4 (d) 3

Solution: Correct answer is (a).

Example 4: The value of $-\dfrac{4}{3}-\dfrac{-1}{3}$ is

(a) – 2 (b) – 3 (c) 2 (d) –1

Solution: Correct answer is (d).

In Examples 5 and 6, fill in the blanks to make the statements true.

Example 5: There are ______ number of rational numbers between two rational numbers.

Solution: Unlimited

Example 6: The rational number ________ is neither positive nor negative.

Solution: 0 (Zero).

In Examples 7 to 9, state whether the statements are True or False.

Example 7: In any rational number $\frac{p}{q}$, denominator is always a non-zero integer.

Solution: True.

Reading Strategy: Read a Lesson for Understanding

You need to be actively involved as you work through each lesson in your textbook. To begin with, find the lesson's objective given as main concepts and results.

Lesson Features

Lesson Features	
Learn to write rational numbers in equivalent forms	Identify the objectives of the lesson and look through the lesson to get a feel for how the objectives are met.
Example	Work through each example. The examples help to demonstrate the lesson objectives.
Think and Discuss	Check your understanding of the lesson by answering the exercise/questions.

Example 8: "To reduce the rational number to its standard form, we divide its numerator and denominator by their HCF".

Solution: True.

Example 9: "All rational numbers are integers".

Solution: False.

Solve

• Choose an Operation

To decide whether to add, subtract, multiply, or divide to solve a problem, you need to determine the action taking place in the problem.

$\div$	$-$
• Divided by • Quotient • Divided into	• Subtracted from • Minus • Difference • Less than • Decreased by
$+$	$\times$
• Added to • Plus • Sum • More than	• Multiplied by • Times • Product • Groups of

Example 10: List three rational numbers between $\frac{4}{5}$ and $\frac{5}{6}$.

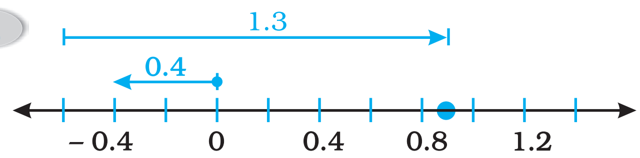
Solution: We convert the rational numbers $\frac{4}{5}$ and $\frac{5}{6}$ into rational numbers with the same denominators.

$$\frac{4}{5} = \frac{4}{5} \times \frac{6}{6} = \frac{24}{30}; \qquad \frac{5}{6} = \frac{5}{6} \times \frac{5}{5} = \frac{25}{30}$$

Using a Number Line to Add Rational Numbers

Use a number line to find each sum.

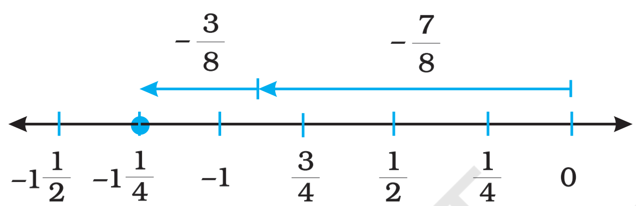
A



Move left from zero 0.4 units
From – 0.4, move right
1.3 units

You finish at 0.9,. So – 0.4 + 1.3 = 0.9

B $\dfrac{-7}{8} + -\dfrac{3}{8}$



Move left $\frac{7}{8}$ units. From
$-\frac{7}{8}$ move left 3/8 units.

You finish at $-1\frac{1}{4}$. So $-\frac{7}{8} + \left(-\frac{3}{8}\right) = -1\frac{1}{4}$

So, $\dfrac{24}{30} = \dfrac{24}{30} \times \dfrac{4}{4}$ and $\dfrac{25}{30} = \dfrac{25}{30} \times \dfrac{4}{4}$

$= \dfrac{96}{120}$ $= \dfrac{100}{120}$

or

Here, $\dfrac{96}{120} < \dfrac{97}{120} < \dfrac{98}{120} < \dfrac{99}{120} < \dfrac{100}{120}$. So, the required numbers are $\dfrac{97}{120}, \dfrac{98}{120}$ and $\dfrac{99}{120}$

Alternate solution A rational number between $\dfrac{4}{5}$ and $\dfrac{5}{6}$ is

$= \dfrac{1}{2}\left(\dfrac{4}{5} + \dfrac{5}{6}\right) = \dfrac{49}{60}$

another rational number

$= \dfrac{1}{2}\left(\dfrac{4}{5} + \dfrac{49}{60}\right) = \dfrac{97}{120}$

one more rational number

$$= \frac{1}{2}\left(\frac{49}{60}+\frac{5}{6}\right)=\frac{99}{120}=\frac{33}{40}$$

Therefore, three rational numbers between $\frac{4}{5}$ and $\frac{5}{6}$ are

$\frac{49}{60}, \frac{97}{120}$ and $\frac{33}{40}$

Note: There can be many set of answers.

