Question 8.1:

Answer the following:

You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?

An astronaut inside a small space ship orbiting around the earth cannot detect gravity. If the space station orbiting around the earth has a large size, can he hope to detect gravity?

If you compare the gravitational force on the earth due to the sun to that due to the moon, you would find that the Sun's pull is greater than the moon's pull. (You can check this yourself using the data available in the succeeding exercises). However, the tidal effect of the moon's pull is greater than the tidal effect of sun. Why?

Answer

Answer: (a) No (b) Yes

Gravitational influence of matter on nearby objects cannot be screened by any means. This is because gravitational force unlike electrical forces is independent of the nature of the material medium. Also, it is independent of the status of other objects.

If the size of the space station is large enough, then the astronaut will detect the change in Earth's gravity (g).

Tidal effect depends inversely upon the cube of the distance while, gravitational force depends inversely on the square of the distance. Since the distance between the Moon and the Earth is smaller than the distance between the Sun and the Earth, the tidal effect of the Moon's pull is greater than the tidal effect of the Sun's pull.

Question 8.2:

Choose the correct alternative:

Acceleration due to gravity increases/decreases with increasing altitude.

Acceleration due to gravity increases/decreases with increasing depth. (assume the earth to be a sphere of uniform density).

Acceleration due to gravity is independent of mass of the earth/mass of the body.

The formula $-G\,Mm(1/r_2 - 1/r_1)$ is more/less accurate than the formula $mg(r_2 - r_1)$ for the difference of potential energy between two points r_2 and r_1 distance away from the centre of the earth.

Answer

Answer:

Decreases

Decreases

Mass of the body

More

Explanation:

Acceleration due to gravity at depth h is given by the relation:

$$g_h = \left(1 - \frac{2h}{R_e}\right)g$$

Where,

R_e = Radius of the Earth

g = Acceleration due to gravity on the surface of the Earth

It is clear from the given relation that acceleration due to gravity decreases with an increase in height.

Acceleration due to gravity at depth d is given by the relation:

$$g_d = \left(1 - \frac{d}{R_e}\right)g$$

It is clear from the given relation that acceleration due to gravity decreases with an increase in depth.

Acceleration due to gravity of body of mass m is given by the relation:

$$g = \frac{GM}{R^2}$$

Where,

G = Universal gravitational constant

M = Mass of the Earth

R = Radius of the Earth

Hence, it can be inferred that acceleration due to gravity is independent of the mass of the body.

Gravitational potential energy of two points r_2 and r_1 distance away from the centre of the Earth is respectively given by:

$$V(r_1) = -\frac{GmM}{r_1}$$

$$V(r_2) = -\frac{GmM}{r_2}$$

$\therefore$ Difference in potential energy, $V = V(r_2) - V(r_1) = -GmM\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$

Hence, this formula is more accurate than the formula $mg(r_2 - r_1)$.

Question 8.3:

Suppose there existed a planet that went around the sun twice as fast as the earth.What would be its orbital size as compared to that of the earth?

Answer

Answer: Lesser by a factor of 0.63

Time taken by the Earth to complete one revolution around the Sun,

T_e = 1 year

Orbital radius of the Earth in its orbit, $R_e = 1$ AU

Time taken by the planet to complete one revolution around the Sun, $T_P = \frac{1}{2}T_e = \frac{1}{2}$ year

Orbital radius of the planet = R_p

From Kepler's third law of planetary motion, we can write:

$$\left(\frac{R_P}{R_e}\right)^3 = \left(\frac{T_P}{T_e}\right)^2$$

$$\frac{R_P}{R_e} = \left(\frac{T_P}{T_e}\right)^{\frac{2}{3}}$$

$$= \left(\frac{\frac{1}{2}}{1}\right)^{\frac{2}{3}} = (0.5)^{\frac{2}{3}} = 0.63$$

Hence, the orbital radius of the planet will be 0.63 times smaller than that of the Earth.

Question 8.4:

Io, one of the satellites of Jupiter, has an orbital period of 1.769 days and the radius of the orbit is 4.22×10^8 m. Show that the mass of Jupiter is about one-thousandth that of the sun.

