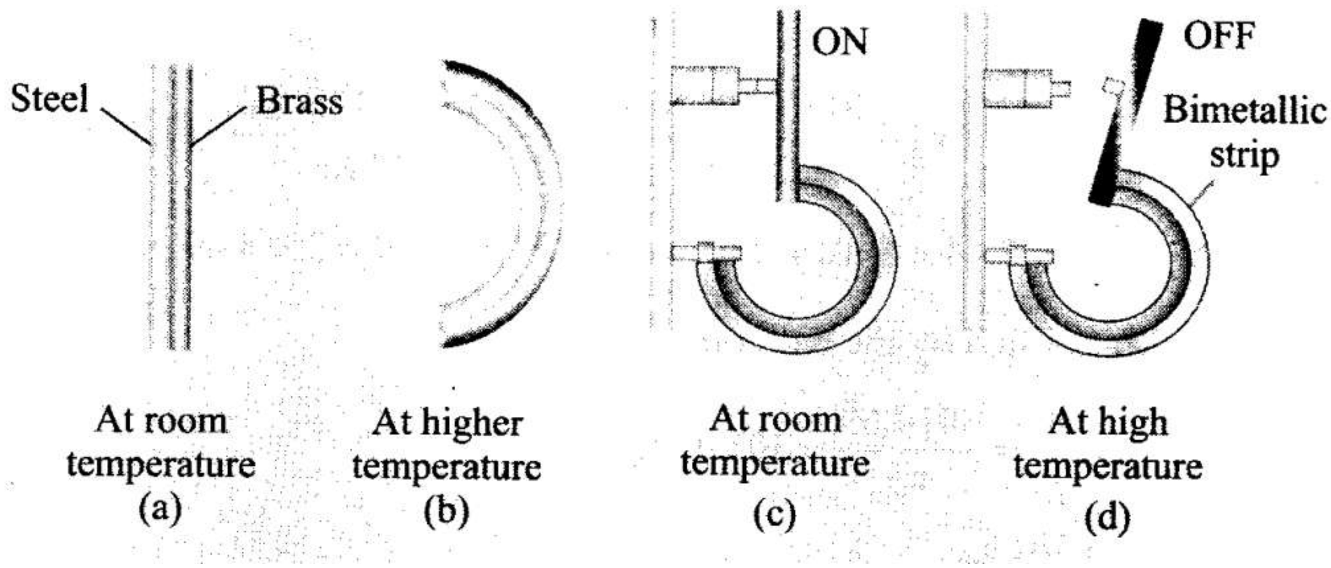
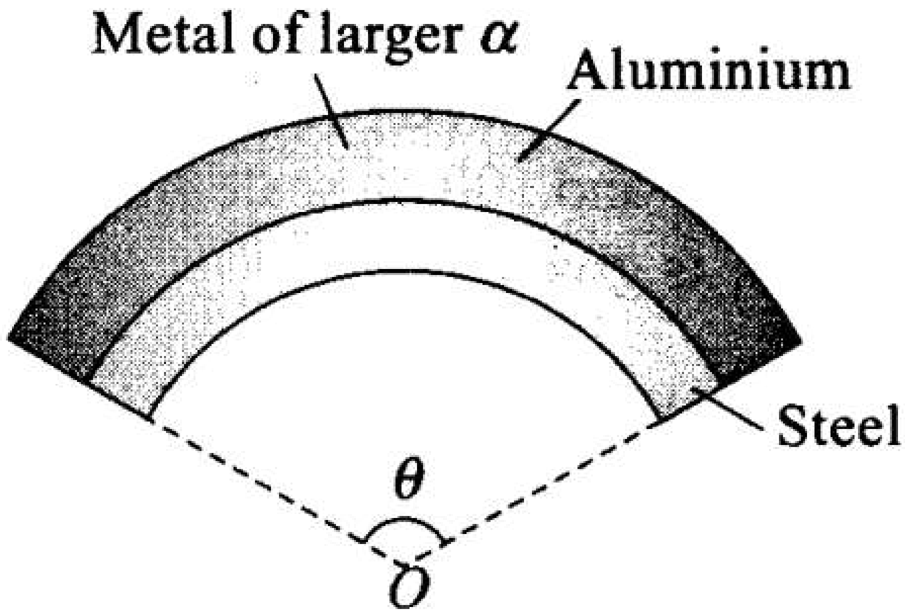
Q1. A bimetallic strip is made of aluminium and steel (aAl > asteel) On heating, the strip will
(a) remain straight (bj get twisted
(c) will bend with aluminium on concave side.
(d) will bend with steel on concave side
Sol: (d)
Key concept: Bi-metallic strip-. Two strips of equal lengths but of different materials (different coefficient of linear expansion) when join together, it is called "bi-metallic strip", and can be used in thermostat to break or make electrical contact. This strip has the characteristic property of bending on heating due to unequal linear expansion of the two metal. The strip will bend with metal of greater a on outer side, i.e. convex side.

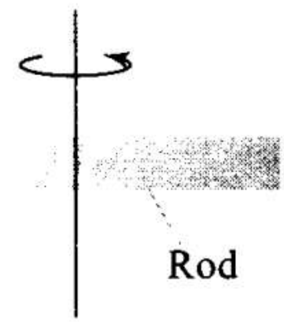


On heating, the metallic strip with higher coefficient of linear expansion ($\propto_{Al}$) will expand more.
According to the question, $\propto_{Al} > \propto_{steel}$, so aluminum will expand more. So, it should have larger radius of curvature. Hence, aluminium will be on convex side. The metal of smaller $\propto$ (i.e., steel) bends on inner side, i.e., concave side.



Q2. A uniform metallic rod rotates about its perpendicular bisector with constant angular speed. If it is heated uniformly to raise its temperature slightly
(a) its speed of rotation increases
(b) its speed of rotation decreases
(c) its speed of rotation remains same
(d) its speed increases because its moment of inertia increases
Sol: (b) When the rod is heated uniformly to raise its temperature slightly, it expands. So, moment of inertia of the rod will increase.
Moment of inertia of a uniform rod about its perpendicular bisector

Moment of inertia of a uniform rod about its perpendicular bisector,



$$I = \frac{1}{12} ML^2$$

ΔT = Increase in the temperature of the rod.

$\therefore$ Changed length, $L' = L(1 + \alpha\Delta T)$ ...(i)

$\therefore$ New moment of inertia of rod,

$$I' = \frac{ML'^2}{12} = \frac{M}{12} L[1 + \alpha\Delta T]^2$$

$$= \frac{ML^2}{12}[1 + 2\alpha\Delta T + \alpha^2(\Delta T)^2]$$

[Using (i)]

$\therefore$ $I' = I[1 + 2\alpha\Delta T]$ $(\because \alpha^2(\Delta T)^2$ is very small)

If the temperature increases, moment of inertia will increase.

