1. Prove that $\dfrac{\tan A+\sec A-1}{\tan A-\sec A+1}=\dfrac{1+\sin A}{\cos A}$

Sol. $\text{L.H.S.}=\dfrac{\tan A+\sec A-1}{\tan A-\sec A+1}$

$$=\frac{\tan A+\sec A-(\sec^2 A-\tan^2 A)}{(\tan A-\sec A+1)} \qquad [\because \sec^2 A-\tan^2 A=1]$$

$$=\frac{(\tan A+\sec A)-(\sec A+\tan A)(\sec A-\tan A)}{(1-\sec A+\tan A)}$$

$$=\frac{(\sec A+\tan A)(1-\sec A+\tan A)}{1-\sec A+\tan A}$$

$$=\sec A+\tan A=\frac{1}{\cos A}+\frac{\sin A}{\cos A}=\frac{1+\sin A}{\cos A}=\text{R.H.S.}$$

2. If $\dfrac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}=y$, then prove that $\dfrac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}$ is also equal to y.

Sol. We have, $\dfrac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}=y$

Now, $\dfrac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}=\dfrac{(1-\cos\alpha+\sin\alpha)}{(1+\sin\alpha)}\cdot\dfrac{(1+\cos\alpha+\sin\alpha)}{(1+\cos\alpha+\sin\alpha)}$

$$=\frac{(1+\sin\alpha)^2-\cos^2\alpha}{(1+\sin\alpha)(1+\sin\alpha+\cos\alpha)}$$

$$=\frac{1+\sin^2\alpha+2\sin\alpha-1+\sin^2\alpha}{(1+\sin\alpha)(1+\sin\alpha+\cos\alpha)}$$

$$= \frac{2 \sin \alpha(1 + \sin \alpha)}{(1 + \sin \alpha)(1 + \sin \alpha + \cos \alpha)}$$

$$= \frac{2 \sin \alpha}{1 + \sin \alpha + \cos \alpha} = y$$

3. If $m \sin \theta = n \sin (\theta + 2\alpha)$, then prove that $\tan (\theta + \alpha) \cot \alpha = \frac{m+n}{m-n}$.

Sol. We have, $m \sin \theta = n \sin (\theta + 2\alpha)$

$$\therefore \qquad \frac{\sin(\theta + 2\alpha)}{\sin \theta} = \frac{m}{n}$$

Using componendo and dividendo, we get

$$\frac{\sin(\theta + 2\alpha) + \sin \theta}{\sin(\theta + 2\alpha) - \sin \theta} = \frac{m+n}{m-n}$$

$$\Rightarrow \qquad \frac{2 \sin\left(\frac{\theta + 2\alpha + \theta}{2}\right) \cdot \cos\left(\frac{\theta + 2\alpha - \theta}{2}\right)}{2 \cos\left(\frac{\theta + 2\alpha + \theta}{2}\right) \cdot \sin\left(\frac{\theta + 2\alpha - \theta}{2}\right)} = \frac{m+n}{m-n}$$

$$\Rightarrow \qquad \frac{\sin(\theta + \alpha) \cdot \cos \alpha}{\cos(\theta + \alpha) \cdot \sin \alpha} = \frac{m+n}{m-n}$$

$$\Rightarrow \qquad \tan (\theta + \alpha) \cdot \cot \alpha = \frac{m+n}{m-n}$$

Q4. If cos (α +) =4/5 and sin (α-)=5/13 , where α lie between 0 and π/4, then find the value of tan 2α.

Sol. We have $\cos(\alpha+\beta) = \frac{4}{5}$ and $\sin(\alpha-\beta) = \frac{5}{13}$

$\Rightarrow \quad \tan(\alpha+\beta) = \pm\frac{3}{4}$

and $\quad \tan(\alpha-\beta) = \pm\frac{5}{12}$

Since $\alpha \in (0, \pi/2)$, $2\alpha \in (0, \pi)$, for which $\tan 2\alpha > 0$

Now, $\tan 2\alpha = \tan[(\alpha+\beta)+(\alpha-\beta)]$

$$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\cdot\tan(\alpha-\beta)} = \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\cdot\frac{5}{12}} = \frac{36+20}{48-15} = \frac{56}{33}$$

As other values of $\tan(\alpha+\beta)$ and $\tan(\alpha-\beta)$ gives negative value of $\tan 2\alpha$.