Answer

Orbital period of $I_0, T_{Io} = 1.769 \text{ days} = 1.769 \times 24 \times 60 \times 60$ s

Orbital radius of I_0, $R_{Io} = 4.22 \times 10^8$ m

Satellite I_0 is revolving around the Jupiter

Mass of the latter is given by the relation:

$$M_J = \frac{4\pi^2 R_{Io}^3}{GT_{Io}^2} \qquad \text{... (i)}$$

Where,

M_J = Mass of Jupiter

G = Universal gravitational constant

Orbital period of the Earth,

$T_e = 365.25 \text{ days} = 365.25 \times 24 \times 60 \times 60 \text{ s}$

Orbital radius of the Earth,

$R_e = 1 \text{ AU} = 1.496 \times 10^{11} \text{ m}$

Mass of Sun is given as:

$$M_s = \frac{4\pi^2 R_e^3}{GT_e^2} \qquad \text{... (ii)}$$

$$\therefore \frac{M_s}{M_J} = \frac{4\pi^2 R_e^3}{GT_e^2} \times \frac{GT_{Io}^2}{4\pi^2 R_{Io}^3} = \frac{R_e^3}{R_{Io}^3} \times \frac{T_{Io}^2}{T_e^2}$$

$$= \left(\frac{1.769 \times 24 \times 60 \times 60}{365.25 \times 24 \times 60 \times 60}\right)^2 \times \left(\frac{1.496 \times 10^{11}}{4.22 \times 10^8}\right)^3$$

$$= 1045.04$$

$$\therefore \frac{M_S}{M_J} \sim 1000$$

$$M_S \sim 1000 \times M_J$$

Hence, it can be inferred that the mass of Jupiter is about one-thousandth that of the Sun.

Question 8.5:

Let us assume that our galaxy consists of 2.5×10^{11} stars each of one solar mass. How long will a star at a distance of 50,000 ly from the galactic centre take to complete one revolution? Take the diameter of the Milky Way to be 10^5 ly.

Answer

Mass of our galaxy Milky Way, $M = 2.5 \times 10^{11}$ solar mass

Solar mass = Mass of Sun = 2.0×10^{36} kg

Mass of our galaxy, $M = 2.5 \times 10^{11} \times 2 \times 10^{36} = 5 \times 10^{41}$ kg

Diameter of Milky Way, $d = 10^5$ ly

Radius of Milky Way, $r = 5 \times 10^4$ ly

1 ly = 9.46×10^{15} m

$\therefore r = 5 \times 10^4 \times 9.46 \times 10^{15}$

$= 4.73 \times 10^{20}$ m

Since a star revolves around the galactic centre of the Milky Way, its time period is given by the relation:

$$T = \left(\frac{4\pi^2 r^3}{GM}\right)^{\frac{1}{2}}$$

$$= \left(\frac{4 \times (3.14)^2 \times (4.73)^3 \times 10^{60}}{6.67 \times 10^{-11} \times 5 \times 10^{41}}\right)^{\frac{1}{2}} = \left(\frac{39.48 \times 105.82 \times 10^{30}}{33.35}\right)^{\frac{1}{2}}$$

$$= (125.27 \times 10^{30})^{\frac{1}{2}} = 1.12 \times 10^{16} \text{ s}$$

$$1 \text{ year} = 365 \times 24 \times 60 \times 60 \text{ s}$$

$$1 \text{ s} = \frac{1}{365 \times 24 \times 60 \times 60} \text{ years}$$

$$\therefore 1.12 \times 10^{16} \text{ s} = \frac{1.12 \times 10^{16}}{365 \times 24 \times 60 \times 60}$$

$$= 3.55 \times 10^{8} \text{ years}$$

Question 8.6:

Choose the correct alternative:

If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic/potential energy.

The energy required to launch an orbiting satellite out of earth's gravitational influence is more/less than the energy required to project a stationary object at the same height (as the satellite) out of earth's influence.

Answer

Answer:

Kinetic energy

Less

Total mechanical energy of a satellite is the sum of its kinetic energy (always positive) and potential energy (may be negative). At infinity, the gravitational potential energy of the satellite is zero. As the Earth-satellite system is a bound system, the total energy of the satellite is negative.

Thus, the total energy of an orbiting satellite at infinity is equal to the negative of its kinetic energy.

An orbiting satellite acquires a certain amount of energy that enables it to revolve around the Earth. This energy is provided by its orbit. It requires relatively lesser energy to move out of the influence of the Earth's gravitational field than a stationary object on the Earth's surface that initially contains no energy.

Question 8.7:

Does the escape speed of a body from the earth depend on

the mass of the body,

the location from where it is projected,

the direction of projection,

the height of the location from where the body is launched?