No external torque is acting on the system, so angular momentum should be conserved.

$$L = \text{Angular momentum} = I\omega = \text{constant}$$

$\Rightarrow$ $I'\omega' = I\omega$

where, I = initial M.I. of rod before heating

ω = initial angular velocity

I' = final M.I after heating

ω' = final angular velocity after heating

Due to expansion of the rod $I' > I$.

$\Rightarrow$ $\frac{\omega'}{\omega} = \frac{I}{I'} < 1$

$\Rightarrow$ $\omega' < \omega$

So, angular velocity (speed of rotation) decreases.

Q3. The graph between two temperature scales A and B is shown in figure between upper fixed point and lower fixed point there are 150 equal division on scale A and 100 on scale B. The relationship for

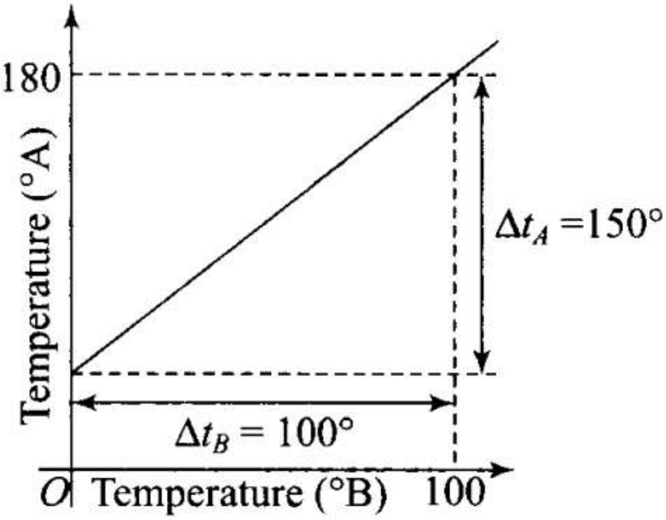
conversion between the two scales is given by

(a) $\frac{t_A - 180}{100} = \frac{t_B}{150}$

(b) $\frac{t_A - 30}{150} = \frac{t_B}{100}$

(c) $\frac{t_B - 180}{150} = \frac{t_A}{100}$

(d) $\frac{t_B - 40}{100} = \frac{t_A}{180}$



Sol: Key concept: Temperature on one scale can be converted into other scale by using the following identity.

Reading on any scale – LFP /UFP – LFP = Constant for all scales

where, LFP —> Lower fixed point

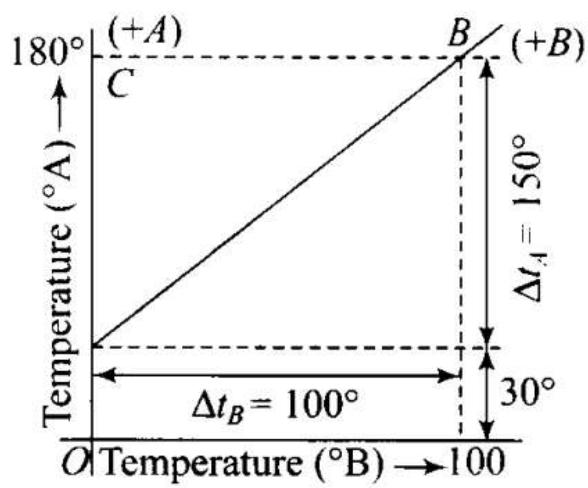
UFP —>Upper fixed point

From the graph it is clear that the lowest point for scale A is 30° and highest point for the scale A is 180°. Lowest point for scale B is 0° and highest point for scale B is 100°. Hence, the relation between the two scales A and B is given by

$$\frac{T_A - (\text{LFP})_A}{(\text{UFP})_A - (\text{LFP})_A} = \frac{T_B - (\text{LFP})_B}{(\text{UFP})_B - (\text{LFP})_B}$$

$$\Rightarrow \quad \frac{T_A - 30}{180 - 30} = \frac{T_B - 0}{100 - 0}$$

$$\Rightarrow \quad \frac{t_A - 30}{150} = \frac{t_B}{100}$$



Q4. An aluminium sphere is dipped into water.

Which of the following is true?

(a) Buoyancy will be less in water at 0°C than that in water at 4°C.

(b) Buoyancy will be more in water at 0°C than that in water at 4°C.

(c) Buoyancy in water at 0°C will be same as that in water at 4°C.

(d) Buoyancy may be more or less in water at 4°C depending on the radius of the sphere.

Sol: (a)

Key concept: Liquids generally increase in volume with increasing temperature but in case of water, it expands on heating if its temperature is greater than 4°C. The density of water reaches a maximum value of 1.000 g/cm3 at 4°C.

This behaviour of water in the range from 0°C to 4°C is called anomalous expansion.

Let volume of the sphere be V and ρ be its density, then we can write buoyant force at 0°C.

$$F_{0°C} = V\rho_{0°}g \quad (\rho_{0°} = \text{density at } 0°C)$$

Buoyancy at 4°C

$$F_{4°C} = V\rho_{4°}g$$

$$\Rightarrow \quad \frac{F_{4°C}}{F_{0°C}} = \frac{\rho_{4°C}}{\rho_{0°C}} > 1 \qquad (\because \rho_{4°C} > \rho_{0°C})$$

$$\Rightarrow \quad F_{4°C} > F_{0°C}$$

Hence, buoyancy will be less in water at 0°C than that in water at 4°C.