5. If $\tan x = \frac{b}{a}$, then find the value of $\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$.

Sol. We have $\tan x = \frac{b}{a}$

Now, $\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} = \frac{(a+b)+(a-b)}{\sqrt{a^2-b^2}}$

$$= \frac{2a}{a\sqrt{1-\left(\frac{b}{a}\right)^2}}$$

$$= \frac{2}{\sqrt{1-\tan^2 x}}$$

$$= \frac{2\cos x}{\sqrt{\cos^2 x - \sin^2 x}}$$

$$= \frac{2\cos x}{\sqrt{\cos 2x}}$$

Q6. Prove that cos cos /2- cos 3 cos 9/2 = sin 7/2 sin 4 .

Sol. L.H.S. $= \cos\theta \cos\frac{\theta}{2} - \cos 3\theta \cos\frac{9\theta}{2}$

$$= \frac{1}{2}\left[2\cos\theta \cdot \cos\frac{\theta}{2} - 2\cos 3\theta \cdot \cos\frac{9\theta}{2}\right]$$

$$= \frac{1}{2}\left(\cos\frac{3\theta}{2} + \cos\frac{\theta}{2} - \cos\frac{15\theta}{2} - \cos\frac{3\theta}{2}\right)$$

$$= \frac{1}{2}\left[\cos\frac{\theta}{2} - \cos\frac{15\theta}{2}\right] = \frac{1}{2}\left[2\sin\left(\frac{\theta+15\theta}{4}\right) \cdot \sin\left(\frac{15\theta-\theta}{4}\right)\right]$$

$$= \sin 4\theta \cdot \sin\frac{7\theta}{2} = \text{R.H.S.}$$

Q7. If a cos θ + b sin θ =m and a sin θ -b cosθ = n, then show that $a^2 + b^2$-$m^2 + n^2$

Sol: We have, a cos θ + b sin θ = m (i)

and a sin θ -bcos θ = n (ii)

On squaring Eqs. (i) and (ii) and then adding the resulting equations, we get

$$m^2 + n^2 = (a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2$$

$$= a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta \cdot \cos\theta + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta \cdot \cos\theta$$

$$= a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta) = a^2 + b^2$$

Q8. Find the value of tan 22°30′

Sol. We know that, $\tan\frac{\theta}{2}=\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}=\frac{2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}=\frac{\sin\theta}{1+\cos\theta}$

$$\therefore \qquad \tan 22°30' = \frac{\sin 45°}{1+\cos 45°}=\frac{\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}=\frac{1}{\sqrt{2}+1}$$

Q9. Prove that sin 4A = 4 sin A $\cos^3$A – 4 cos A $\sin^3$ A.

Sol: L.H.S. = sin 4A

= 2 sin 2A- cos 2A = 2(2 sin A cosA)($\cos^2$ A – $\sin^2$ A)

= 4 sin A • $\cos^3$ A – 4 cos A $\sin^3$ A = R.H.S.

Q10. If tan + sin = m and tan – sin = n, then prove that m^2-n^2 = 4 sin tan

Sol: We have, tan + sin = m (i)

And tan -sin =n (ii)

Now, m + n = 2 tan

And m – n = 2 sin.

(m + n)(m -n) = 4 sin 6

tan m^2 -n^2 = 4 *sin -tan*

Q11. If tan (A + B) =p and tan (A – B) = q, then show that tan 2A = p+q / 1 – pq

Sol: We have tan (A + B) =p and tan (A – B) = q

tan2A = tan [(A + B) + (A-B)]

$$=\frac{\tan(A+B)+\tan(A-B)}{1-\tan(A+B)\tan(A-B)}=\frac{p+q}{1-pq}$$

Q12. If cos + cos = 0 = sin + sin β, then prove that cos 2 + cos 2β = -2 cos (α +).