ADDING AND SUBTRACTING WITH LIKE DENOMINATORS

Words	Numbers	Formula
To add or subtract rational numbers with the same denominator, add or subtract the numerators and keep the same denominator.	$\frac{1}{5}+\left(\frac{-4}{5}\right)=\frac{1+(-4)}{5}$ $=\frac{-3}{5}$, or $\frac{-3}{5}$	$\frac{a}{d}+\frac{b}{d}=\frac{a+b}{d}$

Example 11: Which of the following pairs represent equivalent rational numbers ?

(i) $\frac{7}{12}$ and $\frac{28}{48}$ (ii) $\frac{-2}{-3}$ and $\frac{-16}{24}$

Solution:

(i) $\frac{7}{12}$ and $\frac{28}{48}$

Now, first rational number is $\frac{7}{12}$ and it is already in the standard form because there is no common factor in 7 and 12 other than 1.

So, $\frac{7}{12}$ is in its standard form ______(a)

Now, Consider $\frac{28}{48}$

$28 = 2 \times 2 \times 7$

$48 = 2 \times 2 \times 2 \times 2 \times 3$

$\text{HCF} = 2 \times 2 = 4$

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Now, to reduce the rational numbers to its standard form, we divide the numerator and denominator by their HCF.

First we take HCF of 28 and 48:

Now, $\frac{28}{48} = \frac{28 \div 4}{48 \div 4} = \frac{7}{12}$ ______ (b)

From (a) and (b), we can say that the rational numbers $\frac{7}{12}$ and $\frac{28}{48}$ are equivalent.

(ii) $\frac{-2}{-3}$ and $\frac{-16}{24}$

First we multiply the numerator and denominator of $\frac{-2}{-3}$ by (–1), we get

$\frac{-2}{-3} = \frac{(-2)\times(-1)}{(-3)\times(-1)} = \frac{2}{3}$ ______ (a)

Now it is in its standard form.

Now, Consider $\frac{16}{24}$

HCF of 16 and 24 is 2 × 2 × 2 = 8

16 = 2 × 2 × 2 × 2

24 = 2 × 2 × 2 × 3

HCF = 2 × 2 × 2 = 8

So, $\frac{-16}{24} = \frac{-16 \div 8}{24 \div 8} = \frac{-2}{3}$ ______ (b)

From (a) and (b), we can say that the rational numbers $\frac{-2}{-3}$ and $\frac{-16}{24}$ are not equivalent.

Action	Operation
Combining numbers or putting numbers together	Addition
Taking away or finding out how far apart two numbers are	Subtraction
Combining groups	Multiplication
Splitting things into equal groups or finding how many equal groups you can make	Division

Example 12: Write four more rational numbers to complete the pattern:

$$\frac{-1}{3}, \frac{-2}{6}, \frac{-3}{9}, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_.$$

Solution: By observing the above pattern, we find that denominator is multiple of 3. So we will increase this pattern in this way.

$$\frac{-2}{6} = \frac{-1\times2}{3\times2}, \frac{-3}{9} = \frac{-1\times3}{3\times3}, \frac{-4}{12} = \frac{-1\times4}{3\times4}$$

$$\frac{-1\times1}{3\times1} = \frac{-1}{3},$$

$$\frac{-1\times4}{3\times4} = \frac{-4}{12}$$

Thus, we observe a pattern in these numbers.

So, the other numbers would be

$$\frac{-1\times5}{3\times5} = \frac{-5}{15}, \frac{-1\times6}{3\times6} = \frac{-6}{18}, \frac{-1\times7}{3\times7} = \frac{-7}{21} \text{ and } \frac{-1\times8}{3\times8} = \frac{-8}{24}$$

DIVIDING RATIONAL NUMBERS IN FRACTION FORM

Words	Numbers	Algebra
To divide by a fraction, multiply by the reciprocal	$\frac{1}{7} \div \frac{4}{5} = \frac{1}{7} \times \frac{5}{4} = \frac{5}{28}$	$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$

Example 13: Find the sum of $-4\frac{5}{6}$ and $-7\frac{3}{4}$.

Solution:

$$-4\frac{5}{6} + \left(-7\frac{3}{4}\right)$$

$$= \frac{-29}{6} + \left(\frac{-31}{4}\right) = \frac{-29}{6} + \frac{-31}{4}$$

$$= \frac{-29 \times 2}{12} + \frac{-31 \times 3}{12}$$. [Since LCM of 6 and 4 is 12].

$$= \frac{-29 \times 2 - 31 \times 3}{12}$$

$$= \frac{-58 - 93}{12}$$

$$= \frac{-151}{12}$$

So, the required sum is $\frac{-151}{12}$.

Think and Discuss

1. **Give an example** of two denominators with no common factors.
2. **Tell** if $-2\frac{1}{5} - \left(-2\frac{3}{16}\right)$ is positive or negative. Explain.
3. **Explain** how to add $2\frac{2}{5} + 9\frac{1}{2}$, without first writing them as improper fractions.