Answer

No

No

No

Yes

Escape velocity of a body from the Earth is given by the relation:

$$v_{esc} = \sqrt{2gR} \qquad \text{... (i)}$$

g = Acceleration due to gravity

R = Radius of the Earth

It is clear from equation (i) that escape velocity v_{esc} is independent of the mass of the body and the direction of its projection. However, it depends on gravitational potential at the point from where the body is launched. Since this potential marginally depends on the height of the point, escape velocity also marginally depends on these factors.

Question 8.8:

A comet orbits the Sun in a highly elliptical orbit. Does the comet have a constant (a) linear speed, (b) angular speed, (c) angular momentum, (d) kinetic energy, (e) potential energy, (f) total energy throughout its orbit? Neglect any mass loss of the comet when it comes very close to the Sun.

Answer

No

No

Yes

No

No

Yes

Angular momentum and total energy at all points of the orbit of a comet moving in a highly elliptical orbit around the Sun are constant. Its linear speed, angular speed, kinetic, and potential energy varies from point to point in the orbit.

Question 8.9:

Which of the following symptoms is likely to afflict an astronaut in space (a) swollen feet, (b) swollen face, (c) headache, (d) orientational problem?

Answer

Answer: (b), (c), and (d)

Legs hold the entire mass of a body in standing position due to gravitational pull. In space, an astronaut feels weightlessness because of the absence of gravity. Therefore, swollen feet of an astronaut do not affect him/her in space.

A swollen face is caused generally because of apparent weightlessness in space. Sense organs such as eyes, ears nose, and mouth constitute a person's face. This symptom can affect an astronaut in space.

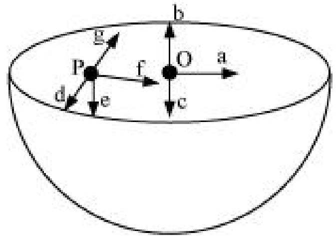
Headaches are caused because of mental strain. It can affect the working of an astronaut in space.

Space has different orientations. Therefore, orientational problem can affect an astronaut in space.

Question 8.10:

Choose the correct answer from among the given ones:

The gravitational intensity at the centre of a hemispherical shell of uniform mass density has the direction indicated by the arrow (see Fig 8.12) (i) a, (ii) b, (iii) c, (iv) O.

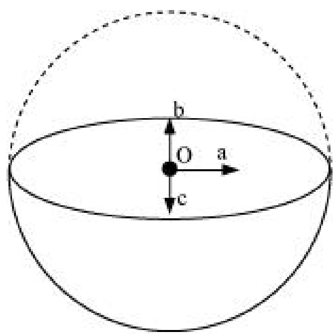


Answer

Answer: (iii)

Gravitational potential (V) is constant at all points in a spherical shell. Hence, the gravitational potential gradient $\left(\frac{dV}{dr}\right)$ is zero everywhere inside the spherical shell. The gravitational potential gradient is equal to the negative of gravitational intensity. Hence, intensity is also zero at all points inside the spherical shell. This indicates that gravitational forces acting at a point in a spherical shell are symmetric.

If the upper half of a spherical shell is cut out (as shown in the given figure), then the net gravitational force acting on a particle located at centre O will be in the downward direction.



Since gravitational intensity at a point is defined as the gravitational force per unit mass at that point, it will also act in the downward direction. Thus, the gravitational intensity at centre O of the given hemispherical shell has the direction as indicated by arrow **c**.

Question 8.11:

Choose the correct answer from among the given ones:

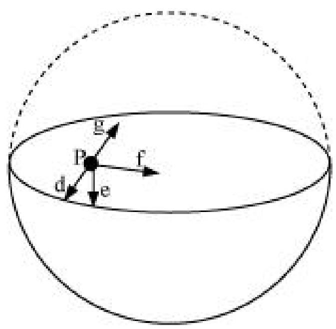
For the problem 8.10, the direction of the gravitational intensity at an arbitrary point P is indicated by the arrow (i) d, (ii) e, (iii) f, (iv) g.

Answer

Answer: (ii)

Gravitational potential (V) is constant at all points in a spherical shell. Hence, the gravitational potential gradient $\left(\frac{dV}{dr}\right)$ is zero everywhere inside the spherical shell. The gravitational potential gradient is equal to the negative of gravitational intensity. Hence, intensity is also zero at all points inside the spherical shell. This indicates that gravitational forces acting at a point in a spherical shell are symmetric.

If the upper half of a spherical shell is cut out (as shown in the given figure), then the net gravitational force acting on a particle at an arbitrary point P will be in the downward direction.