Q5. As the temperature is increased, the period of a pendulum

(a) increases as its effective length increases even though its centre of mass still remains at the centre of the bob

(b) decreases as its effective length increases even though its centre of mass ‘ still remains at the centre of the bob

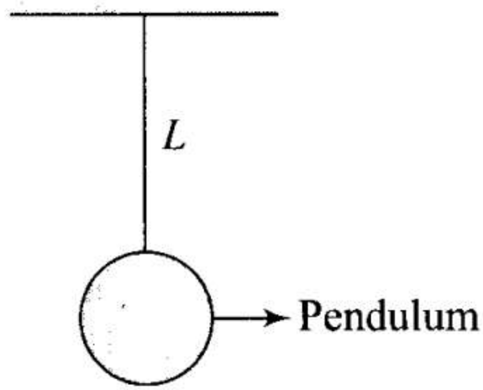
(c) increases as its effective length increases due to shifting to centre of mass below the centre of the bob

(d) decreases as its effective length remains same but the centre of mass shifts above the centre of the bob

Sol: (a) A pendulum clock keeps proper time at temperature θ_0. If temperature is increased to θ ($>\theta_0$), then due to linear expansion, length of pendulum increases and hence its time period will increase

Let $T = 2\pi\sqrt{\frac{L_0}{g}}$ at temperature θ_0 and

$T' = 2\pi\sqrt{\frac{L}{g}}$ at temperature θ.



$$\frac{T'}{T} = \sqrt{\frac{L'}{L}} = \sqrt{\frac{L[1+\alpha\Delta\theta]}{L}} = 1 + \frac{1}{2}\alpha\Delta\theta$$

Therefore change (loss or gain) in time per unit time lapsed is $\frac{T'-T}{T} = \frac{1}{2}\alpha\Delta\theta$

Fractional change in time period $\frac{\Delta T}{T} = \frac{1}{2}\alpha\Delta\theta$

So, as the temperature increases, length of pendulum increases and hence time period of pendulum increases. Due to increment in its time period, a pendulum clock becomes slow in summer and will lose time.

Q6. Heat is associated with
(a) kinetic energy of random motion of molecules
(b) kinetic energy of orderly motion of molecules
(c) total kinetic energy of random and orderly motion of molecules
(d) kinetic energy of random motion in some cases and kinetic energy of orderly motion in other
Sol: (a) When a body is heated its temperature rises and in liquids and gases vibration of molecules about their mean position increases, hence kinetic energy associated with random motion of molecules increases. So, thermal energy or heat associated with the random and translatory motions of molecules.

Q7. The radius of a metal sphere at room temperature Tis Rand the coefficient of linear expansion of the metal is .The sphere heated a little by a temperature ΔTso that its new temperature is T + ΔT.The increase in the volume of the sphere is approximately.

(a) $2\pi R\alpha\Delta T$ (b) $\pi R^2\alpha\Delta T$ (c) $4\pi R^3\alpha\Delta T/3$ (d) $4\pi R^3\alpha\Delta T$

Sol. (d)

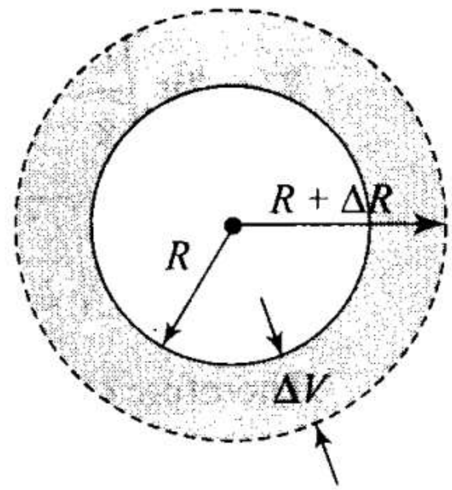
Key concept: The co-efficient α, β and γ for a solid are related to each other as follows:

$$\alpha = \frac{\beta}{2} = \frac{\gamma}{3} \Rightarrow \alpha : \beta : \gamma = 1:2:3$$

The values of α, β and γ are independent of the units of length, area and volume respectively.

For anisotropic solids $\gamma = \alpha_x + \alpha_y + \alpha_z$ where α_x, α_y and α_z represent the mean coefficients of linear expansion along three mutually perpendicular directions.

As the temperature increases radius of the sphere increases as shown. So, the volume of the sphere increases.



Original volume of the sphere $V_0 = \frac{4}{3}\pi R^3$

Coefficient of linear expansion = α

Hence, coefficient of volume expansion = 3α

$\therefore$ Change in volume is $\Delta V = V_0\gamma\Delta T$

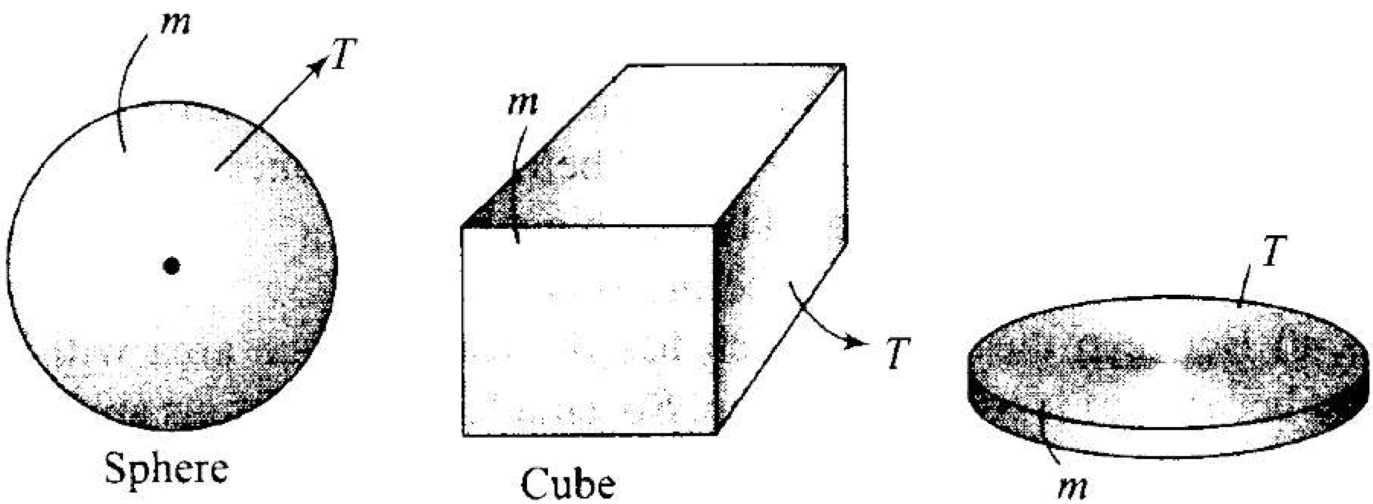
$$\Delta V = \frac{4}{3}\pi R^3(3\alpha)\Delta T$$

Increase in volume = $4\pi R^3\alpha\Delta T$

ρ = mass / volume

So, the volume of all object will also be same.

Here the cooling will be done in the form of radiations that is according to Stefan’s law. Since, emissive power is directly proportional to the surface. Here, for given volume, sphere has least surface area and circular plate of greatest surface area.



As thickness of the plate is least, hence surface area of the plate is maximum. According to Stefan's law, heat loss (cooling) is directly proportional to the surface area.