Example 14: Find the product of $-2\frac{3}{4}$ and $5\frac{6}{7}$.

Solution: $-2\frac{3}{4} \times 5\frac{6}{7} = \frac{-11}{4} \times \frac{41}{7}$

Now, product of two rational numbers

$$= \frac{\text{Product of numerators}}{\text{Product of denominators}}$$

So, $-2\frac{3}{4} \times 5\frac{6}{7} = \frac{-11}{4} \times \frac{41}{7} = \frac{-11 \times 41}{4 \times 7} = \frac{-451}{28}$

Think and Discuss

1. **Explain** how you can be sure that a fraction is simplified.
2. **Give** the sign of a rational number in which the numerator is negative and the denominator is negative.

Example 15: Match column I to column II in the following:

	Column I	Column II
(i)	$\frac{3}{4} \div \frac{3}{4}$	(a) -1
(ii)	$\frac{1}{2} \div \frac{4}{3}$	(b) $\frac{-2}{3}$
(iii)	$\frac{2}{3} \div (-1)$	(c) $\frac{3}{2}$
(iv)	$\frac{3}{4} \div \frac{1}{2}$	(d) $\frac{3}{8}$
(v)	$\frac{5}{7} \div \left(\frac{-5}{7}\right)$	(e) 1

Solution: (i) $\leftrightarrow$ (e), (ii) $\leftrightarrow$ (d), (iii) $\leftrightarrow$ (b), (iv) $\leftrightarrow$ (c), (v) $\leftrightarrow$ (a)

Application on Problem Solving Strategy

Example 16

Find the reciprocal of $\frac{2}{11} \div -\frac{5}{55}$.

Solution : **Understand and Explore the Problem**

- What are you trying to find?
  The reciprocal of the given number.

Plan a Strategy

- You know the division of rational numbers and the meaning of reciprocal. Apply this knowledge to find the reciprocal.

Solve

- **Given expression** = $\frac{2}{11} \div \frac{-5}{55} = \frac{2}{\cancel{11}} \times -\frac{\overset{\cancel{5}}{\cancel{55}}}{\cancel{5}} = -2$
- Now find out the reciprocal of – 2

  The reciprocal of – 2 is $-\frac{1}{2}$.

Revise

- Multiply – 2 and $-\frac{1}{2}$ and check whether you get 1 or not.

  $-2 \times -\frac{1}{2} = 1$

  Hence, our answer is correct, since we know that the product of a rational number with its reciprocal is always 1.

Think and Discuss

1. **Can you find** the reciprocal of $\frac{2}{11} \times \frac{5}{55}$?
2. **Can you compare** this reciprocal with the orignal number?

(C) Exercise

In each of the following questions 1 to 12, there are four options, out of which, only one is correct. Write the correct one.

1. A rational number is defined as a number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and

   (a) $q = 0$ (b) $q = 1$ (c) $q \neq 1$ (d) $q \neq 0$

2. Which of the following rational numbers is positive?

(a) $\frac{-8}{7}$ (b) $\frac{19}{-13}$ (c) $\frac{-3}{-4}$ (d) $\frac{-21}{13}$

3. Which of the following rational numbers is negative?

(a) $-\left(\frac{-3}{7}\right)$ (b) $\frac{-5}{-8}$ (c) $\frac{9}{8}$ (d) $\frac{3}{-7}$

4. In the standard form of a rational number, the common factor of numerator and denominator is always:

(a) 0 (b) 1 (c) – 2 (d) 2

5. Which of the following rational numbers is equal to its reciprocal?

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 0

6. The reciproal of $\frac{1}{2}$ is

(a) 3 (b) 2 (c) – 1 (d) 0

7. The standard form of $\frac{-48}{60}$ is

(a) $\frac{48}{60}$ (b) $\frac{-60}{48}$ (c) $\frac{-4}{5}$ (d) $\frac{-4}{-5}$

Number	Reciprocal	Product
$\frac{3}{4}$	$\frac{4}{3}$	$\frac{3}{4}\left(\frac{4}{3}\right)=1$
$-\frac{5}{12}$	$-\frac{12}{5}$	$-\frac{5}{12}\left(-\frac{12}{5}\right)=1$
6	$\frac{1}{6}$	$6\left(\frac{1}{6}\right)=1$

8. Which of the following is equivalent to $\frac{4}{5}$?

(a) $\frac{5}{4}$ (b) $\frac{16}{25}$ (c) $\frac{16}{20}$ (d) $\frac{15}{25}$

9. How many rational numbers are there between two rational numbers?

(a) 1 (b) 0 (c) unlimited (d) 100

10. In the standard form of a rational number, the denominator is always a

(a) 0 (b) negative integer

(c) positive integer (d) 1

11. To reduce a rational number to its standard form, we divide its numerator and denominator by their

(a) LCM (b) HCF (c) product (d) multiple

12. Which is greater number in the following:

(a) $\frac{-1}{2}$ (b) 0 (c) $\frac{1}{2}$ (d) -2

RULES FOR MULTIPYING TWO RATIONAL NUMBERS

If the signs of the factors are the same, the product is positive.

