

## STRAIGHT LINES

### 10.1 Overview

#### 10.1.1 Slope of a line

If  $\theta$  is the angle made by a line with positive direction of  $x$ -axis in anticlockwise direction, then the value of  $\tan \theta$  is called the **slope of the line** and is denoted by  $m$ .

The slope of a line passing through points  $P(x_1, y_1)$  and  $Q(x_2, y_2)$  is given by

$$m = \tan \theta = \frac{y_2 - y_1}{x_2 - x_1}$$

**10.1.2 Angle between two lines** The angle  $\theta$  between the two lines having slopes  $m_1$  and  $m_2$  is given by

$$\tan \theta = \pm \frac{(m_1 - m_2)}{1 + m_1 m_2}$$

If we take the acute angle between two lines, then  $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

If the lines are parallel, then  $m_1 = m_2$ .

If the lines are perpendicular, then  $m_1 m_2 = -1$ .

**10.1.3 Collinearity of three points** If three points  $P(h, k)$ ,  $Q(x_1, y_1)$  and  $R(x_2, y_2)$

are such that slope of  $PQ$  = slope of  $QR$ , i.e.,  $\frac{y_1 - k}{x_1 - h} = \frac{y_2 - y_1}{x_2 - x_1}$

or  $(h - x_1)(y_2 - y_1) = (k - y_1)(x_2 - x_1)$  then they are said to be collinear.

#### 10.1.4 Various forms of the equation of a line

- (i) If a line is at a distance  $a$  and parallel to  $x$ -axis, then the equation of the line is  $y = \pm a$ .
- (ii) If a line is parallel to  $y$ -axis at a distance  $b$  from  $y$ -axis then its equation is  $x = \pm b$

- (iii) Point-slope form : The equation of a line having slope  $m$  and passing through the point  $(x_0, y_0)$  is given by  $y - y_0 = m(x - x_0)$
- (iv) Two-point-form : The equation of a line passing through two points  $(x_1, y_1)$  and  $(x_2, y_2)$  is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

- (v) Slope intercept form : The equation of the line making an intercept  $c$  on  $y$ -axis and having slope  $m$  is given by

$$y = mx + c$$

Note that the value of  $c$  will be positive or negative as the intercept is made on the positive or negative side of the  $y$ -axis, respectively.

- (vi) Intercept form : The equation of the line making intercepts  $a$  and  $b$  on  $x$ - and  $y$ -axis respectively is given by  $\frac{x}{a} + \frac{y}{b} = 1$ .

- (vii) Normal form : Suppose a non-vertical line is known to us with following data:

- (a) Length of the perpendicular (normal)  $p$  from origin to the line.  
 (b) Angle  $\omega$  which normal makes with the positive direction of  $x$ -axis.

Then the equation of such a line is given by  $x \cos \omega + y \sin \omega = p$

### 10.1.5 General equation of a line

Any equation of the form  $Ax + By + C = 0$ , where  $A$  and  $B$  are simultaneously not zero, is called the general equation of a line.

#### Different forms of $Ax + By + C = 0$

The general form of the line can be reduced to various forms as given below:

- (i) Slope intercept form : If  $B \neq 0$ , then  $Ax + By + C = 0$  can be written as

$$y = \frac{-A}{B}x + \frac{-C}{B} \quad \text{or} \quad y = mx + c, \quad \text{where} \quad m = \frac{-A}{B} \quad \text{and} \quad c = \frac{-C}{B}$$

If  $B = 0$ , then  $x = \frac{-C}{A}$  which is a vertical line whose slope is not defined and  $x$ -intercept

$$\text{is } \frac{-C}{A}.$$

(ii) Intercept form : If  $C \neq 0$ , then  $Ax + By + C = 0$  can be written as  $\frac{x}{\frac{-C}{A}} + \frac{y}{\frac{-C}{B}}$

$$= 1 \text{ or } \frac{x}{a} + \frac{y}{b} = 1, \text{ where } a = \frac{-C}{A} \text{ and } b = \frac{-C}{B}.$$

If  $C = 0$ , then  $Ax + By + C = 0$  can be written as  $Ax + By = 0$  which is a line passing through the origin and therefore has zero intercepts on the axes.

(iii) Normal Form : The normal form of the equation  $Ax + By + C = 0$  is  $x \cos \omega + y \sin \omega = p$  where,

$$\cos \omega = \pm \frac{A}{\sqrt{A^2 + B^2}}, \sin \omega = \pm \frac{B}{\sqrt{A^2 + B^2}} \text{ and } p = \pm \frac{C}{\sqrt{A^2 + B^2}}.$$

**Note:** Proper choice of signs is to be made so that  $p$  should be always positive.

**10.1.6 Distance of a point from a line** The perpendicular distance (or simply distance)  $d$  of a point  $P(x_1, y_1)$  from the line  $Ax + By + C = 0$  is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

#### Distance between two parallel lines

The distance  $d$  between two parallel lines  $y = mx + c_1$  and  $y = mx + c_2$  is given by

$$d = \frac{|c_1 - c_2|}{\sqrt{1 + m^2}}.$$

**10.1.7 Locus and Equation of Locus** The curve described by a point which moves under certain given condition is called its locus. To find the locus of a point  $P$  whose coordinates are  $(h, k)$ , express the condition involving  $h$  and  $k$ . Eliminate variables if any and finally replace  $h$  by  $x$  and  $k$  by  $y$  to get the locus of  $P$ .