Sol. We have, $\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$

$\Rightarrow \quad (\cos\alpha + \cos\beta)^2 - (\sin\alpha + \sin\beta)^2 = 0$

$\Rightarrow \quad \cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta - \sin^2\alpha - \sin^2\beta - 2\sin\alpha\sin\beta = 0$

$\Rightarrow \quad (\cos^2\alpha - \sin^2\alpha) + (\cos^2\beta - \sin^2\beta) = -2(\cos\alpha\cos\beta - \sin\alpha\sin\beta)$

$\Rightarrow \quad \cos 2\alpha + \cos 2\beta = -2\cos(\alpha + \beta)$

13. If $\dfrac{\sin(x+y)}{\sin(x-y)} = \dfrac{a+b}{a-b}$, then show that $\dfrac{\tan x}{\tan y} = \dfrac{a}{b}$.

Sol. Given that, $\dfrac{\sin(x+y)}{\sin(x-y)} = \dfrac{a+b}{a-b}$

Using componendo and dividendo, we get

$$\Rightarrow \quad \frac{\sin(x+y) + \sin(x-y)}{\sin(x+y) - \sin(x-y)} = \frac{a+b+a-b}{a+b-a+b}$$

$$\Rightarrow \quad \frac{2\sin\left(\dfrac{x+y+x-y}{2}\right)\cdot\cos\left(\dfrac{x+y-x+y}{2}\right)}{2\cos\left(\dfrac{x+y+x-y}{2}\right)\cdot\sin\left(\dfrac{x+y-x+y}{2}\right)} = \frac{2a}{2b}$$

$$\Rightarrow \quad \frac{\sin x\cdot\cos y}{\cos x\cdot\sin y} = \frac{a}{b} \Rightarrow \frac{\tan x}{\tan y} = \frac{a}{b}$$

14. If $\tan\theta = \dfrac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$, then show that $\sin\alpha + \cos\alpha = \sqrt{2}\cos\theta$.

Sol. We have, $\tan\theta = \dfrac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$

$$\Rightarrow \quad 1 + \tan^2\theta = 1 + \frac{(\sin\alpha - \cos\alpha)^2}{(\sin\alpha + \cos\alpha)^2}$$

$$\Rightarrow \quad \sec^2\theta = \frac{(\sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha + \sin^2\alpha + \cos^2\alpha - 2\sin\alpha\cos\alpha)}{(\sin\alpha + \cos\alpha)^2}$$

$$\Rightarrow \quad \sec^2\theta = \frac{2}{(\sin\alpha + \cos\alpha)^2}$$

$$\Rightarrow \quad (\sin\alpha + \cos\alpha)^2 = 2\cos^2\theta \quad \Rightarrow \quad \sin\alpha + \cos\alpha = \sqrt{2}\cos\theta$$

Q15. If sin θ+ cos θ =1, then find the general value of θ

Sol. $\sin\theta + \cos\theta = 1$

$$\Rightarrow \quad \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad \sin\theta\cos\frac{\pi}{4} + \cos\theta\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \sin\left(\theta + \frac{\pi}{4}\right) = \sin\frac{\pi}{4} \quad \Rightarrow \quad \theta + \frac{\pi}{4} = n\pi + (-1)^n\frac{\pi}{4}, n \in Z$$

$$\therefore \quad \theta = n\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{4}, n \in Z$$

Q16. Find the most general value of θ satisfying the equation tan θ = -1 and cos θ = 1/√2 .

Sol: We have tan θ = -1 and cos θ =1/√2 .

So, θ lies in IV quadrant.

θ = 7/4

So, general solution is θ = 7π/4 + 2 n π, n∈ Z

Q17. If cot θ + tan θ = 2 cosec θ, then find the general value of θ

Sol: Given that, cot θ + tan θ = 2 cosec θ

$$\Rightarrow \quad \frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}=\frac{2}{\sin\theta}$$

$$\Rightarrow \quad \frac{\cos^2+\sin^2\theta}{\sin\theta\cdot\cos\theta}=\frac{2}{\sin\theta}\Rightarrow\frac{1}{\cos\theta}=2$$

$$\Rightarrow \quad \cos\theta=\frac{1}{2} \quad \Rightarrow \quad \cos\theta=\cos\frac{\pi}{3}$$

$$\therefore \quad \theta=2n\pi\pm\frac{\pi}{3}, n\in Z$$

Q18. If 2 $\sin^2$ θ =3 cos θ, where 0≤θ≤2, then find the value of θ

Sol. We have, $2\sin^2\theta=3\cos\theta$

$$\Rightarrow \quad 2-2\cos^2\theta=3\cos\theta$$

$$\Rightarrow \quad 2\cos^2\theta+3\cos\theta-2=0$$

$$\Rightarrow \quad (\cos\theta+2)(2\cos\theta-1)=0$$

$$\therefore \quad \cos\theta=\frac{1}{2}=\cos\frac{\pi}{3}$$

$$\therefore \quad \theta=\frac{\pi}{3} \text{ or } 2\pi-\frac{\pi}{3}$$

$$\therefore \quad \theta=\frac{\pi}{3},\frac{5\pi}{3}$$

Q19. If sec x cos 5x + 1 = 0, where 0 < x <π/2 , then find the value of x.