Since gravitational intensity at a point is defined as the gravitational force per unit mass at that point, it will also act in the downward direction. Thus, the gravitational intensity at an arbitrary point P of the hemispherical shell has the direction as indicated by arrow **e**.

Question 8.12:

A rocket is fired from the earth towards the sun. At what distance from the earth's centre is the gravitational force on the rocket zero? Mass of the sun = 2×10^{30} kg, mass of the earth = 6×10^{24} kg. Neglect the effect of other planets etc. (orbital radius = 1.5×10^{11} m).

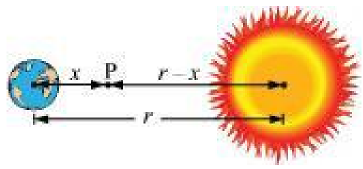
Answer

Mass of the Sun, $M_s = 2 \times 10^{30}$ kg

Mass of the Earth, $M_e = 6 \times 10^{24}$ kg

Orbital radius, $r = 1.5 \times 10^{11}$ m

Mass of the rocket = m



Let x be the distance from the centre of the Earth where the gravitational force acting on satellite P becomes zero.

From Newton's law of gravitation, we can equate gravitational forces acting on satellite P under the influence of the Sun and the Earth as:

$$\frac{GmM_s}{(r-x)^2} = Gm\frac{M_e}{x^2}$$

$$\left(\frac{r-x}{x}\right)^2 = \frac{M_s}{M_e}$$

$$\frac{r-x}{x} = \left(\frac{2\times10^{30}}{60\times10^{24}}\right)^{\frac{1}{2}} = 577.35$$

$$1.5\times10^{11} - x = 577.35x$$

$$578.35x = 1.5\times10^{11}$$

$$x = \frac{1.5\times10^{11}}{578.35} = 2.59\times10^{8}\ \text{m}$$

Question 8.13:

How will you 'weigh the sun', that is estimate its mass? The mean orbital radius of the earth around the sun is 1.5×10^8 km.

Answer

Orbital radius of the Earth around the Sun, $r = 1.5 \times 10^{11}$ m

Time taken by the Earth to complete one revolution around the Sun,

$T = 1$ year = 365.25 days

$= 365.25 \times 24 \times 60 \times 60$ s

Universal gravitational constant, G = 6.67×10^{-11} Nm^2 kg^{-2}

Thus, mass of the Sun can be calculated using the relation,

$$M = \frac{4\pi^2 r^3}{GT^2}$$

$$= \frac{4 \times (3.14)^2 \times (1.5 \times 10^{11})^3}{6.67 \times 10^{-11} \times (365.25 \times 24 \times 60 \times 60)^2}$$

$$= \frac{133.24 \times 10}{6.64 \times 10^4} = 2.0 \times 10^{30} \text{ kg}$$

Hence, the mass of the Sun is 2×10^{30} kg.

Question 8.14:

A Saturn year is 29.5 times the earth year. How far is the Saturn from the sun if the earth is 1.50×10^8 km away from the sun?

Answer

Distance of the Earth from the Sun, $r_e = 1.5 \times 10^8$ km $= 1.5 \times 10^{11}$ m

Time period of the Earth = T_e

Time period of Saturn, $T_s = 29.5\ T_e$

Distance of Saturn from the Sun = r_s

From Kepler's third law of planetary motion, we have

$$T = \left(\frac{4\pi^2 r^3}{GM}\right)^{\frac{1}{2}}$$

For Saturn and Sun, we can write

$$\frac{r_s^3}{r_e^3} = \frac{T_s^2}{T_e^2}$$

$$r_s = r_e\left(\frac{T_s}{T_e}\right)^{\frac{2}{3}}$$

$$= 1.5\times10^{11}\left(\frac{29.5T_e}{T_e}\right)^{\frac{2}{3}}$$

$$= 1.5\times10^{11}\left(29.5\right)^{\frac{2}{3}}$$

$$= 1.5\times10^{11}\times9.55$$

$$= 14.32\times10^{11}\,\text{m}$$

Hence, the distance between Saturn and the Sun is 1.43×10^{12} m..

Question 8.15:

A body weighs 63 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth?