$H_{sphere} : H_{cube} : H_{plate} = A_{sphere} : A_{cube} : A_{plate}$
As A_{plate} is maximum, hence the plate will cool fastest.
As the sphere is having minimum surface area, hence the sphere cools slowest.

More Than One Correct Answer Type
Q9. Mark the correct options.
(a) A system X is in thermal equilibrium with Y but not with Z. The systems Y and Z may be in thermal equilibrium with each other.
(b) A system X is in thermal equilibrium with Y but not with Z. The systems Y and Z are not in thermal equilibrium with each other.
(c) A system X is neither in thermal equilibrium with Y nor with Z. The systems Y and Z must be in thermal equilibrium with each other.
(d) A system X is neither in thermal equilibrium with Y nor with Z. The systems Y-and Z may be in thermal equilibrium with each other.
Sol: (b, d)
Key concept: Two bodies are said to be in thermal equilibrium with each other**,** when no heat flows from one body to the other. That is when both the bodies are at the same temperature.

According to the problem,

(b) If two systems X and Y are in thermal equilibrium, *i.e.*, $T_x = T_y$ and x is not in thermal equilibrium with z,
i.e., $T_x \neq T_z$
then clearly, $T_y \neq T_z$
Hence, y and z are not in thermal equilibrium.

(d) if X is not in thermal equilibrium with Y, *i.e*, $T_x \neq T_y$ and also X is not in thermal equilibrium with Y, *i.e.*, $T_x \neq T_z$
Then we cannot say about equilibrium of Y and Z, they may or may not be in equilibrium.

Q10. Gulab jamuns (assumed to be spherical) are to be heated in an oven. They are available in two sizes, one twice bigger (in radius) than the other. Pizzas (assumed to be discs) are also to be heated in oven. They are also in two sizes, one twice bigger (in radius) than the other. All four are put together to be heated to oven temperature. Choose the correct option from the following.

(a) Both size gulab jamuns will get heated in the same time

(b) Smaller gulab jamuns are heated before bigger ones

(c) Smaller pizzas are heated before bigger ones

(d) Bigger pizzas are heated before smaller

Sol: (b, c) Between these four which has the least surface area will be heated first because of less heat radiation. So, smaller gulab jamuns are having least surface area, hence they will be heated first. Similarly, smaller pizzas are heated before bigger ones because they are of small surface areas.

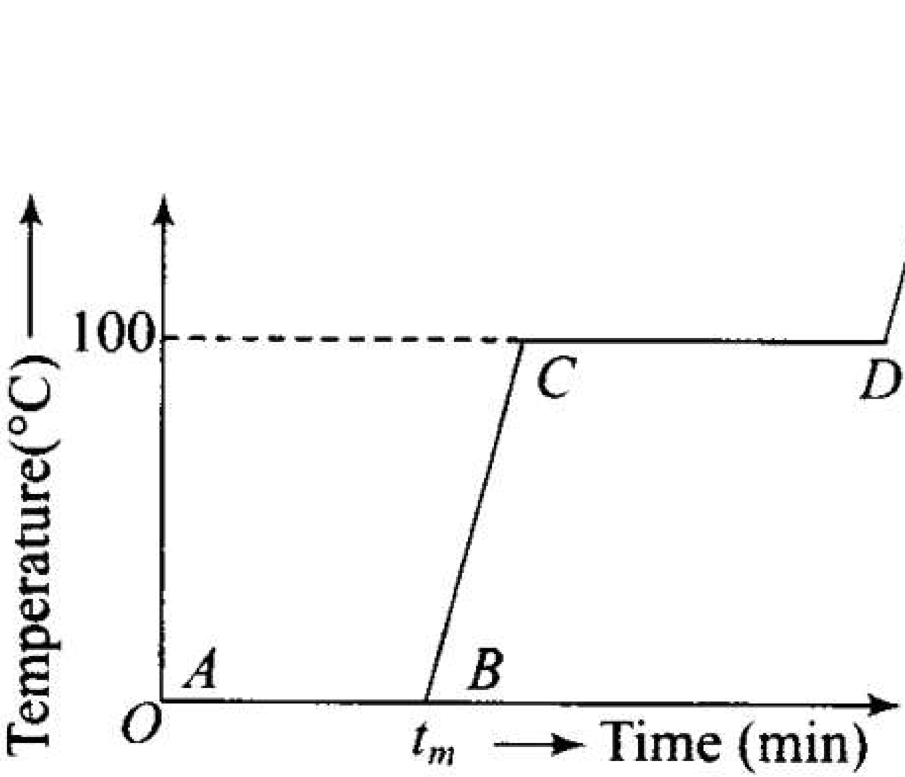
Q11. Refer to the plot of temperature versus time (figure) showing the changes in the state if ice on heating (not to scale). Which of the following is correct?

(a) The region AB represents ice and water in thermal equilibrium

(b) At B water starts boiling

(c) At C all the water gets converted into steam

(d) C to D represents water and steam in equilibrium at boiling point



Sol: (a, d) During phase change process, temperature of the system remains constant.
(a) In region AB, a phase change takes place, heat is supplied and ice melts but temperature of the system is 0°C. it remains constant during process. The heat supplied is used to break bonding between molecules.
(b) In region CD, again a phase change takes place from a liquid to a vapour state during which temperature remains constant. It shows water and steam are in equilibrium at boiling point.

Q12. A glass full of hot milk is poured on the table. It begins to cool gradually. Which of the following is correct?
(a) The rate of cooling is constant till milk attains the temperature of the surrounding.
(b) The temperature of milk falls off exponentially with time.
(c) While cooling, there is a flow of heat from milk to the surrounding as well as from surrounding to the milk but the net flow of heat is from milk to the surrounding and that is why it cools.
(d) All three phenomenon, conduction, convection and radiation are responsible for the loss of heat from milk to the surroundings.
Sol: (b, c, d) When hot milk spread on the table heat is transferred to the surroundings by conduction, convection and radiation. Because the surface area of poured milk on a table is more than the surface area of milk filled in a glass. Hence, its temperature falls off exponentially according to Newton's law of cooling. Heat also will be transferred from surroundings to the milk but will be lesser than that of transferred from milk to the surroundings. So, option (b), (c) and (d) are correct.

Very Short Answer Type Questions

Q13. Is the bulb of a thermometer made of diathermic **or adiabatic wall?**

Sol: The bulb of a thermometer is made up of diathermic wall because diathermic walls allow exchange of heat energy between two systems but adiabatic walls do not. So it receives heat from the body to measure the temperature of body.