Sol. We have, $\sec x \cos 5x + 1 = 0$

$$\Rightarrow \quad \frac{\cos 5x}{\cos x} + 1 = 0$$

$$\Rightarrow \quad \cos 5x + \cos x = 0 \quad \Rightarrow \quad 2\cos 3x \cos 2x = 0$$

$$\Rightarrow \quad \cos 3x = 0 \text{ or } \cos 2x = 0$$

If $\cos 3x = 0$, then $3x = \frac{\pi}{2}, \frac{3\pi}{2}$

If $\cos 2x = 0$, then $2x = \frac{\pi}{2}$

$$\Rightarrow \quad x = \frac{\pi}{6}, \frac{\pi}{4}$$

Long Answer Type Questions

Q20. If sin(θ + α) = a and sin(θ + β) = b , then prove that cos2(α – β) – 4abcos(α – β) = 1-2a^2 -2b^2

Sol: We have sin(θ + α) = a —(i)
sin(θ + β) = b —–-(ii)

$\therefore \quad \cos(\theta+\alpha) = \sqrt{1-a^2}$ and $\cos(\theta+\beta) = \sqrt{1-b^2}$

$\therefore \quad \cos(\alpha-\beta) = \cos[(\theta+\alpha)-(\theta+\beta)]$

$$= \cos(\theta+\beta)\cos(\theta+\alpha) + \sin(\theta+\alpha)\sin(\theta+\beta)$$

$$= \sqrt{1-a^2}\,\sqrt{1-b^2} + ab = ab + \sqrt{1-a^2-b^2+a^2b^2}$$

$\Rightarrow \quad \cos 2(\alpha-\beta) - 4ab\cos(\alpha-\beta)$

$$= 2\cos^2(\alpha-\beta) - 1 - 4ab\cos(\alpha-\beta)$$

$$= 2\cos(\alpha-\beta)[\cos(\alpha-\beta) - 2ab] - 1$$

$$= 2(ab + \sqrt{1-a^2-b^2+a^2b^2})(ab + \sqrt{1-a^2-b^2+a^2b^2} - 2ab) - 1$$

$$= 2[(\sqrt{1-a^2-b^2+a^2b^2} + ab)(\sqrt{1-a^2-b^2+a^2b^2} - ab)] - 1$$

$$= 2[1-a^2-b^2+a^2b^2-a^2b^2] - 1 = 2-2a^2-2b^2-1 = 1-2a^2-2b^2$$

21. If $\cos(\theta+\varphi) = m\cos(\theta-\varphi)$, then prove that $\tan\theta = \dfrac{1-m}{1+m}\cot\phi$.

Sol. Given that, $\cos(\theta+\phi) = m\cos(\theta-\phi)$

$$\Rightarrow \quad \frac{\cos(\theta+\phi)}{\cos(\theta-\phi)} = \frac{m}{1}$$

Using componendo and dividendo rule, we get

$$\frac{\cos(\theta-\phi)-\cos(\theta+\phi)}{\cos(\theta-\phi)+\cos(\theta+\phi)} = \frac{1-m}{1+m}$$

$$\Rightarrow \quad \frac{2\sin\left(\dfrac{\theta+\phi-\theta+\phi}{2}\right)\cdot\sin\left(\dfrac{\theta-\phi+\theta+\phi}{2}\right)}{2\cos\left(\dfrac{\theta-\phi+\theta+\phi}{2}\right)\cdot\cos\left(\dfrac{\theta-\phi-\theta-\phi}{2}\right)} = \frac{1-m}{1+m}$$

$$\Rightarrow \quad \frac{\sin\theta\cdot\sin\phi}{\cos\theta\cdot\cos\phi} = \frac{1-m}{1+m}$$

$$\Rightarrow \quad \tan\theta = \left(\frac{1-m}{1+m}\right)\cot\phi$$

Q22. Find the value of the expression

$$3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right].$$

Sol. $3\left[\sin^4\left(\dfrac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]-2\left[\sin^6\left(\dfrac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right]$