Answer

Weight of the body, W = 63 N

Acceleration due to gravity at height h from the Earth's surface is given by the relation:

$$g' = \frac{g}{\left(\frac{1+h}{R_e}\right)^2}$$

Where,

g = Acceleration due to gravity on the Earth's surface

Re = Radius of the Earth

$$For\ h = \frac{R_e}{2}$$

$$g' = \frac{g}{\left(1 + \frac{R_e}{2 \times R_e}\right)^2} = \frac{g}{\left(1 + \frac{1}{2}\right)^2} = \frac{4}{9} g$$

Weight of a body of mass m at height h is given as:

$$W' = mg'$$

$$= m \times \frac{4}{9} g = \frac{4}{9} \times mg$$

$$= \frac{4}{9} W$$

$$= \frac{4}{9} \times 63 = 28 \text{ N}$$

Question 8.16:

Assuming the earth to be a sphere of uniform mass density, how much would a body weigh half way down to the centre of the earth if it weighed 250 N on the surface?

Answer

Weight of a body of mass m at the Earth's surface, $W = mg = 250$ N

Body of mass m is located at depth, $d = \frac{1}{2} R_e$

Where,

R_e = Radius of the Earth

Acceleration due to gravity at depth g (d) is given by the relation:

$$g' = \left(1 - \frac{d}{R_e}\right) g$$
$$= \left(1 - \frac{R_e}{2 \times R_e}\right) g = \frac{1}{2} g$$

Weight of the body at depth d,

$$W' = mg'$$
$$= m \times \frac{1}{2} g = \frac{1}{2} mg = \frac{1}{2} W$$
$$= \frac{1}{2} \times 250 = 125 \text{ N}$$

Question 8.17:

A rocket is fired vertically with a speed of 5 km s^{-1} from the earth's surface. How far from the earth does the rocket go before returning to the earth? Mass of the earth = 6.0 × 10^{24} kg; mean radius of the earth = 6.4 × 10^{6} m; G= 6.67 × 10^{-11} N m^{2} kg$^{-2.}$

<u>**Answer**</u>

Answer: 8 × 10^{6} m from the centre of the Earth

Velocity of the rocket, v = 5 km/s = 5 × 10^{3} m/s

Mass of the Earth, $M_e = 6.0 \times 10^{24}$ kg

Radius of the Earth, $R_e = 6.4 \times 10^{6}$ m

Height reached by rocket mass, $m = h$

At the surface of the Earth,

Total energy of the rocket = Kinetic energy + Potential energy

$$= \frac{1}{2}mv^2 + \left(\frac{-GM_e m}{R_e}\right)$$

At highest point *h*,

$v = 0$

And, Potential energy $= -\frac{GM_e m}{R_e + h}$

Total energy of the rocket $= 0 + \left(-\frac{GM_e m}{R_e + h}\right) = -\frac{GM_e m}{R_e + h}$

From the law of conservation of energy, we have

Total energy of the rocket at the Earth's surface = Total energy at height *h*

$$\frac{1}{2}mv^2 + \left(-\frac{GM_e m}{R_e}\right) = -\frac{GM_e m}{R_e + h}$$

$$\frac{1}{2}v^2 = GM_e\left(\frac{1}{R_e} - \frac{1}{R_e + h}\right)$$

$$= GM_e\left(\frac{R_e + h - R_e}{R_e\left(R_e + h\right)}\right)$$

$$\frac{1}{2}v^2 = \frac{GM_e h}{R_e\left(R_e + h\right)} \times \frac{R_e}{R_e}$$

$$\frac{1}{2} \times v^2 = \frac{gR_e h}{R_e + h}$$

Where $g = \frac{GM}{R_e^2} = 9.8 \text{ m/s}^2$ (Acceleration due to gravity on the Earth's surface)

$$\therefore v^2\left(R_e + h\right) = 2gR_e h$$

$$v^2 R_e = h\left(2gR_e - v^2\right)$$

$$h = \frac{R_e v^2}{2gR_e - v^2}$$

$$= \frac{6.4 \times 10^6 \times \left(5 \times 10^3\right)^2}{2 \times 9.8 \times 6.4 \times 10^6 - \left(5 \times 10^3\right)^2}$$

$$h = \frac{6.4 \times 25 \times 10^{12}}{100.44 \times 10^6} = 1.6 \times 10^6 \text{ m}$$

Height achieved by the rocket with respect to the centre of the Earth

$= R_e + h$
$= 6.4\times10^6 + 1.6\times10^6$
$= 8.0\times10^6\ \text{m}$

Question 8.18:

The escape speed of a projectile on the earth's surface is 11.2 km s^{-1}. A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.