$(+) \cdot (+) = (+)$ or $(–) \cdot (–) = (+)$

If the signs of the factors are different, the product is negative

$(+) \cdot (–) = (–)$ or $(–) \cdot (+) = (–)$

In Questions 13 to 46, fill in the blanks to make the statements true.

13. $-\frac{3}{8}$ is a ______ rational number.

14. 1 is a ______ rational number.

15. The standard form of $\frac{-8}{-36}$ is ______.

16. The standard form of $\frac{18}{-24}$ is ______.

17. On a number line, $\frac{-1}{2}$ is to the ______ of zero (0).

18. On a number line, $\frac{4}{3}$ is to the ______ of zero (0).

19. $-\frac{1}{2}$ is ______ than $\frac{1}{5}$.

20. $-\frac{3}{5}$ is ______ than 0.

21. $\frac{-16}{24}$ and $\frac{20}{-16}$ represent ______ rational numbers.

22. $\frac{-27}{45}$ and $\frac{-3}{5}$ represent ______ rational numbers.

23. Additive inverse of $\frac{2}{3}$ is ______.

24. $\frac{-3}{5}+\frac{2}{5}=$ ______.

25. $\frac{-5}{6}+\frac{-1}{6}=$ ______.

26. $\frac{3}{4}\times\left(\frac{-2}{3}\right)=$ ______.

27. $\frac{-5}{3}\times\left(\frac{-3}{5}\right)=$ ______.

28. $\frac{-6}{7}=\frac{\_\_}{42}$

29. $\frac{1}{2}=\frac{6}{\_\_}$

30. $\frac{-2}{9}-\frac{7}{9}=$ ______.

In questions 31 to 35, fill in the boxes with the correct symbol >,< or =.

31. $\frac{7}{-8} \square \frac{8}{9}$

32. $\frac{3}{7} \square \frac{-5}{6}$

33. $\frac{5}{6} \square \frac{8}{4}$

34. $\frac{-9}{7} \square \frac{4}{-7}$

35. $\frac{8}{8} \square \frac{2}{2}$

36. The reciprocal of ______ does not exist.

37. The reciprocal of 1 is ______.

38. $\frac{-3}{7} \div \left(\frac{-7}{3}\right) =$ ______.

39. $0 \div \left(\frac{-5}{6}\right) =$ ______.

40. $0 \times \left(\frac{-5}{6}\right) =$ ______.

41. ______ $\times \left(\frac{-2}{5}\right) = 1$.

42. The standard form of rational number -1 is ______.

43. If m is a common divisor of a and b, then $\frac{a}{b} = \frac{a \div m}{\_\_\_\_}$

44. If p and q are positive integers, then $\frac{p}{q}$ is a ______ rational number and $\frac{p}{-q}$ is a ______ rational number.

45. Two rational numbers are said to be equivalent or equal, if they have the same ______ form.

46. If $\frac{p}{q}$ is a rational number, then q cannot be ______.

State whether the statements given in question 47 to 65 are True or False.

47. Every natural number is a rational number but every rational number need not be a natural number.

48. Zero is a rational number.

49. Every integer is a rational number but every rational number need not be an integer.

50. Every negative integer is not a negative rational number.

51. If $\frac{p}{q}$ is a rational number and m is a non-zero integer, then $\frac{p}{q} = \frac{p \times m}{q \times m}$

52. If $\frac{p}{q}$ is a rational number and m is a non-zero common divisor of p and q, then $\frac{p}{q} = \frac{p \div m}{q \div m}$.

53. In a rational number, denominator always has to be a non-zero integer.

54. If $\frac{p}{q}$ is a rational number and m is a non-zero integer, then $\frac{p \times m}{q \times m}$ is a rational number not equivalent to $\frac{p}{q}$.

55. Sum of two rational numbers is always a rational number.

56. All decimal numbers are also rational numbers.

57. The quotient of two rationals is always a rational number.

58. Every fraction is a rational number.

59. Two rationals with different numerators can never be equal.

60. 8 can be written as a rational number with any integer as denominator.

61. $\frac{4}{6}$ is equivalent to $\frac{2}{3}$.

62. The rational number $\frac{-3}{4}$ lies to the right of zero on the number line.

63. The rational numbers $\frac{-12}{-5}$ and $\frac{-7}{17}$ are on the opposite sides of zero on the number line.

64. Every rational number is a whole number.

65. Zero is the smallest rational number.

66. Match the following:

Column I	**Column II**
(i) $\frac{a}{b} \div \frac{a}{b}$	(a) $\frac{-a}{b}$
(ii) $\frac{a}{b} \div \frac{c}{d}$	(b) -1
(iii) $\frac{a}{b} \div (-1)$	(c) 1
(iv) $\frac{a}{b} \div \frac{-a}{b}$	(d) $\frac{bc}{ad}$
(v) $\frac{b}{a} \div \left(\frac{d}{c}\right)$	(e) $\frac{ad}{bc}$

67. Write each of the following rational numbers with positive denominators: $\frac{5}{-8}, \frac{15}{-28}, \frac{-17}{-13}$.