Q14. A student records the initial length l , **change in temperature** ∆ **T and change in length** ∆ **l of a rod as follows:**

S. No	l(m)	∆T(°C)	∆l(m)
1.	2	10	4×10^{-4}
2.	1	10	4×10^{-4}
3.	2	20	2×10^{-4}
4.	3	10	6×10^{-4}

If the first observation is correct, what can you say about observations 2, 3 and 4.

Sol: If the first observation is correct, hence from the 1st observation we get the coefficient of linear expansion

$$\alpha = \frac{\Delta l}{l \times \Delta T} = \frac{4 \times 10^{-4}}{2 \times 10} = 2 \times 10^{-5}\ {}^\circ C^{-1}$$

For 2nd observation, $\Delta l = \alpha l \Delta T$

$$= 2 \times 10^{-5} \times 1 \times 10$$
$$= 2 \times 10^{-4}\ m \neq 4 \times 10^{-4}\ m$$

which is incorrect.

For 3rd observation, $\Delta l = \alpha l \Delta T$

$$= 2 \times 10^{-5} \times 2 \times 20$$
$$= 8 \times 10^{-4}\ m \neq 2 \times 10^{-4}\ m$$

which is incorrect.

For 4th observation, $\Delta l = \propto l \Delta T$

$= 2 \times 10^{-5} \times 3 \times 10 = 6 \times 10^{-4}$ m [i.e., observed value is correct]

Q15. Why does a metal bar appear hotter than a wooden bar at the same temperature? Equivalently it also appears cooler than wooden bar if they are both colder than room temperature.
Sol:
Key concept: Kirchhoffs Law:
The ratio of emissive power to absorptive power is same for all surfaces at the same temperature and is equal to the emissive power of a perfectly black body at that temperature.

Thus if $a_{practical}$ and $E_{practical}$ represent the absorptive and emissive power of a given surface, while a_{black} and E_{black} for a perfectly black body, then according to law $\frac{E_{practical}}{a_{practical}} = \frac{E_{black}}{a_{black}}$

But for a perfectly black body $a_{black} = 1$, so $\frac{E_{practical}}{a_{practical}} = E_{black}$

If emissive and absorptive powers are considered for a particular wavelength λ, $\left(\frac{E_\lambda}{a_\lambda}\right)_{practical} = (E_\lambda)_{black}$

Now since $(E_\lambda)_{Black}$ is constant at a given temperature, according to this law if a surface is a good absorber of a particular wavelength it is also a good emitter of that wavelength.
This in turn implies that a good absorber is a good emitter (or radiator).
The conductivities of metals are very high compared to wood. Due to difference in conductivity, if one touch the hot metal with a finger, heat from the surrounding flows faster to the finger from metals and so one feels the heat.
Similarly, when one touches a cold metal the heat from the finger flows away to the surroundings faster. So we can say that a good radiator can be a good absorber.

Q16. Calculate the temperature which has numeral value on Celsius and Fahrenheit scale.
Sol: To construct a scale of temperature, two fixed points are taken. First fixed point is the freezing point of water, it is called lower fixed point. The second fixed point is the boiling point of water, it is called upper fixed point. Temperature on one scale can be converted into other scale by using the following identity.

$$\frac{\text{Reading on any scale} - (\text{LFP})}{(\text{UFP}) - (\text{LFP})} = \text{Constant for all scales}$$

$$\frac{C-0}{100} = \frac{F-32}{212-32}$$

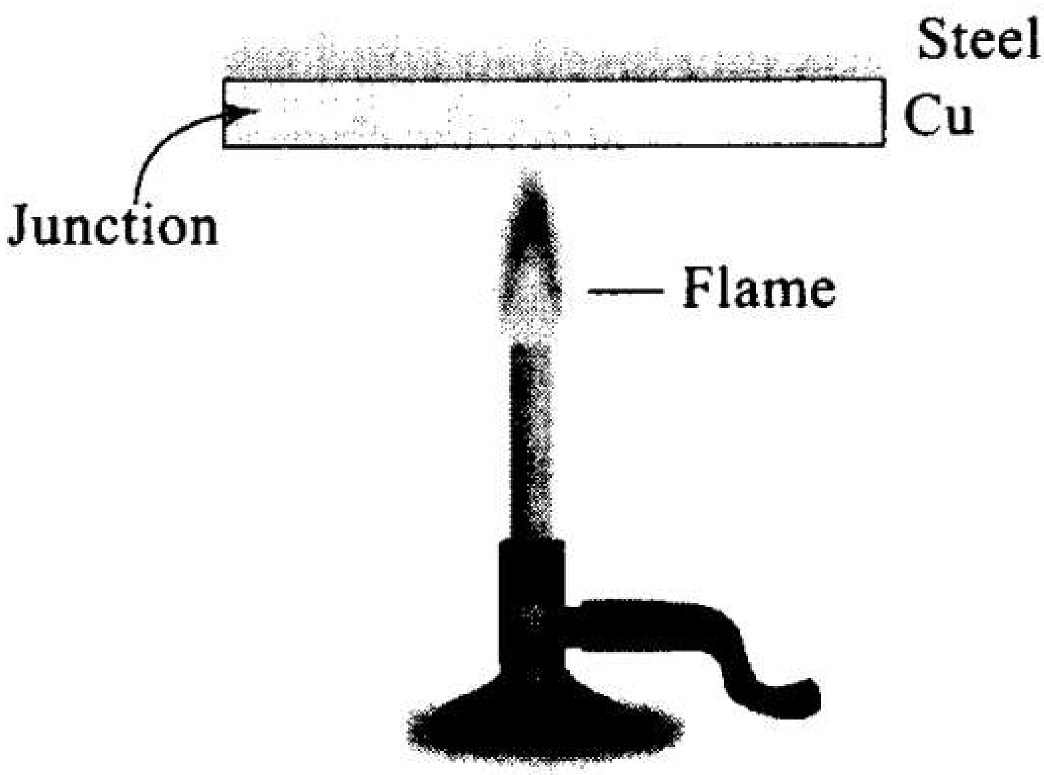
Let T be the value of temperature having same value on Celsius and Fahrenheit scale, *i.e.* $F = C = T$

$$\frac{T-32}{180} = \frac{T}{100}$$

$$\Rightarrow \quad T - 32 = \frac{9}{5}T \Rightarrow \frac{4}{5}T = -32$$

$$\Rightarrow \quad T = -40°C = -40°F$$

Q17. These days people use steel utensils with copper bottom. This is supposed to be good for uniform heating of food. Explain this effect using the fact that copper is the better conductor. Junction