Answer

Escape velocity of a projectile from the Earth, v_{esc} = 11.2 km/s

Projection velocity of the projectile, $v_p = 3v_{esc}$

Mass of the projectile = m

Velocity of the projectile far away from the Earth = v_f

Total energy of the projectile on the Earth $= \frac{1}{2}mv_p^2 - \frac{1}{2}mv_{esc}^2$

Gravitational potential energy of the projectile far away from the Earth is zero.

Total energy of the projectile far away from the Earth = $\frac{1}{2}mv_f^2$

From the law of conservation of energy, we have

$$\frac{1}{2}mv_p^2 - \frac{1}{2}mv_{esc}^2 = \frac{1}{2}mv_f^2$$
$$v_f = \sqrt{v_p^2 - v_{esc}^2}$$
$$= \sqrt{(3v_{esc})^2 - (v_{esc})^2}$$
$$= \sqrt{8}\, v_{esc}$$
$$= \sqrt{8}\times 11.2 = 31.68 \text{ km/s}$$

Question 8.19:

A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of the earth's gravitational influence? Mass of the satellite = 200 kg; mass of the earth = 6.0×10^{24} kg; radius of the earth = 6.4×10^{6} m; G = 6.67×10^{-11} N m^2 kg^{-2}.

Answer

Mass of the Earth, $M = 6.0 \times 10^{24}$ kg

Mass of the satellite, $m = 200$ kg

Radius of the Earth, $R_e = 6.4 \times 10^{6}$ m

Universal gravitational constant, G = 6.67×10^{-11} Nm^{2}kg^{-2}

Height of the satellite, $h = 400$ km $= 4 \times 10^{5}$ m $= 0.4 \times 10^{6}$ m

Total energy of the satellite at height h $= \frac{1}{2}mv^2 + \left(\frac{-GM_e m}{R_e + h}\right)$

Orbital velocity of the satellite, $v = \sqrt{\frac{GM_e}{R_e + h}}$

Total energy of height, h $= \frac{1}{2}m\left(\frac{GM_e}{R_e + h}\right) - \frac{GM_e m}{R_e + h} = -\frac{1}{2}\left(\frac{GM_e m}{R_e + h}\right)$

The negative sign indicates that the satellite is bound to the Earth. This is called bound energy of the satellite.

Energy required to send the satellite out of its orbit = – (Bound energy)

$$= \frac{1}{2} \frac{GM_e m}{(R_e + h)}$$

$$= \frac{1}{2} \times \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 200}{(6.4 \times 10^6 + 0.4 \times 10^6)}$$

$$= \frac{1}{2} \times \frac{6.67 \times 6 \times 2 \times 10}{6.8 \times 10^6} = 5.9 \times 10^9 \text{ J}$$

Question 8.20:

Two stars each of one solar mass (= 2× 10^{30} kg) are approaching each other for a head on collision. When they are a distance 109 km, their speeds are negligible. What is the speed with which they collide? The radius of each star is 104 km. Assume the stars to remain undistorted until they collide. (Use the known value of G).

Answer

Mass of each star, $M = 2 \times 10^{30}$ kg

Radius of each star, $R = 10^4$ km $= 10^7$ m

Distance between the stars, $r = 10^9$ km $= 10^{12}$m

For negligible speeds, $v = 0$ total energy of two stars separated at distance r

$$= \frac{-GMM}{r} + \frac{1}{2}mv^2$$

$$= \frac{-GMM}{r} + 0 \qquad \text{... (i)}$$

Now, consider the case when the stars are about to collide:

Velocity of the stars = v

Distance between the centers of the stars = $2R$

Total kinetic energy of both stars $= \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = Mv^2$

Total potential energy of both stars $= \frac{-GMM}{2R}$

Total energy of the two stars = $Mv^2 - \frac{GMM}{2R}$ ... (ii)

Using the law of conservation of energy, we can write:

$$Mv^2 - \frac{GMM}{2R} = \frac{-GMM}{r}$$
$$v^2 = \frac{-GM}{r} + \frac{GM}{2R} = GM\left(-\frac{1}{r} + \frac{1}{2R}\right)$$
$$= 6.67\times10^{-11}\times2\times10^{30}\left[-\frac{1}{10^{12}} + \frac{1}{2\times10^{7}}\right]$$
$$= 13.34\times10^{19}\left[-10^{-12} + 5\times10^{-8}\right]$$

$$\sim 13.34\times10^{19}\times5\times10^{-8}$$
$$\sim 6.67\times10^{12}$$
$$v = \sqrt{6.67\times10^{12}} = 2.58\times10^{6}\,\text{m/s}$$

Question 8.21:

Two heavy spheres each of mass 100 kg and radius 0.10 m are placed 1.0 m apart on a horizontal table. What is the gravitational force and potential at the mid point of the line joining the centers of the spheres? Is an object placed at that point in equilibrium? If so, is the equilibrium stable or unstable?