68. Express $\frac{3}{4}$ as a rational number with denominator:

(i) 36 (ii) – 80

69. Reduce each of the following rational numbers in its lowest form:

(i) $\frac{-60}{72}$ (ii) $\frac{91}{-364}$

70. Express each of the following rational numbers in its standard form:

(i) $\frac{-12}{-30}$ (ii) $\frac{14}{-49}$ (iii) $\frac{-15}{35}$ (iv) $\frac{299}{-161}$

71. Are the rational numbers $\frac{-8}{28}$ and $\frac{32}{-112}$ equivalent? Give reason.

72. Arrange the rational numbers $\frac{-7}{10}, \frac{5}{-8}, \frac{2}{-3}, \frac{-1}{4}, \frac{-3}{5}$ in ascending order.

73. Represent the following rational numbers on a number line: $\frac{3}{8}, \frac{-7}{3}, \frac{22}{-6}$.

74. If $\frac{-5}{7} = \frac{x}{28}$, find the value of x.

75. Give three rational numbers equivalent to:

(i) $\frac{-3}{4}$ (ii) $\frac{7}{11}$

76. Write the next three rational numbers to complete the pattern:

(i) $\frac{4}{-5}, \frac{8}{-10}, \frac{12}{-15}, \frac{16}{-20}$, ______, ______, ______.

(ii) $\frac{-8}{7}, \frac{-16}{14}, \frac{-24}{21}, \frac{-32}{28}$, ______, ______, ______.

77. List four rational numbers between $\frac{5}{7}$ and $\frac{7}{8}$.

78. Find the sum of

(i) $\frac{8}{13}$ and $\frac{3}{11}$ (ii) $\frac{7}{3}$ and $\frac{-4}{3}$

79. Solve:

(i) $\frac{29}{4} - \frac{30}{7}$ (ii) $\frac{5}{13} - \frac{-8}{26}$

80. Find the product of:

(i) $\frac{-4}{5}$ and $\frac{-5}{12}$ (ii) $\frac{-22}{11}$ and $\frac{-21}{11}$

81. Simplify:

(i) $\frac{13}{11} \times \frac{-14}{5} + \frac{13}{11} \times \frac{-7}{5} + \frac{-13}{11} \times \frac{34}{5}$ (ii) $\frac{6}{5} \times \frac{3}{7} - \frac{1}{5} \times \frac{3}{7}$

82. Simplify:

(i) $\frac{3}{7} \div \left(\frac{21}{-55}\right)$ (ii) $1 \div \left(-\frac{1}{2}\right)$

83. Which is greater in the following?

(i) $\frac{3}{4}, \frac{7}{8}$ (ii) $-3\frac{5}{7}, 3\frac{1}{9}$

84. Write a rational number in which the numerator is less than '-7×11' and the denominator is greater than '$12 + 4$'.

85. If $x = \frac{1}{10}$ and $y = \frac{-3}{8}$, then evaluate $x + y$, $x - y$, $x \times y$ and $x \div y$.

86. Find the reciprocal of the following:

(i) $\left(\frac{1}{2}\times\frac{1}{4}\right)+\left(\frac{1}{2}\times 6\right)$ (ii) $\frac{20}{51}\times\frac{4}{91}$

(iii) $\frac{3}{13}\div\frac{-4}{65}$ (iv) $\left(-5\times\frac{12}{15}\right)-\left(-3\times\frac{2}{9}\right)$

87. Complete the following table by finding the sums:

$+$	$-\frac{1}{9}$	$\frac{4}{11}$	$\frac{-5}{6}$
$\frac{2}{3}$			
$-\frac{5}{4}$		$\frac{-39}{44}$	
$-\frac{1}{3}$			

88. Write each of the following numbers in the form $\frac{p}{q}$, where p and q are integers:

(a) six-eighths

(b) three and half

(c) opposite of 1

(d) one-fourth

(e) zero

(f) opposite of three-fifths

89. If $p = m\times t$ and $q = n\times t$, then $\frac{p}{q}=\frac{\square}{\square}$

90. Given that $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers with different denominators and both of them are in standard form. To compare these rational numbers we say that:

(a) $\frac{\square}{\square} < \frac{\square}{\square}$, if $p\times s < r\times q$

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(b) $\frac{p}{q} = \frac{r}{s}$, if ________ = ________

(c) $\frac{\square}{\square} > \frac{\square}{\square}$, if $p \times s > r \times q$

91. In each of the following cases, write the rational number whose numerator and denominator are respectively as under:

(a) 5 – 39 and 54 – 6

(b) (–4) × 6 and 8 ÷ 2

(c) 35 ÷ (–7) and 35 –18

(d) 25 + 15 and 81 ÷ 40

92. Write the following as rational numbers in their standard forms:

(a) 35% (b) 1.2 (c) $-6\frac{3}{7}$

(d) 240 ÷ (– 840) (e) 115 ÷ 207

93. Find a rational number exactly halfway between:

(a) $\frac{-1}{3}$ and $\frac{1}{3}$ (b) $\frac{1}{6}$ and $\frac{1}{9}$ (c) $\frac{5}{-13}$ and $\frac{-7}{9}$ (d) $\frac{1}{15}$ and $\frac{1}{12}$

94. Taking $x = \frac{-4}{9}, y = \frac{5}{12}$ and $z = \frac{7}{18}$, find :

(a) the rational number which when added to x gives y.