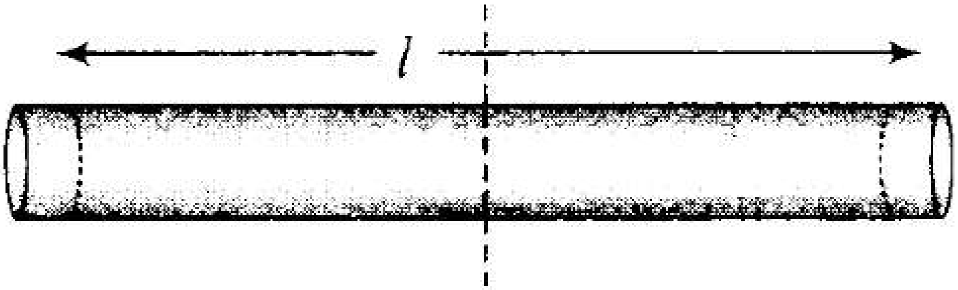
Sol: The copper bottom of the steel utensil gets heated quickly.

Because of the reason that copper is a good conductor of heat as compared to steel. But steel does not conduct as quickly, thereby allowing food inside to get heated uniformly.

Short Answer Type Questions

Q18. Find out the increase in moment of inertia I of a uniform rod (coefficient of linear expansion a) about its perpendicular bisector when its temperature is slightly increased by ∆T.

Sol: Moment of inertia of a uniform rod of mass M and length l about its perpendicular bisector



$$I = \frac{1}{12}Ml^2$$

Increase in length of the rod when temperature is increased by Δt, is given by

$$L' = L(1 + \alpha\Delta T)$$

∴ New moment of inertia of the rod

$$I' = \frac{ML'^2}{12} = \frac{M}{12}L^2[1 + \alpha\Delta T]^2$$

$$= \frac{ML^2}{12}[1 + 2\alpha\Delta T + \alpha^2(\Delta T)^2]$$

∴ As change in length Δl is very small, therefore, neglecting $(\Delta l)^2$ ($\because$ $\alpha^2(\Delta T)^2$ is very small) we get

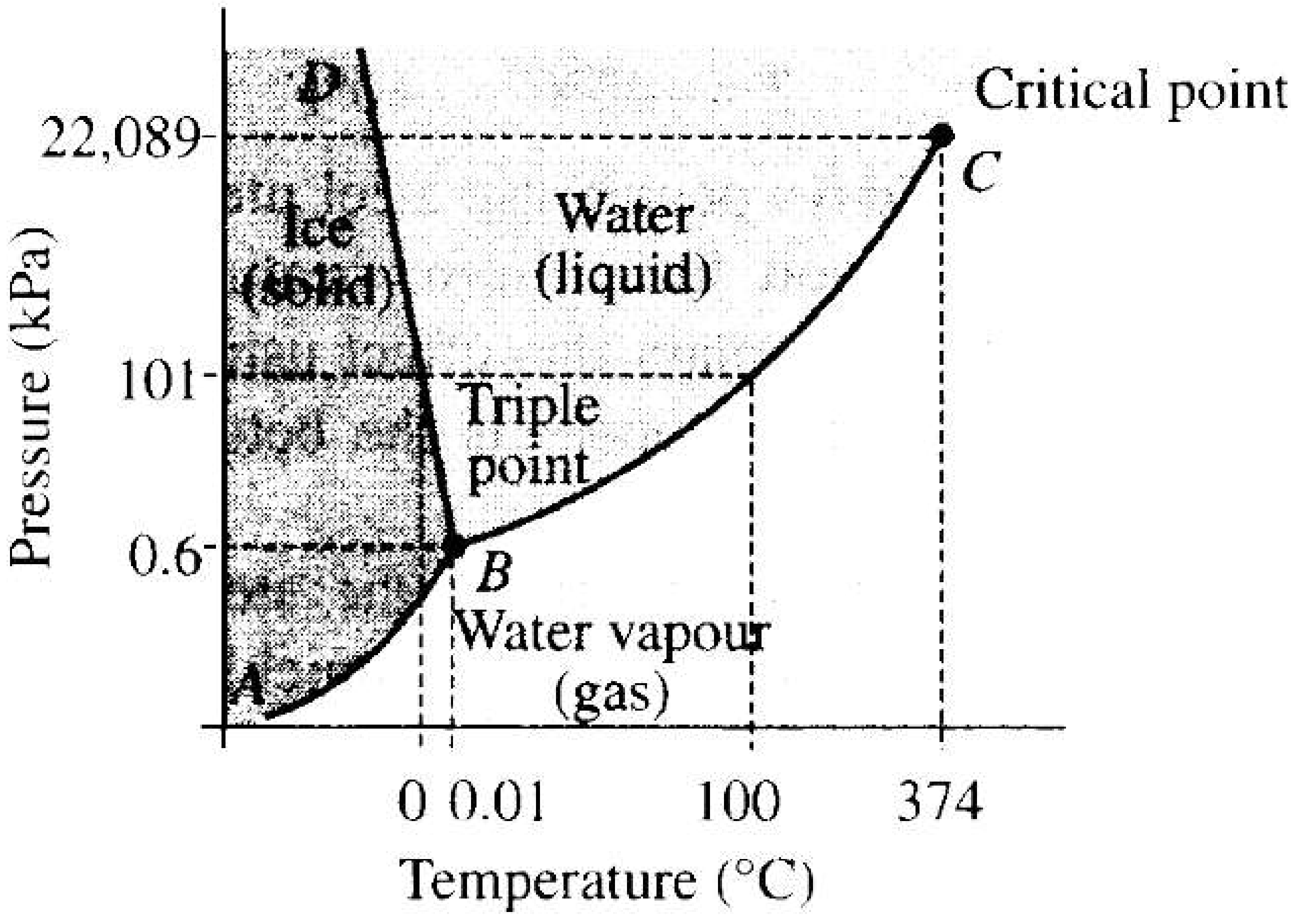
$$I' = I[1 + 2\alpha\Delta T]$$

∴ Increase in moment of inertia

$$= I' - I = I[1 + 2\alpha\Delta T] - I = 2\alpha I\Delta T$$

Q19. During summers in India, one Of the common practice to keep cool is to make ice balls of crushed ice, dip it in flavored sugar syrup and sip it. For this a stick is inserted into crushed ice and is squeezed in the palm to make it into the ball. Equivalently in winter in those areas where it snows, people make snow balls and throw around. Explain the formation of ball **out of crushed ice or snow in the light of p-T diagram of water.**

Sol : Given diagram shows the variation of pressure with temperature for water. When the pressure is increased in solid state (at 0°, 1 atm), ice changes into liquid state while decreasing pressure in liquid state (at 0°, 1 atm), water changes to ice.