**10.1.8 Intersection of two given lines** Two lines  $a_1x + b_1y + c_1 = 0$  and  $a_2x + b_2y + c_2 = 0$  are

(i) intersecting if  $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

- (ii) parallel and distinct if  $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
- (iii) coincident if  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

**Remarks**

- (i) The points  $(x_1, y_1)$  and  $(x_2, y_2)$  are on the same side of the line or on the opposite side of the line  $ax + by + c = 0$ , if  $ax_1 + by_1 + c$  and  $ax_2 + by_2 + c$  are of the same sign or of opposite signs respectively.
- (ii) The condition that the lines  $a_1x + b_1y + c_1 = 0$  and  $a_2x + b_2y + c_2 = 0$  are perpendicular is  $a_1a_2 + b_1b_2 = 0$ .
- (iii) The equation of any line through the point of intersection of two lines  $a_1x + b_1y + c_1 = 0$  and  $a_2x + b_2y + c_2 = 0$  is  $a_1x + b_1y + c_1 + k(ax_2 + by_2 + c_2) = 0$ . The value of  $k$  is determined from extra condition given in the problem.

**10.2 Solved Examples****Short Answer Type**

**Example 1** Find the equation of a line which passes through the point  $(2, 3)$  and makes an angle of  $30^\circ$  with the positive direction of  $x$ -axis.

**Solution** Here the slope of the line is  $m = \tan\theta = \tan 30^\circ = \frac{1}{\sqrt{3}}$  and the given point is  $(2, 3)$ . Therefore, using point slope formula of the equation of a line, we have

$$y - 3 = \frac{1}{\sqrt{3}}(x - 2) \quad \text{or} \quad x - \sqrt{3}y + (3\sqrt{3} - 2) = 0.$$

**Example 2** Find the equation of the line where length of the perpendicular segment from the origin to the line is 4 and the inclination of the perpendicular segment with the positive direction of  $x$ -axis is  $30^\circ$ .

**Solution** The normal form of the equation of the line is  $x \cos \omega + y \sin \omega = p$ . Here  $p = 4$  and  $\omega = 30^\circ$ . Therefore, the equation of the line is

$$x \cos 30^\circ + y \sin 30^\circ = 4$$

$$x \frac{\sqrt{3}}{2} + y \frac{1}{2} = 4 \quad \text{or} \quad \sqrt{3}x + y = 8$$

**Example 3** Prove that every straight line has an equation of the form  $Ax + By + C = 0$ , where  $A$ ,  $B$  and  $C$  are constants.

**Proof** Given a straight line, either it cuts the  $y$ -axis, or is parallel to or coincident with it. We know that the equation of a line which cuts the  $y$ -axis (i.e., it has  $y$ -intercept) can be put in the form  $y = mx + b$ ; further, if the line is parallel to or coincident with the  $y$ -axis, its equation is of the form  $x = x_1$ , where  $x = 0$  in the case of coincidence. Both of these equations are of the form given in the problem and hence the proof.

**Example 4** Find the equation of the straight line passing through  $(1, 2)$  and perpendicular to the line  $x + y + 7 = 0$ .

**Solution** Let  $m$  be the slope of the line whose equation is to be found out which is perpendicular to the line  $x + y + 7 = 0$ . The slope of the given line  $y = (-1)x - 7$  is  $-1$ . Therefore, using the condition of perpendicularity of lines, we have  $m \times (-1) = -1$  or  $m = 1$  (Why?)

Hence, the required equation of the line is  $y - 1 = (1)(x - 2)$  or  $y - 1 = x - 2 \Rightarrow x - y - 1 = 0$ .

**Example 5** Find the distance between the lines  $3x + 4y = 9$  and  $6x + 8y = 15$ .

**Solution** The equations of lines  $3x + 4y = 9$  and  $6x + 8y = 15$  may be rewritten as

$$3x + 4y - 9 = 0 \quad \text{and} \quad 3x + 4y - \frac{15}{2} = 0$$

Since, the slope of these lines are same and hence they are parallel to each other. Therefore, the distance between them is given by

$$\left| \frac{9 - \frac{15}{2}}{\sqrt{3^2 + 4^2}} \right| = \frac{3}{10}$$

**Example 6** Show that the locus of the mid-point of the distance between the axes of the variable line  $x \cos \alpha + y \sin \alpha = p$  is  $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$  where  $p$  is a constant.

**Solution** Changing the given equation of the line into intercept form, we have

$\frac{x}{\frac{p}{\cos \alpha}} + \frac{y}{\frac{p}{\sin \alpha}} = 1$  which gives the coordinates  $\left(\frac{p}{\cos \alpha}, 0\right)$  and  $\left(0, \frac{p}{\sin \alpha}\right)$ , where the

line intersects  $x$ -axis and  $y$ -axis, respectively.

Let  $(h, k)$  denote the mid-point of the line segment joining the points  $\left(\frac{p}{\cos \alpha}, 0\right)$  and  $\left(0, \frac{p}{\sin \alpha}\right)$

Then  $h = \frac{p}{2 \cos \alpha}$  and  $k = \frac{p}{2 \sin \alpha}$  (Why?)