(b) the rational number which subtracted from y gives z.

(c) the rational number which when added to z gives us x.

(d) the rational number which when multiplied by y to get x.

(e) the reciprocal of $x + y$.

(f) the sum of reciprocals of x and y.

(g) $(x \div y) \times z$ (h) $(x - y) + z$

(i) $x + (y + z)$ (j) $x \div (y \div z)$

(k) $x - (y + z)$

95. What should be added to $\frac{-1}{2}$ to obtain the nearest natural number?

96. What should be subtracted from $\frac{-2}{3}$ to obtain the nearest integer?

97. What should be multiplied with $\frac{-5}{8}$ to obtain the nearest integer?

98. What should be divided by $\frac{1}{2}$ to obtain the greatest negative integer?

99. From a rope 68 m long, pieces of equal size are cut. If length of one piece is $4\frac{1}{4}$ m, find the number of such pieces.

100. If 12 shirts of equal size can be prepared from 27m cloth, what is length of cloth required for each shirt?

101. Insert 3 equivalent rational numbers between

(i) $\frac{-1}{2}$ and $\frac{1}{5}$ (ii) 0 and –10

102. Put the (√), wherever applicable

Number	Natural Number	Whole Number	Integer	Fraction	Rational Number
(a) – 114					
(b) $\frac{19}{27}$					
(c) $\frac{623}{1}$					
(d) $-19\frac{3}{4}$					
(e) $\frac{73}{71}$					
(f) 0					

103. '*a*' and '*b*' are two different numbers taken from the numbers 1 – 50. What is the largest value that $\frac{a-b}{a+b}$ can have? What is the largest value that $\frac{a+b}{a-b}$ can have?

104. 150 students are studying English, Maths or both. 62 per cent of the students are studying English and 68 per cent are studying Maths. How many students are studying both?

105. A body floats $\frac{2}{9}$ of its volume above the surface. What is the ratio of the body submerged volume to its exposed volume? Re-write it as a rational number.

Find the odd one out of the following and give reason.

106. (a) $\frac{4}{3}\times\frac{3}{4}$ (b) $\frac{-3}{2}\times\frac{-2}{3}$

(c) $2\times\frac{1}{2}$ (d) $\frac{-1}{3}\times\frac{3}{1}$

107. (a) $\frac{4}{-9}$ (b) $\frac{-16}{36}$

(c) $\frac{-20}{-45}$ (d) $\frac{28}{-63}$

108. (a) $\frac{-4}{3}$ (b) $\frac{-7}{6}$

(c) $\frac{-10}{3}$ (d) $\frac{-8}{7}$

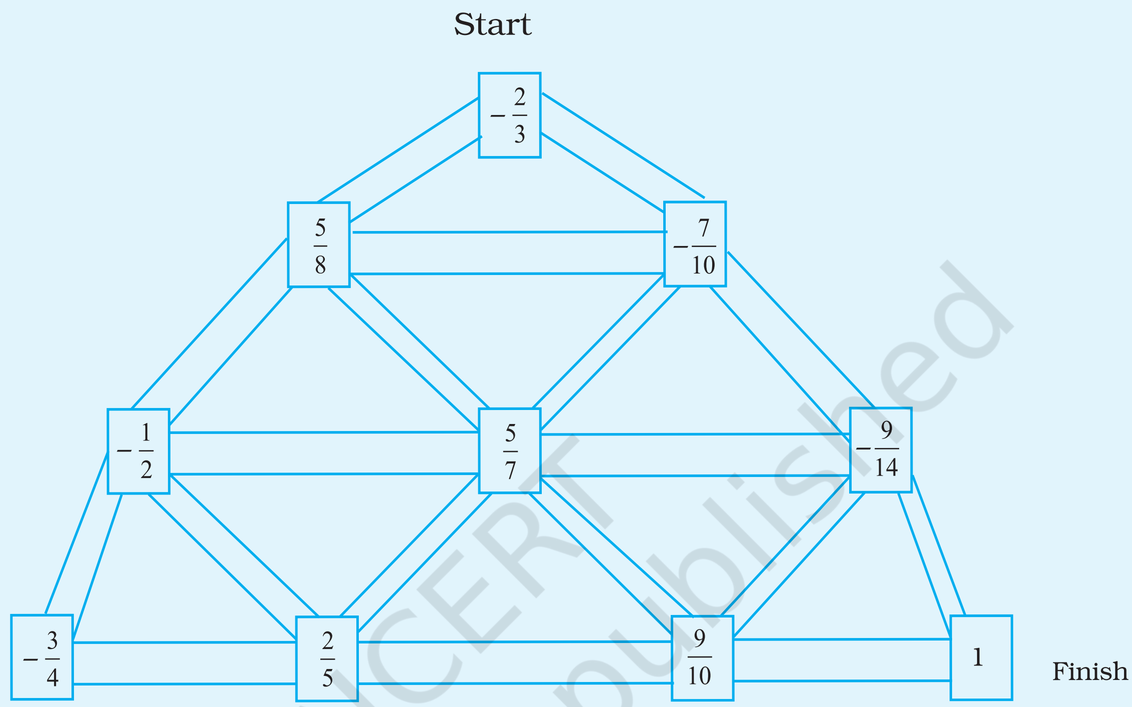
109. (a) $\frac{-3}{7}$ (b) $\frac{-9}{15}$