Answer

Answer:

0;

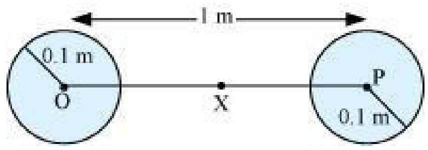
-2.7×10^{-8} J /kg;

Yes;

Unstable

Explanation:

The situation is represented in the given figure:



Mass of each sphere, M = 100 kg

Separation between the spheres, r = 1m

X is the mid point between the spheres. Gravitational force at point X will be zero. This is because gravitational force exerted by each sphere will act in opposite directions.

Gravitational potential at point X:

$$= \frac{-GM}{\left(\frac{r}{2}\right)} - \frac{GM}{\left(\frac{r}{2}\right)} = -4\frac{GM}{r}$$

$$= \frac{4\times 6.67\times 10^{-11}\times 100}{1}$$

$$= -2.67\times 10^{-8} \text{ J/kg}$$

Any object placed at point X will be in equilibrium state, but the equilibrium is unstable. This is because any change in the position of the object will change the effective force in that direction.

Question 8.22:

As you have learnt in the text, a geostationary satellite orbits the earth at a height of nearly 36,000 km from the surface of the earth. What is the potential due to earth's gravity at the site of this satellite? (Take the potential energy at infinity to be zero). Mass of the earth = 6.0×10^{24} kg, radius = 6400 km.

Answer

Mass of the Earth, $M = 6.0 \times 10^{24}$ kg

Radius of the Earth, $R = 6400$ km $= 6.4 \times 10^{6}$ m

Height of a geostationary satellite from the surface of the Earth,

$h = 36000$ km $= 3.6 \times 10^{7}$ m

Gravitational potential energy due to Earth's gravity at height *h,*

$$= \frac{-GM}{(R+h)}$$

$$= -\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{3.6 \times 10^{7} + 0.64 \times 10^{7}}$$
$$= -\frac{6.67 \times 6}{4.24} \times 10^{13-7}$$
$$= -9.4 \times 10^{6} \text{ J/kg}$$

Question 8.23:

A star 2.5 times the mass of the sun and collapsed to a size of 12 km rotates with a speed of 1.2 rev. per second. (Extremely compact stars of this kind are known as neutron stars. Certain stellar objects called pulsars belong to this category). Will an object placed on its equator remain stuck to its surface due to gravity? (Mass of the sun $= 2 \times 10^{30}$ kg).

Answer

Answer: Yes

A body gets stuck to the surface of a star if the inward gravitational force is greater than the outward centrifugal force caused by the rotation of the star.

Gravitational force, $f_g = \frac{GMm}{R^2}$

Where,

M = Mass of the star $= 2.5 \times 2 \times 10^{30} = 5 \times 10^{30}$ kg

m = Mass of the body

R = Radius of the star = 12 km = 1.2×10^4 m

$$\therefore f_g = \frac{6.67 \times 10^{-11} \times 5 \times 10^{30} \times m}{\left(1.2 \times 10^4\right)^2} = 2.31 \times 10^{11} m \text{ N}$$

Centrifugal force, $f_c = mr\omega^2$
ω = Angular speed = $2\pi\nu$

ν = Angular frequency = 1.2 rev s^{-1}

$f_c = mR\,(2\pi\nu)^2$

$= m \times (1.2 \times 10^4) \times 4 \times (3.14)^2 \times (1.2)^2 = 1.7 \times 10^5 m$ N

Since $f_g > f_c$, the body will remain stuck to the surface of the star.

Question 8.24:

A spaceship is stationed on Mars. How much energy must be expended on the spaceship to launch it out of the solar system? Mass of the space ship = 1000 kg; mass of the Sun = 2×10^{30} kg; mass of mars = 6.4×10^{23} kg; radius of mars = 3395 km; radius of the orbit of mars = 2.28×10^8kg; G= 6.67×10^{-11} m^2kg^{-2}.