This gives  $\cos \alpha = \frac{p}{2h}$  and  $\sin \alpha = \frac{p}{2k}$

Squaring and adding we get

$$\frac{p^2}{4h^2} + \frac{p^2}{4k^2} = 1 \quad \text{or} \quad \frac{1}{h^2} + \frac{1}{k^2} = \frac{4}{p^2}.$$

Therefore, the required locus is  $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$ .

**Example 7** If the line joining two points  $A(2, 0)$  and  $B(3, 1)$  is rotated about  $A$  in anticlockwise direction through an angle of  $15^\circ$ . Find the equation of the line in new position.

**Solution** The slope of the line  $AB$  is  $\frac{1-0}{3-2} = 1$  or  $\tan 45^\circ$  (Why?) (see Fig.). After rotation of the line through  $15^\circ$ , the slope of the line  $AC$  in new position is  $\tan 60^\circ = \sqrt{3}$

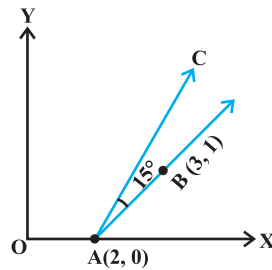


Fig. 10.1

Therefore, the equation of the new line AC is

$$y - 0 = \sqrt{3}(x - 2) \quad \text{or} \quad y - \sqrt{3}x + 2\sqrt{3} = 0$$

### Long Answer Type

**Example 8** If the slope of a line passing through the point A(3, 2) is  $\frac{3}{4}$ , then find points on the line which are 5 units away from the point A.

**Solution** Equation of the line passing through (3, 2) having slope  $\frac{3}{4}$  is given by

$$y - 2 = \frac{3}{4}(x - 3)$$

$$\text{or} \quad 4y - 3x + 1 = 0 \quad (1)$$

Let  $(h, k)$  be the points on the line such that

$$(h - 3)^2 + (k - 2)^2 = 25 \quad (2) \quad (\text{Why?})$$

Also, we have

$$4k - 3h + 1 = 0 \quad (3) \quad (\text{Why?})$$

$$\text{or} \quad k = \frac{3h - 1}{4} \quad (4)$$

Putting the value of  $k$  in (2) and on simplifying, we get

$$25h^2 - 150h - 175 = 0 \quad (\text{How?})$$

$$\text{or} \quad h^2 - 6h - 7 = 0$$

$$\text{or} \quad (h + 1)(h - 7) = 0 \Rightarrow h = -1, h = 7$$

Putting these values of  $k$  in (4), we get  $k = -1$  and  $k = 5$ . Therefore, the coordinates of the required points are either  $(-1, -1)$  or  $(7, 5)$ .

**Example 9** Find the equation to the straight line passing through the point of intersection of the lines  $5x - 6y - 1 = 0$  and  $3x + 2y + 5 = 0$  and perpendicular to the line  $3x - 5y + 11 = 0$ .

**Solution** First we find the point of intersection of lines  $5x - 6y - 1 = 0$  and  $3x + 2y + 5 = 0$  which is  $(-1, -1)$ . Also the slope of the line  $3x - 5y + 11 = 0$  is  $\frac{3}{5}$ . Therefore, the slope of the line perpendicular to this line is  $-\frac{5}{3}$  (Why?). Hence, the equation of the required line is given by

$$y + 1 = \frac{-5}{3}(x + 1)$$

or  $5x + 3y + 8 = 0$

**Alternatively** The equation of any line through the intersection of lines  $5x - 6y - 1 = 0$  and  $3x + 2y + 5 = 0$  is

$$5x - 6y - 1 + k(3x + 2y + 5) = 0 \quad (1)$$

or Slope of this line is  $\frac{-(5+3k)}{-6+2k}$

Also, slope of the line  $3x - 5y + 11 = 0$  is  $\frac{3}{5}$

Now, both are perpendicular

$$\text{so } \frac{-(5+3k)}{-6+2k} \times \frac{3}{5} = -1$$

or  $k = 45$

Therefore, equation of required line is given by

$$5x - 6y - 1 + 45(3x + 2y + 5) = 0$$

or  $5x + 3y + 8 = 0$

**Example 10** A ray of light coming from the point  $(1, 2)$  is reflected at a point  $A$  on the  $x$ -axis and then passes through the point  $(5, 3)$ . Find the coordinates of the point  $A$ .

**Solution** Let the incident ray strike  $x$ -axis at the point  $A$  whose coordinates be  $(x, 0)$ . From the figure, the slope of the reflected ray is given by

$$\tan \theta = \frac{3}{5-x} \quad (1)$$

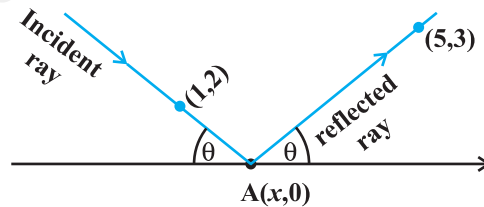


Fig. 10.2



Again, the slope of the incident ray is given by

$$\tan(\pi - \theta) = \frac{-2}{x-1} \quad (\text{Why?})$$

or 
$$-\tan \theta = \frac{-2}{x-1} \quad (2)$$

Solving (1) and (2), we get

$$\frac{3}{5-x} = \frac{2}{x-1} \quad \text{or} \quad x = \frac{13}{5}$$

Therefore, the required coordinates of the point A are  $\left(\frac{13}{5}, 0\right)$ .