(c) $\frac{+24}{20}$ (d) $\frac{+35}{25}$

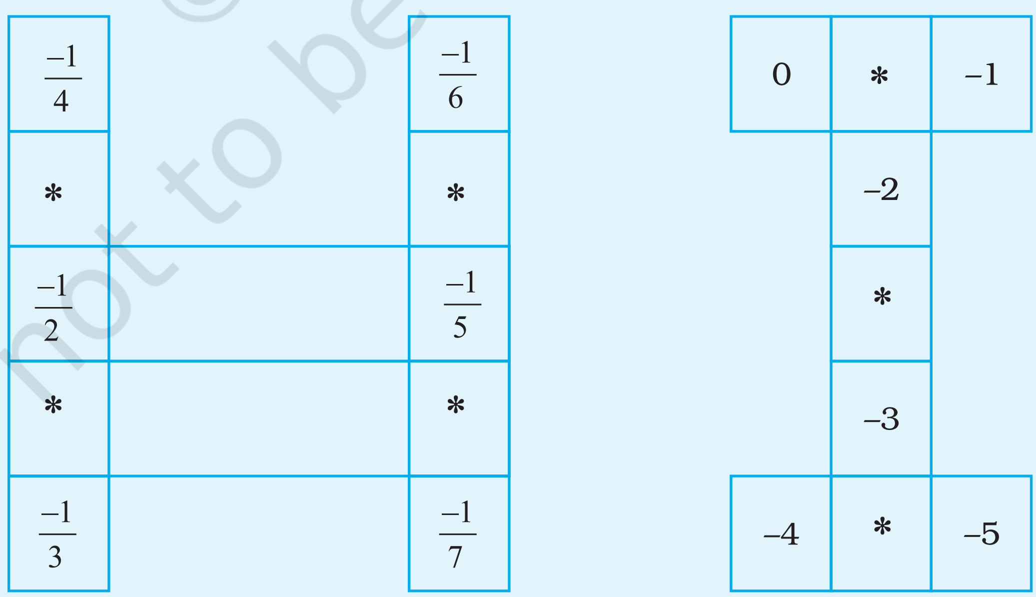
110. What's the Error? Chhaya simplified a rational number in this manner $\frac{-25}{-30}=\frac{-5}{6}$. What error did the student make?

(D) Applications

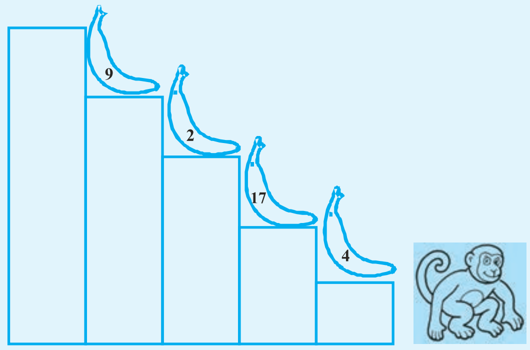
1. Moving from start to finish by going from smaller to bigger rational numbers.



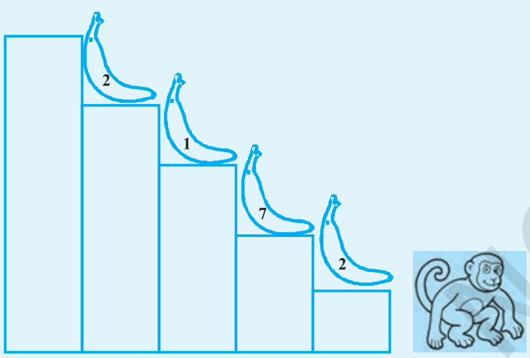
2. Replace '*' by inserting an appropriate rational number between the given rational numbers.