When crushed ice is squeezed, some of it melts, filling up the gap between ice flakes upon releasing pressure. This water freezes, binding all ice flakes and making the ball more stable.

Q20. 100 g of water is super cooled to -10°C. At this point, due to some disturbance mechanised or otherwise **some of it suddenly freezes to ice. What will be the temperature of the resultant mixture and how much mass would freeze?**

$[S_w = 1 \text{ cal / g/°C and } L^w_{Fusion} = 80 \text{ cal/g}]$

Sol. According to the problem, mass of water (m) = 100 g

Change in temperature $\Delta T = 0 - (-10) = 10°C$

Specific heat of water (S_w) = 1 cal/g/°C

Latent heat of fusion of water $L^w_{fusion} = 80$ cal/g

Amount of heat required to change the temperature of 100 g of water at −10°Cto 0°C,

$$Q = mS_w \Delta T$$

$$= 100 \times 1 \times [0 - (-10)] = 1000 \text{ cal}$$

Let m gram of ice be melted.

$$\therefore \quad Q = mL$$

$$m = \frac{Q}{L} = \frac{1000}{80} = 12.5 \text{ g}$$

or

As small mass of ice is melted, thus the temperature of the resultant mixture will remain 0°C which contains 12.5 g of ice and 87.5 g of water.

Q21. One day in the morning Ramesh filled up 1/3 bucket of hot water from geyser, to take bath. Remaining 2/3 was to be filled by cold water (at room temperature) to bring mixture to a comfortable temperature. Suddenly Ramesh had to attend to something which would take some times, say 5-10 min before he could take bath. Now, he had two options (i) fill the remaining bucket completely by cold water and then attend to the work, (ii) first attend to the work and fill the remaining bucket just before taking bath. Which option do you think would have kept water warmer? Explain

Sol: According to the Newton's law of cooling, the rate of loss of heat is directly proportional to the difference of temperature. Or we can say which gives a consequence about rate of fall of temperature of a body with respect to the difference of temperature of body and surroundings.

The first option would have kept water warmer because by adding hot water to cold water, the temperature of the mixture decreases. Due to this temperature difference between the mixed water in the bucket and the surrounding decreases, thereby the decrease in the rate of loss of the heat by the water.

In second option, the hot water in the bucket will lose heat quickly. So if he first attend to the work and fill the remaining bucket with cold water which already lose much heat in 5-10 minutes then the water become more colder as comparison with first case.

Long Answer Type Questions

Q22. We would like to prepare a scale whose length does not change with temperature. It is proposed to prepare a unit scale of this type whose length remains, say 10 cm. We can use a bimetallic strip made of brass and iron each of different length whose length (both components) would change in such a way that difference between their length B remain constant. If α_{iron} = 1.2 x 10 $^{-5}$/K and α_{brass} = 1.8xl0^{-5}/K, what should we take as length **of each strip?**

Sol: According to the problem, L_1-L_b = 10 cm where,

L_1 = length of iron scale

L_b = Length of brass scale

This condition is possible if change in length both the rods is remain same at all temperatures.

Change in length of iron rod,

$$\Delta L = \alpha_I L_I \Delta T$$

Change in length of brass rod,

$$\Delta L = \alpha_B L_B \Delta T$$

As the change will equal in both the rods, so

$$\alpha_I L_I \Delta T = \alpha_B L_B \Delta T$$

$$\Rightarrow \qquad \alpha_I L_I = \alpha_B L_B \Rightarrow \frac{L_I}{L_B} = \frac{\alpha_B}{\alpha_I}$$

Here, $\alpha_\beta = 1.8 \times 10^{-5}\ \text{K}^{-1}, \alpha_I = 1.2 \times 10^{-5}\ \text{K}^{-1}$

$$\therefore \qquad \frac{L_I}{L_B} = \frac{1.8 \times 10^{-5}}{1.2 \times 10^{-5}} = \frac{3}{2}$$

$$L_I = \frac{3}{2} L_B$$

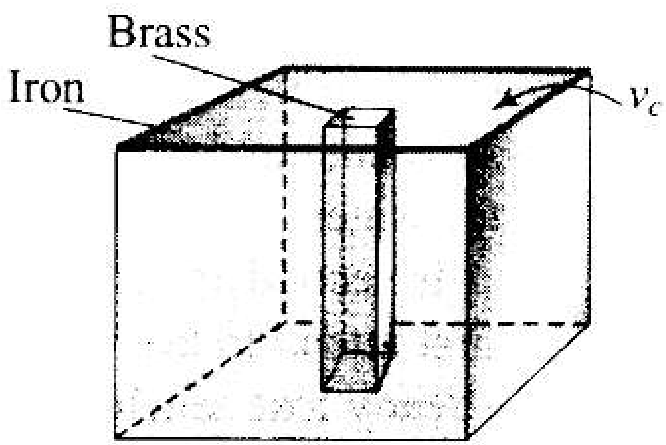
As, $L_I - L_B = 10$ cm

$$\therefore \qquad \frac{3}{2} L_B - L_B = 10 \Rightarrow \frac{1}{2} L_B = 10 \Rightarrow L_B = 20 \text{ cm}.$$

Q23. We would like to make a vessel whose volume does not change with temperature (take a hint from the problem above). We can use brass and iron (β_{vbrass} = 6 x 10 5 / K and β_{viron} = 3.55 x 10^{-5}/K) to create a volume of 100 cc. How do you think you can achieve this?

Sol: Here we are making a vessel whose Brass volume does not change with temperature.

To make the desired vessel, we should have an iron vessel with a brass rod inside as shown in the diagram.



Volume of vessel, $V_0 = 100\ \text{cm}^3$

$= 10^{-4}$ = constant

Volume of iron vessel (V_I) – Volume of brass rod (V_B) = $10^{-4}\ \text{m}^3$

$\Rightarrow \quad V_I - V_B = 10^{-4}\ \text{m}^3$ ...(i)

This condition is possible if

$$\beta_I V_I \Delta T = \beta_B V_B \Delta T$$

$$\therefore \quad V_I = \left(\frac{\beta_B}{\beta_I}\right) V_B = \left(\frac{6 \times 10^{-5}}{3.55 \times 10^{-5}}\right) V_B = 1.69 V_B$$

From equation (i),

$$1.69 V_B - V_B = 10^{-4}$$

$$\Rightarrow \quad V_B = \frac{10^{-4}}{0.69} = 1.449 \times 10^{-4}\ \text{m}^3$$

$$\Rightarrow \quad V_B = 144.9\ \text{cm}^3$$

$$V_I = 1.69 V_B = 1.69 \times 144.9 = 244.9\ \text{cm}^3$$

Therefore, an iron vessel with a volume of 249.9 cm^3 fitted with a brass rod of volume 144.9 cm^3 will serve as a vessel of volume 100 cm^3, which will not change with temperature.