**Example 11** If one diagonal of a square is along the line  $8x - 15y = 0$  and one of its vertex is at  $(1, 2)$ , then find the equation of sides of the square passing through this vertex.

**Solution** Let ABCD be the given square and the coordinates of the vertex D be  $(1, 2)$ . We are required to find the equations of its sides DC and AD.

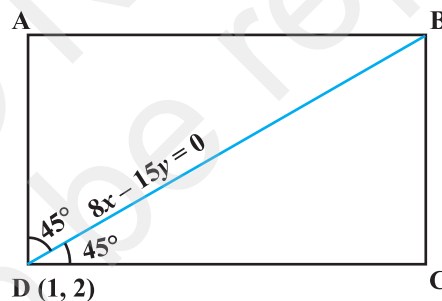


Fig. 10.3

Given that BD is along the line  $8x - 15y = 0$ , so its slope is  $\frac{8}{15}$  (Why?). The angles made by BD with sides AD and DC is  $45^\circ$  (Why?). Let the slope of DC be  $m$ . Then

$$\tan 45^\circ = \frac{m - \frac{8}{15}}{1 + \frac{8m}{15}} \quad (\text{Why?})$$

or  $15 + 8m = 15m - 8$

or  $7m = 23$ , which gives  $m = \frac{23}{7}$

Therefore, the equation of the side DC is given by

$$y - 2 = \frac{23}{7}(x - 1) \text{ or } 23x - 7y - 9 = 0.$$

Similarly, the equation of another side AD is given by

$$y - 2 = \frac{-7}{23}(x - 1) \text{ or } 7x + 23y - 53 = 0.$$

### Objective Type Questions

Each of the Examples 12 to 20 has four possible options out of which only one option is correct. Choose the correct option (M.C.Q.).

**Example 12** The inclination of the line  $x - y + 3 = 0$  with the positive direction of  $x$ -axis is

- (A)  $45^\circ$                       (B)  $135^\circ$                       (C)  $-45^\circ$                       (D)  $-135^\circ$

**Solution** (A) is the correct answer. The equation of the line  $x - y + 3 = 0$  can be rewritten as  $y = x + 3 \Rightarrow m = \tan \theta = 1$  and hence  $\theta = 45^\circ$ .

**Example 13** The two lines  $ax + by = c$  and  $a'x + b'y = c'$  are perpendicular if

- (A)  $aa' + bb' = 0$                       (B)  $ab' = ba'$   
(C)  $ab + a'b' = 0$                       (D)  $ab' + ba' = 0$

**Solution** (A) is correct answer. Slope of the line  $ax + by = c$  is  $\frac{-a}{b}$ ,

and the slope of the line  $a'x + b'y = c'$  is  $\frac{-a'}{b'}$ . The lines are perpendicular if

$$\tan \theta = \frac{3}{5-x} \tag{1}$$

$$\left(\frac{-a}{b}\right)\left(\frac{-a'}{b'}\right) = -1 \text{ or } aa' + bb' = 0 \quad (\text{Why?})$$

**Example 14** The equation of the line passing through (1, 2) and perpendicular to  $x + y + 7 = 0$  is

- (A)  $y - x + 1 = 0$                       (B)  $y - x - 1 = 0$

(C)  $y - x + 2 = 0$

(D)  $y - x - 2 = 0$ .

**Solution** (B) is the correct answer. Let the slope of the line be  $m$ . Then, its equation passing through  $(1, 2)$  is given by

$$y - 2 = m(x - 1) \quad (1)$$

Again, this line is perpendicular to the given line  $x + y + 7 = 0$  whose slope is  $-1$  (Why?)

Therefore, we have  $m(-1) = -1$

or  $m = 1$

Hence, the required equation of the line is obtained by putting the value of  $m$  in (1), i.e.,

$$y - 2 = x - 1$$

or  $y - x - 1 = 0$

**Example 15** The distance of the point  $P(1, -3)$  from the line  $2y - 3x = 4$  is

- (A) 13      (B)  $\frac{7\sqrt{13}}{13}$       (C)  $\sqrt{13}$       (D) None of these

**Solution** (A) is the correct answer. The distance of the point  $P(1, -3)$  from the line  $2y - 3x - 4 = 0$  is the length of perpendicular from the point to the line which is given by

$$\left| \frac{2(-3) - 3 - 4}{\sqrt{13}} \right| = \sqrt{13}$$

**Example 16** The coordinates of the foot of the perpendicular from the point  $(2, 3)$  on the line  $x + y - 11 = 0$  are

- (A)  $(-6, 5)$       (B)  $(5, 6)$       (C)  $(-5, 6)$       (D)  $(6, 5)$

**Solution** (B) is the correct choice. Let  $(h, k)$  be the coordinates of the foot of the perpendicular from the point  $(2, 3)$  on the line  $x + y - 11 = 0$ . Then, the slope of the

perpendicular line is  $\frac{k-3}{h-2}$ . Again the slope of the given line  $x + y - 11 = 0$  is  $-1$

(why?)