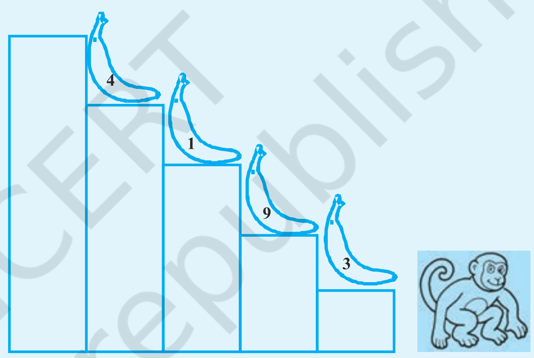
3. Three monkeys are climbing upstairs. They can only move ahead if they eat a banana with the common factor of their numerator and denominator on it. Which of the three monkeys will be able to reach till the end?



$\frac{136}{-124}$



$\frac{-112}{224}$



$\frac{-108}{405}$

4. Crossword Puzzle

Solve the given crossword and then fill up the given boxes. Clues are given below for across as well as downward filling. Also, for across and down clues. clue number is written at the corner of boxes. Answers of clues have to be filled in their respective boxes.

Down 1: $\frac{2}{3}$ and $\frac{-5}{4}$ are ____________ numbers.

Down 2: The ____________ inverse of $\frac{a}{f}$ is $-\frac{a}{f}$.

Down 3: The addition and multiplication of whole numbers, integers and rational numbers is ____________

Down 4: Since, $\frac{1}{0}$ is not defined, hence 0 has no ____________ .

Down 5: Reciprocal is also known as multiplicative __________.

Down 6 : The number line extends __________ on both the sides.

Down 7: The __________ of two integers may not lead to the formation of another integer.

Down 8: The multiplication of a number by its reciprocal gives __________.

Across 1: There are __________ rational numbers between two integers.

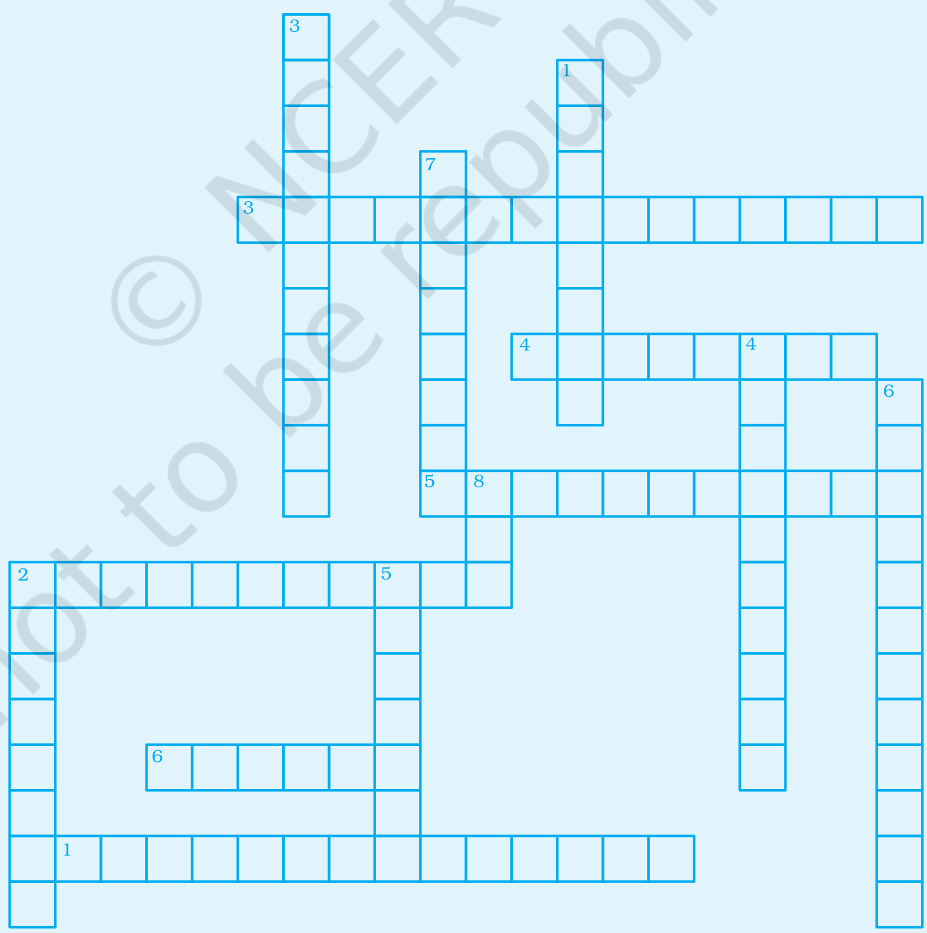
Across 2: The multiplication of rational numbers is __________ commutative and __________.

Across 3: The addition and __________ of two whole numbers lead to the formation of another whole number.

Across 4: All the positive integers excluding 0 are known as __________ numbers.

Across 5: For any rational number a; $a \div 0$ is __________.

Across 6: Rational numbers can be represented on a __________ line.



15-04-2018