$= 3[\cos^4\alpha+\sin^4\alpha] - 2[\cos^6\alpha+\sin^6\alpha]$

$= 3[(\cos^2\alpha+\sin^2\alpha)^2 - 2\cos^2\alpha\cdot\sin^2\alpha] - 2[(\cos^2\alpha+\sin^2\alpha)^3$

$\qquad - 3\cos^2\alpha\cdot\sin^2\alpha(\cos^2\alpha+\sin^2\alpha)]$

$= 3[1-2\cos^2\alpha\cdot\sin^2\alpha] - 2[1-3\cos^2\alpha\cdot\sin^2\alpha] = 3-2 = 1$

Q23. If a cos 2+b sin 2 = c has α and β as its roots, then prove that tan α +tan β = 2b/a+c

Sol. We have, $a \cos 2\theta + b \sin 2\theta = c$ (i)

$$\Rightarrow \quad a\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)+b\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)=c$$

$$\Rightarrow \quad a(1-\tan^2\theta)+2b\tan\theta=c(1+\tan^2\theta)$$

$$\Rightarrow \quad (c+a)\tan^2\theta-2b\tan\theta+c-a=0 \qquad \text{(ii)}$$

Equation (i) has roots α and β.

Thus, equation (ii) has roots $\tan\alpha$ and $\tan\beta$.

$$\therefore \text{ Sum of roots, } \tan\alpha+\tan\beta=\frac{-(-2b)}{a+c}=\frac{2b}{a+c}$$

Q24. If x = sec ϕ-tanϕandy = cosec ϕ + cot ϕ then show that xy + x -y +1=0.

Sol. $x=\sec\phi-\tan\phi \quad \Rightarrow \quad x=\dfrac{1-\sin\phi}{\cos\phi}$

$y=\operatorname{cosec}\phi+\cot\phi \quad \Rightarrow \quad y=\dfrac{1+\cos\phi}{\sin\phi}$

$$\Rightarrow \quad xy+x-y=\frac{1-\sin\phi}{\cos\phi}\frac{1+\cos\phi}{\sin\phi}+\frac{1-\sin\phi}{\cos\phi}-\frac{1+\cos\phi}{\sin\phi}$$

$$=\frac{(1-\sin\phi)(1+\cos\phi)+(1-\sin\phi)\sin\phi-\cos\phi(1+\cos\phi)}{\sin\phi\cos\phi}$$

$$=\frac{1-\sin\phi+\cos\phi-\sin\phi\cos\phi+\sin\phi-\sin^2\phi-\cos\phi-\cos^2\phi}{\sin\phi\cos\phi}$$

$$=\frac{1-\sin\phi\cos\phi-(\sin^2\phi+\cos^2\phi)}{\sin\phi\cos\phi}=-1$$

$$\therefore \quad xy+x-y-1=0$$

Q25. If lies in the first quadrant and cos =8/17 , then find the value of cos (30° +) + cos (45° −) + cos (120° −).

Sol. Given that, $\cos\theta = \frac{8}{17}$

$\Rightarrow \quad \sin\theta = \frac{15}{17}$ [Since θ lies in the first quadrant]

Now, $\cos(30° + \theta) + \cos(45° - \theta) + \cos(120° - \theta)$

$= \cos 30° \cos\theta - \sin 30° \sin\theta + \cos 45° \cos\theta + \sin 45° \sin\theta + \cos 120° \cos\theta + \sin 120° \sin\theta$

$$= \frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta - \frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta$$

$$= \left(\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} - \frac{1}{2}\right)\cos\theta + \left(\frac{1}{\sqrt{2}} - \frac{1}{2} + \frac{\sqrt{3}}{2}\right)\sin\theta$$

$$= \left(\frac{\sqrt{3} + \sqrt{2} - 1}{2}\right)\frac{8}{17} + \left(\frac{\sqrt{2} - 1 + \sqrt{3}}{2}\right)\frac{15}{17}$$

$$= \left(\frac{8\sqrt{3} + 8\sqrt{2} - 8 + 15\sqrt{2} - 15 + 15\sqrt{3}}{34}\right) = \frac{23}{34}(\sqrt{3} + \sqrt{2} - 1)$$