Using the condition of perpendicularity of lines, we have

$$\left( \frac{k-3}{h-2} \right) (-1) = -1 \quad (\text{Why?})$$

or  $k - h = 1 \quad (1)$

Since  $(h, k)$  lies on the given line, we have,

$$h + k - 11 = 0 \text{ or } h + k = 11 \quad (2)$$

Solving (1) and (2), we get  $h = 5$  and  $k = 6$ . Thus  $(5, 6)$  are the required coordinates of the foot of the perpendicular.

**Example 17** The intercept cut off by a line from  $y$ -axis is twice than that from  $x$ -axis, and the line passes through the point  $(1, 2)$ . The equation of the line is

- (A)  $2x + y = 4$  (B)  $2x + y + 4 = 0$   
 (C)  $2x - y = 4$  (D)  $2x - y + 4 = 0$

**Solution** (A) is the correct choice. Let the line make intercept ' $a$ ' on  $x$ -axis. Then, it makes intercept ' $2a$ ' on  $y$ -axis. Therefore, the equation of the line is given by

$$\frac{x}{a} + \frac{y}{2a} = 1$$

It passes through  $(1, 2)$ , so, we have

$$\frac{1}{a} + \frac{2}{2a} = 1 \text{ or } a = 2$$

Therefore, the required equation of the line is given by

$$\frac{x}{2} + \frac{y}{4} = 1 \text{ or } 2x + y = 4$$

**Example 18** A line passes through  $P(1, 2)$  such that its intercept between the axes is bisected at  $P$ . The equation of the line is

- (A)  $x + 2y = 5$  (B)  $x - y + 1 = 0$   
 (C)  $x + y - 3 = 0$  (D)  $2x + y - 4 = 0$

**Solution** The correct choice is (D). We know that the equation of a line making intercepts  $a$  and  $b$  with  $x$ -axis and  $y$ -axis, respectively, is given by

$$\frac{x}{a} + \frac{y}{b} = 1.$$

Here we have  $1 = \frac{a+0}{2}$  and  $2 = \frac{0+b}{2}$ , (Why?)

which give  $a = 2$  and  $b = 4$ . Therefore, the required equation of the line is given by

$$\frac{x}{2} + \frac{y}{4} = 1 \quad \text{or} \quad 2x + y - 4 = 0$$

**Example 19** The reflection of the point  $(4, -13)$  about the line  $5x + y + 6 = 0$  is

- (A)  $(-1, -14)$       (B)  $(3, 4)$       (C)  $(0, 0)$       (D)  $(1, 2)$

**Solution** The correct choice is (A). Let  $(h, k)$  be the point of reflection of the given point  $(4, -13)$  about the line  $5x + y + 6 = 0$ . The mid-point of the line segment joining points  $(h, k)$  and  $(4, -13)$  is given by

$$\left( \frac{h+4}{2}, \frac{k-13}{2} \right) \quad (\text{Why?})$$

This point lies on the given line, so we have

$$5\left(\frac{h+4}{2}\right) + \frac{k-13}{2} + 6 = 0$$

$$\text{or} \quad 5h + k + 19 = 0 \quad (1)$$

Again the slope of the line joining points  $(h, k)$  and  $(4, -13)$  is given by  $\frac{k+13}{h-4}$ . This line

is perpendicular to the given line and hence  $(-5)\left(\frac{k+13}{h-4}\right) = -1$       (Why?)

This gives  $5k + 65 = h - 4$

$$\text{or} \quad h - 5k - 69 = 0 \quad (2)$$

On solving (1) and (2), we get  $h = -1$  and  $k = -14$ . Thus the point  $(-1, -14)$  is the reflection of the given point.

**Example 20** A point moves such that its distance from the point  $(4, 0)$  is half that of its distance from the line  $x = 16$ . The locus of the point is

- (A)  $3x^2 + 4y^2 = 192$       (B)  $4x^2 + 3y^2 = 192$   
 (C)  $x^2 + y^2 = 192$       (D) None of these

**Solution** The correct choice is (A). Let  $(h, k)$  be the coordinates of the moving point. Then, we have

$$\sqrt{(h-4)^2 + k^2} = \frac{1}{2} \left( \frac{h-16}{\sqrt{1^2 + 0}} \right) \quad (\text{Why?})$$

$$\Rightarrow (h - 4)^2 + k^2 = \frac{1}{4} (h - 16)^2$$

$$4(h^2 - 8h + 16 + k^2) = h^2 - 32h + 256$$

$$\text{or } 3h^2 + 4k^2 = 192$$

Hence, the required locus is given by  $3x^2 + 4y^2 = 192$

### 10.3 EXERCISE

#### Short Answer Type Questions

- Find the equation of the straight line which passes through the point  $(1, -2)$  and cuts off equal intercepts from axes.
- Find the equation of the line passing through the point  $(5, 2)$  and perpendicular to the line joining the points  $(2, 3)$  and  $(3, -1)$ .
- Find the angle between the lines  $y = (2 - \sqrt{3})(x + 5)$  and  $y = (2 + \sqrt{3})(x - 7)$ .
- Find the equation of the lines which passes through the point  $(3, 4)$  and cuts off intercepts from the coordinate axes such that their sum is 14.
- Find the points on the line  $x + y = 4$  which lie at a unit distance from the line  $4x + 3y = 10$ .
- Show that the tangent of an angle between the lines  $\frac{x}{a} + \frac{y}{b} = 1$  and  $\frac{x}{a} - \frac{y}{b} = 1$  is  $\frac{2ab}{a^2 - b^2}$ .
- Find the equation of lines passing through  $(1, 2)$  and making angle  $30^\circ$  with  $y$ -axis.
- Find the equation of the line passing through the point of intersection of  $2x + y = 5$  and  $x + 3y + 8 = 0$  and parallel to the line  $3x + 4y = 7$ .
- For what values of  $a$  and  $b$  the intercepts cut off on the coordinate axes by the line  $ax + by + 8 = 0$  are equal in length but opposite in signs to those cut off by the line  $2x - 3y + 6 = 0$  on the axes.
- If the intercept of a line between the coordinate axes is divided by the point  $(-5, 4)$  in the ratio  $1 : 2$ , then find the equation of the line.
- Find the equation of a straight line on which length of perpendicular from the origin is four units and the line makes an angle of  $120^\circ$  with the positive direction of  $x$ -axis.