Answer

Mass of the spaceship, m_s = 1000 kg

Mass of the Sun, $M = 2 \times 10^{30}$ kg

Mass of Mars, $m_m = 6.4 \times 10^{23}$ kg

Orbital radius of Mars, $R = 2.28 \times 10^8$ kg $= 2.28 \times 10^{11}$m

Radius of Mars, r = 3395 km = 3.395×10^6 m

Universal gravitational constant, G = 6.67×10^{-11} m^2kg^{-2}

Potential energy of the spaceship due to the gravitational attraction of the Sun $= \frac{-GMm_s}{R}$

Potential energy of the spaceship due to the gravitational attraction of Mars $= \frac{-GM_m m_s}{r}$

Since the spaceship is stationed on Mars, its velocity and hence, its kinetic energy will be zero.

Total energy of the spaceship $= \frac{-GMm_s}{R} - \frac{-GM_s m_m}{r}$

$$= -Gm_s\left(\frac{M}{R} + \frac{m_m}{r}\right)$$

The negative sign indicates that the system is in bound state.

Energy required for launching the spaceship out of the solar system

= – (Total energy of the spaceship)

$$= Gm_s\left(\frac{M}{R} + \frac{m_m}{r}\right)$$
$$= 6.67\times10^{-11}\times10^3\times\left(\frac{2\times10^{30}}{2.28\times10^{11}} + \frac{6.4\times10^{23}}{3.395\times10^6}\right)$$
$$= 6.67\times10^{-8}\left(87.72\times10^{17} + 1.88\times10^{17}\right)$$
$$= 6.67\times10^{-8}\times89.50\times10^{17}$$
$$= 596.97\times10^9$$
$$= 6\times10^{11}\,\text{J}$$

Question 8.25:

A rocket is fired ‘vertically’ from the surface of mars with a speed of 2 km s–1. If 20% of its initial energy is lost due to Martian atmospheric resistance, how far will the rocket go from the surface of mars before returning to it? Mass of mars = 6.4× 1023 kg; radius of mars = 3395 km; G = 6.67× 10^{-11} N m^2 kg^{-2}.

Answer

Initial velocity of the rocket, $v = 2$ km/s $= 2 \times 10^3$ m/s

Mass of Mars, $M = 6.4 \times 10^{23}$ kg

Radius of Mars, $R = 3395$ km $= 3.395 \times 10^6$ m

Universal gravitational constant, G = 6.67×10^{-11} N m^2 kg^{-2}

Mass of the rocket = m

Initial kinetic energy of the rocket = $\frac{1}{2}mv^2$

Initial potential energy of the rocket $= \frac{-GMm}{R}$

Total initial energy $= \frac{1}{2}mv^2 - \frac{GMm}{R}$

If 20 % of initial kinetic energy is lost due to Martian atmospheric resistance, then only 80 % of its kinetic energy helps in reaching a height.

Total initial energy available $= \frac{80}{100} \times \frac{1}{2}mv^2 - \frac{GMm}{R} = 0.4mv^2 - \frac{GMm}{R}$

Maximum height reached by the rocket = h

At this height, the velocity and hence, the kinetic energy of the rocket will become zero.

Total energy of the rocket at height h $= -\frac{GMm}{(R+h)}$

Applying the law of conservation of energy for the rocket, we can write:

$$0.4mv^2 - \frac{GMm}{R} = \frac{-GMm}{(R+h)}$$

$$0.4v^2 = \frac{GM}{R} - \frac{GM}{R+h}$$

$$= GM\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$= GM\left(\frac{R+h-R}{R(R+h)}\right)$$

$$= \frac{GMh}{R(R+h)}$$

$$\frac{R+h}{h} = \frac{GM}{0.4v^2R}$$

$$\frac{R}{h} + 1 = \frac{GM}{0.4v^2R}$$

$$\frac{R}{h} = \frac{GM}{0.4v^2R} - 1$$

$$h = \frac{R}{\frac{GM}{0.4v^2R} - 1}$$

$$= \frac{0.4R^2v^2}{GM - 0.4v^2R}$$

$$= \frac{0.4\times(3.395\times10^6)^2\times(2\times10^3)^2}{6.67\times10^{-11}\times6.4\times10^{23} - 0.4\times(2\times10^3)^2\times(3.395\times10^6)}$$

$$= \frac{18.442\times10^{18}}{42.688\times10^{12} - 5.432\times10^{12}} = \frac{18.442}{37.256}\times10^6$$

$$= 495\times10^3\text{ m} = 495\text{ km}$$