Important points:

- Solids can expand in one dimension (linear expansion), two dimensions (superficial expansion) and three dimensions (volume expansion) while liquids and gases usually suffers change in volume only.
- Thermal expansion is minimum in case of solids but maximum in case of gases because intermoleeular force is maximum in solids but minimum in gases.

Q24. Calculate the stress developed inside a tooth cavity filled with copper when hot tea at temperature of 57°C is drunk. You can take body (tooth) temperature to be 37°C and a = 1.7x 10^{-5}/°C bulk modulus for copper = 140x $10^9 N/m^2$.

Sol. According to the problem, decrease in temperature

$$(\Delta t) = 57 - 37 = 20°C$$

Coefficient of linear expansion

$$(\alpha) = 1.7 \times 10^{-5}/°C$$

Bulk modulus for copper $(B) = 140 \times 10^9 \text{ N/m}^2$

Coefficient of cubical expansion

$$(\gamma) = 3\alpha = 5.1 \times 10^{-5}/°C$$

Let initial volume of the cavity be V and its volume increases by ΔV due to increase in temperature.

$$\therefore \quad \Delta V = \gamma V \Delta t \Rightarrow \frac{\Delta V}{V} = \gamma \Delta t$$

We know, $B = \dfrac{\text{stress}}{\text{volume strain}}$

$$\therefore \quad \text{Thermal stress} = B \times \left(\frac{\Delta V}{V}\right) = B(\gamma \Delta T)$$

$$= B(3\alpha\Delta T) \qquad (\because \gamma = 3\alpha)$$

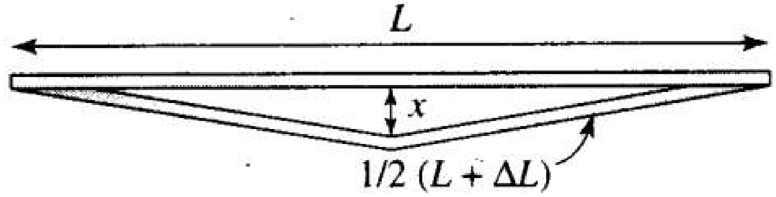
$$= 140 \times 10^9 \times 3 \times 1.7 \times 10^{-5} \times 20$$

$$= 1.428 \times 10^8 \text{ Nm}^{-2}$$

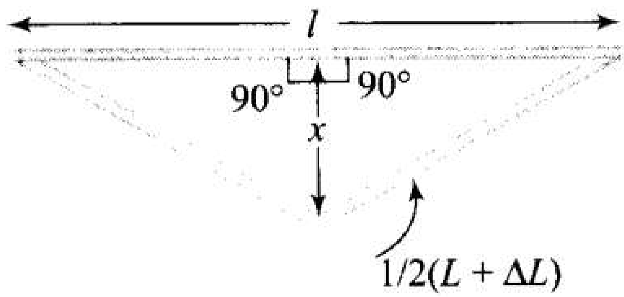
This is about 10^3 times of atmospheric pressure

Q25. A rail track made of steel having length 10 m is clamped on a railway line at its two ends (figure). On a summer day due to rise in temperature by 20°C. It is deformed as shown in figure. Find x (displacement of the centre) if27

$\alpha_{steel} = 1.2 \times 10^{-5}/°C$



Sol. Diagram shows the deformation of a railway track due to rise in temperature.



Applying Pythagoras theorem in right angled triangle,

$$x^2 = \left(\frac{L+\Delta L}{2}\right)^2 - \left(\frac{L}{2}\right)^2$$

$$\Rightarrow \quad x = \sqrt{\left(\frac{L+\Delta L}{2}\right)^2 - \left(\frac{L}{2}\right)^2}$$

$$= \sqrt{\left(\frac{L}{2}\right)^2 + \frac{2L\Delta L}{4} + \left(\frac{\Delta L}{2}\right)^2 - \left(\frac{L}{2}\right)^2}$$

$$= \frac{1}{2}\sqrt{(L^2 + \Delta L^2 + 2L\Delta L) - L^2} = \frac{1}{2}\sqrt{(\Delta L^2 + 2L\Delta L)}$$

As increase in length ΔL is very small, therefore, neglecting $(\Delta L)^2$, we get

$$x = \frac{\sqrt{2L\Delta L}}{2} \quad \ldots\text{(i)}$$

But $\quad \Delta L = L\alpha\Delta t \quad \ldots\text{(ii)}$

According to the problem, $L = 10$ m,

$\alpha = 1.2 \times 10^{-5}\ °C^{-1}$, $\Delta T = 20°C$

Substituting value of ΔL in Eq. (i) from Eq. (ii)

$$x = \frac{1}{2}\sqrt{2L \times L\alpha\Delta t} = \frac{1}{2}L\sqrt{2\alpha\Delta t}$$

$$= \frac{10}{2} \times \sqrt{2 \times 1.2 \times 10^{-5} \times 20}$$

$$= 5 \times \sqrt{4 \times 1.2 \times 10^{-4}} = 5 \times 2 \times 1.1 \times 10^{-2} = 0.11\text{ m} = 11\text{ cm}$$

Q26. A thin rod. having length L_0 at 0°C and coefficient of linear expansion α has its two ends maintained at temperatures θ_1, and θ_2, Find its new length.

Sol. When temperature of a rod varies linearly, then average temperature of the middle point of the rod can be taken as mean of temperatures at the two ends. According to the diagram,

According to the diagram,

$$\theta = \frac{\theta_1 + \theta_2}{2}$$

Let temperature varies linearly in the rod from its one end to other end from θ_1 to θ_2. Let θ be the temperature of the mid-point of the rod.

Therefore, average temperature of the mid-point of the rod is

$$\Rightarrow \qquad \theta = \frac{\theta_1 + \theta_2}{2}$$

Using relation, $L = L_0(1 + \alpha\theta)$

$$\text{or} \qquad L = L_0\left[1 + \alpha\left(\frac{\theta_1 + \theta_2}{2}\right)\right]$$