[**Hint:** Use normal form, here  $\omega = 30^\circ$ .]

12. Find the equation of one of the sides of an isosceles right angled triangle whose hypotenuse is given by  $3x + 4y = 4$  and the opposite vertex of the hypotenuse is  $(2, 2)$ .

### Long Answer Type

13. If the equation of the base of an equilateral triangle is  $x + y = 2$  and the vertex is  $(2, -1)$ , then find the length of the side of the triangle.

[**Hint:** Find length of perpendicular ( $p$ ) from  $(2, -1)$  to the line and use  $p = l \sin 60^\circ$ , where  $l$  is the length of side of the triangle].

14. A variable line passes through a fixed point  $P$ . The algebraic sum of the perpendiculars drawn from the points  $(2, 0)$ ,  $(0, 2)$  and  $(1, 1)$  on the line is zero. Find the coordinates of the point  $P$ .

[**Hint:** Let the slope of the line be  $m$ . Then the equation of the line passing through the fixed point  $P(x_1, y_1)$  is  $y - y_1 = m(x - x_1)$ . Taking the algebraic sum of perpendicular distances equal to zero, we get  $y - 1 = m(x - 1)$ . Thus  $(x_1, y_1)$  is  $(1, 1)$ .]

15. In what direction should a line be drawn through the point  $(1, 2)$  so that its point of intersection with the line  $x + y = 4$  is at a distance  $\frac{\sqrt{6}}{3}$  from the given point.

16. A straight line moves so that the sum of the reciprocals of its intercepts made on axes is constant. Show that the line passes through a fixed point.

[**Hint:**  $\frac{x}{a} + \frac{y}{b} = 1$  where  $\frac{1}{a} + \frac{1}{b} = \text{constant} = \frac{1}{k}$  (say). This implies that

$\frac{k}{a} + \frac{k}{b} = 1 \Rightarrow$  line passes through the fixed point  $(k, k)$ .]

17. Find the equation of the line which passes through the point  $(-4, 3)$  and the portion of the line intercepted between the axes is divided internally in the ratio  $5 : 3$  by this point.

18. Find the equations of the lines through the point of intersection of the lines  $x - y + 1 = 0$  and  $2x - 3y + 5 = 0$  and whose distance from the point  $(3, 2)$  is  $\frac{7}{5}$ .

19. If the sum of the distances of a moving point in a plane from the axes is 1, then find the locus of the point. [**Hint:** Given that  $|x| + |y| = 1$ , which gives four sides of a square.]

20.  $P_1, P_2$  are points on either of the two lines  $y - \sqrt{3}|x| = 2$  at a distance of 5 units from their point of intersection. Find the coordinates of the foot of perpendiculars drawn from  $P_1, P_2$  on the bisector of the angle between the given lines.

[Hint: Lines are  $y = \sqrt{3}x + 2$  and  $y = -\sqrt{3}x + 2$  according as  $x \geq 0$  or  $x < 0$ . y-axis is the bisector of the angles between the lines.  $P_1, P_2$  are the points on these lines at a distance of 5 units from the point of intersection of these lines which have a point on y-axis as common foot of perpendiculars from these points. The y-coordinate of the foot of the perpendicular is given by  $2 + 5 \cos 30^\circ$ .]

21. If  $p$  is the length of perpendicular from the origin on the line  $\frac{x}{a} + \frac{y}{b} = 1$  and  $a^2, p^2, b^2$  are in A.P, then show that  $a^4 + b^4 = 0$ .

### Objective Type Questions

Choose the correct answer from the given four options in Exercises 22 to 41

22. A line cutting off intercept  $-3$  from the y-axis and the tangent at angle to the x-axis is  $\frac{3}{5}$ , its equation is
- (A)  $5y - 3x + 15 = 0$  (B)  $3y - 5x + 15 = 0$   
 (C)  $5y - 3x - 15 = 0$  (D) None of these
23. Slope of a line which cuts off intercepts of equal lengths on the axes is
- (A)  $-1$  (B)  $-0$   
 (C)  $2$  (D)  $\sqrt{3}$
24. The equation of the straight line passing through the point  $(3, 2)$  and perpendicular to the line  $y = x$  is
- (A)  $x - y = 5$  (B)  $x + y = 5$   
 (C)  $x + y = 1$  (D)  $x - y = 1$
25. The equation of the line passing through the point  $(1, 2)$  and perpendicular to the line  $x + y + 1 = 0$  is
- (A)  $y - x + 1 = 0$  (B)  $y - x - 1 = 0$   
 (C)  $y - x + 2 = 0$  (D)  $y - x - 2 = 0$
26. The tangent of angle between the lines whose intercepts on the axes are  $a, -b$  and  $b, -a$ , respectively, is



- (A)  $\frac{a^2 - b^2}{ab}$  (B)  $\frac{b^2 - a^2}{2}$   
 (C)  $\frac{b^2 - a^2}{2ab}$  (D) None of these

27. If the line  $\frac{x}{a} + \frac{y}{b} = 1$  passes through the points  $(2, -3)$  and  $(4, -5)$ , then  $(a, b)$  is  
 (A)  $(1, 1)$  (B)  $(-1, 1)$  (C)  $(1, -1)$  (D)  $(-1, -1)$
28. The distance of the point of intersection of the lines  $2x - 3y + 5 = 0$  and  $3x + 4y = 0$  from the line  $5x - 2y = 0$  is  
 (A)  $\frac{130}{17\sqrt{29}}$  (B)  $\frac{13}{7\sqrt{29}}$  (C)  $\frac{130}{7}$  (D) None of these
29. The equations of the lines which pass through the point  $(3, -2)$  and are inclined at  $60^\circ$  to the line  $\sqrt{3}x + y = 1$  is  
 (A)  $y + 2 = 0, \sqrt{3}x - y - 2 - 3\sqrt{3} = 0$   
 (B)  $x - 2 = 0, \sqrt{3}x - y + 2 + 3\sqrt{3} = 0$   
 (C)  $\sqrt{3}x - y - 2 - 3\sqrt{3} = 0$   
 (D) None of these
30. The equations of the lines passing through the point  $(1, 0)$  and at a distance  $\frac{\sqrt{3}}{2}$  from the origin, are  
 (A)  $\sqrt{3}x + y - \sqrt{3} = 0, \sqrt{3}x - y - \sqrt{3} = 0$   
 (B)  $\sqrt{3}x + y + \sqrt{3} = 0, \sqrt{3}x - y + \sqrt{3} = 0$   
 (C)  $x + \sqrt{3}y - \sqrt{3} = 0, x - \sqrt{3}y - \sqrt{3} = 0$   
 (D) None of these.
31. The distance between the lines  $y = mx + c_1$  and  $y = mx + c_2$  is  
 (A)  $\frac{c_1 - c_2}{\sqrt{m^2 + 1}}$  (B)  $\frac{|c_1 - c_2|}{\sqrt{1 + m^2}}$  (C)  $\frac{c_2 - c_1}{\sqrt{1 + m^2}}$  (D) 0

32. The coordinates of the foot of perpendiculars from the point (2, 3) on the line  $y = 3x + 4$  is given by
- (A)  $\left(\frac{37}{10}, \frac{-1}{10}\right)$  (B)  $\left(\frac{-1}{10}, \frac{37}{10}\right)$  (C)  $\left(\frac{10}{37}, -10\right)$  (D)  $\left(\frac{2}{3}, -\frac{1}{3}\right)$
33. If the coordinates of the middle point of the portion of a line intercepted between the coordinate axes is (3, 2), then the equation of the line will be
- (A)  $2x + 3y = 12$  (B)  $3x + 2y = 12$  (C)  $4x - 3y = 6$  (D)  $5x - 2y = 10$
34. Equation of the line passing through (1, 2) and parallel to the line  $y = 3x - 1$  is
- (A)  $y + 2 = x + 1$  (B)  $y + 2 = 3(x + 1)$   
(C)  $y - 2 = 3(x - 1)$  (D)  $y - 2 = x - 1$
35. Equations of diagonals of the square formed by the lines  $x = 0, y = 0, x = 1$  and  $y = 1$  are
- (A)  $y = x, y + x = 1$  (B)  $y = x, x + y = 2$   
(C)  $2y = x, y + x = \frac{1}{3}$  (D)  $y = 2x, y + 2x = 1$
36. For specifying a straight line, how many geometrical parameters should be known?
- (A) 1 (B) 2 (C) 4 (D) 3
37. The point (4, 1) undergoes the following two successive transformations :
- (i) Reflection about the line  $y = x$   
(ii) Translation through a distance 2 units along the positive  $x$ -axis
- Then the final coordinates of the point are
- (A) (4, 3) (B) (3, 4) (C) (1, 4) (D)  $\left(\frac{7}{2}, \frac{7}{2}\right)$
38. A point equidistant from the lines  $4x + 3y + 10 = 0, 5x - 12y + 26 = 0$  and  $7x + 24y - 50 = 0$  is
- (A) (1, -1) (B) (1, 1) (C) (0, 0) (D) (0, 1)
39. A line passes through (2, 2) and is perpendicular to the line  $3x + y = 3$ . Its  $y$ -intercept is
- (A)  $\frac{1}{3}$  (B)  $\frac{2}{3}$  (C) 1 (D)  $\frac{4}{3}$

40. The ratio in which the line  $3x + 4y + 2 = 0$  divides the distance between the lines  $3x + 4y + 5 = 0$  and  $3x + 4y - 5 = 0$  is  
 (A) 1 : 2            (B) 3 : 7            (C) 2 : 3            (D) 2 : 5
41. One vertex of the equilateral triangle with centroid at the origin and one side as  $x + y - 2 = 0$  is  
 (A)  $(-1, -1)$       (B)  $(2, 2)$             (C)  $(-2, -2)$       (D)  $(2, -2)$
- [Hint: Let ABC be the equilateral triangle with vertex A  $(h, k)$  and let D  $(\alpha, \beta)$  be the point on BC. Then  $\frac{2\alpha + h}{3} = 0 = \frac{2\beta + k}{3}$ . Also  $\alpha + \beta - 2 = 0$  and

$$\left(\frac{k-0}{h-0}\right) \times (-1) = -1].$$

Fill in the blank in Exercises 42 to 47.

42. If  $a, b, c$  are in A.P., then the straight lines  $ax + by + c = 0$  will always pass through \_\_\_\_.
43. The line which cuts off equal intercept from the axes and pass through the point  $(1, -2)$  is \_\_\_\_.
44. Equations of the lines through the point  $(3, 2)$  and making an angle of  $45^\circ$  with the line  $x - 2y = 3$  are \_\_\_\_.
45. The points  $(3, 4)$  and  $(2, -6)$  are situated on the \_\_\_\_ of the line  $3x - 4y - 8 = 0$ .
46. A point moves so that square of its distance from the point  $(3, -2)$  is numerically equal to its distance from the line  $5x - 12y = 3$ . The equation of its locus is \_\_\_\_.
47. Locus of the mid-points of the portion of the line  $x \sin \theta + y \cos \theta = p$  intercepted between the axes is \_\_\_\_.

State whether the statements in Exercises 48 to 56 are true or false. Justify.

48. If the vertices of a triangle have integral coordinates, then the triangle can not be equilateral.
49. The points A  $(-2, 1)$ , B  $(0, 5)$ , C  $(-1, 2)$  are collinear.
50. Equation of the line passing through the point  $(a \cos^3 \theta, a \sin^3 \theta)$  and perpendicular to the line  $x \sec \theta + y \operatorname{cosec} \theta = a$  is  $x \cos \theta - y \sin \theta = a \sin 2\theta$ .
51. The straight line  $5x + 4y = 0$  passes through the point of intersection of the straight lines  $x + 2y - 10 = 0$  and  $2x + y + 5 = 0$ .
52. The vertex of an equilateral triangle is  $(2, 3)$  and the equation of the opposite side is  $x + y = 2$ . Then the other two sides are  $y - 3 = (2 \pm \sqrt{3})(x - 2)$ .

- 53.** The equation of the line joining the point (3, 5) to the point of intersection of the lines  $4x + y - 1 = 0$  and  $7x - 3y - 35 = 0$  is equidistant from the points (0, 0) and (8, 34).
- 54.** The line  $\frac{x}{a} + \frac{y}{b} = 1$  moves in such a way that  $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$ , where  $c$  is a constant. The locus of the foot of the perpendicular from the origin on the given line is  $x^2 + y^2 = c^2$ .
- 55.** The lines  $ax + 2y + 1 = 0$ ,  $bx + 3y + 1 = 0$  and  $cx + 4y + 1 = 0$  are concurrent if  $a, b, c$  are in G.P.
- 56.** Line joining the points (3, -4) and (-2, 6) is perpendicular to the line joining the points (-3, 6) and (9, -18).

Match the questions given under Column  $C_1$  with their appropriate answers given under the Column  $C_2$  in Exercises 57 to 59.

**57.**

**Column  $C_1$**

- (a) The coordinates of the points P and Q on the line  $x + 5y = 13$  which are at a distance of 2 units from the line  $12x - 5y + 26 = 0$  are
- (b) The coordinates of the point on the line  $x + y = 4$ , which are at a unit distance from the line  $4x + 3y - 10 = 0$  are
- (c) The coordinates of the point on the line joining A(-2, 5) and B(3, 1) such that  $AP = PQ = QB$  are

**Column  $C_2$**

- (i) (3, 1), (-7, 11)
- (ii)  $\left(-\frac{1}{3}, \frac{11}{3}\right), \left(\frac{4}{3}, \frac{7}{3}\right)$
- (iii)  $\left(1, \frac{12}{5}\right), \left(-3, \frac{16}{5}\right)$

- 58.** The value of the  $\lambda$ , if the lines  $(2x + 3y + 4) + \lambda(6x - y + 12) = 0$  are

**Column  $C_1$**

- (a) parallel to y-axis is

**Column  $C_2$**

- (i)  $\lambda = -\frac{3}{4}$

- (b) perpendicular to  $7x + y - 4 = 0$  is (ii)  $\lambda = -\frac{1}{3}$
- (c) passes through  $(1, 2)$  is (iii)  $\lambda = -\frac{17}{41}$
- (d) parallel to  $x$  axis is (iv)  $\lambda = 3$

**59.** The equation of the line through the intersection of the lines  $2x - 3y = 0$  and  $4x - 5y = 2$  and

**Column C<sub>1</sub>**

- (a) through the point  $(2, 1)$  is
- (b) perpendicular to the line  $x + 2y + 1 = 0$  is
- (c) parallel to the line  $3x - 4y + 5 = 0$  is
- (d) equally inclined to the axes is

**Column C<sub>2</sub>**

- (i)  $2x - y = 4$
- (ii)  $x + y - 5 = 0$
- (iii)  $x - y - 1 = 0$
- (iv)  $3x - 4y - 1 = 0$



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