Q26. Find the value of the expression $\cos^4 \pi/8 + \cos^4 3\pi/8 + \cos^4 5\pi/8 + \cos^4 7\pi/8$

Sol. $\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{5\pi}{8} + \cos^4 \frac{7\pi}{8}$

$$= \cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \left(\pi - \frac{3\pi}{8}\right) + \cos^4 \left(\pi - \frac{\pi}{8}\right)$$

$$= \cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{\pi}{8}$$

$$= 2\left[\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8}\right]$$

$$= 2\left[\cos^4 \frac{\pi}{8} + \cos^4 \left(\frac{\pi}{2} - \frac{\pi}{8}\right)\right]$$

$$= 2\left[\cos^4 \frac{\pi}{8} + \sin^4 \frac{\pi}{8}\right]$$

$$= 2\left[\left(\cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8}\right)^2 - 2\cos^2 \frac{\pi}{8} \cdot \sin^2 \frac{\pi}{8}\right]$$

$$= 2\left[1 - 2\cos^2 \frac{\pi}{8} \cdot \sin^2 \frac{\pi}{8}\right]$$

$$= 2 - \left(2\sin\frac{\pi}{8} \cdot \cos\frac{\pi}{8}\right)^2$$

$$= 2 - \left(\sin\frac{2\pi}{8}\right)^2 = 2 - \left(\frac{1}{\sqrt{2}}\right)^2 = 2 - \frac{1}{2} = \frac{3}{2}$$

Q27. Find the general solution of the equation $5\cos^2 + 7\sin^2 - 6 = 0$.

Sol. We have $5\cos^2\theta + 7\sin^2\theta - 6 = 0$

$\Rightarrow \quad 5\cos^2\theta + 7(1 - \cos^2\theta) - 6 = 0 \quad \Rightarrow \quad 5\cos^2\theta + 7 - 7\cos^2\theta - 6 = 0$

$\Rightarrow \quad \cos^2\theta = 1/2 \quad \Rightarrow \quad \cos^2\theta = \cos^2\frac{\pi}{4}$

$\therefore \quad \theta = n\pi \pm \frac{\pi}{4}, n \in Z$

Q28. Find the general solution of'the equation sin x – 3 sin 2x + sin 3x = cos x – 3 cos 2x + cos 3x.

Sol: We have, (sin x + sin 3x) – 3 sin 2x = (cos x + cos 3x) – 3 cos 2x

=> 2 sin 2x cos x – 3 sin 2x = 2 cos 2x.cos x – 3 cos 2x

=> sin 2x(2 cos x – 3) = cos 2x(2 cos x – 3)

=> sin 2x = cos 2x (As cos x ≠ 3/2)

=> tan 2x = 1 => tan 2x = tan π/4

=> 2x = nπ + π/4 , n∈Z

x = nπ/2 +π/8 , n∈Z

Q29. Find the general solution of the equation (√3- l)cos + (√3+ 1)sin = 2.

Sol. $(\sqrt{3}-1)\cos\theta+(\sqrt{3}+1)\sin\theta=2$ (i)

Put $\sqrt{3}-1=r\sin\alpha$ and $\sqrt{3}+1=r\cos\alpha$

$\therefore$ $r^2=(\sqrt{3}-1)^2+(\sqrt{3}+1)^2 \Rightarrow r^2=8 \Rightarrow r=2\sqrt{2}$

Also, $\tan\alpha=\dfrac{\sqrt{3}-1}{\sqrt{3}+1}=2-\sqrt{3} \quad \Rightarrow \alpha=\dfrac{\pi}{12}$

From eq. (i), we have

$$r\sin\alpha\cos\theta+r\cos\alpha\sin\theta=2 \quad \Rightarrow \quad r\sin(\theta+\alpha)=2$$

$$\Rightarrow \quad \sin(\theta+\alpha)=\frac{1}{\sqrt{2}} \quad \Rightarrow \quad \sin(\theta+\alpha)=\sin\frac{\pi}{4}$$

$$\Rightarrow \quad \theta+\alpha=n\pi+(-1)^n\frac{\pi}{4},\ n\in Z$$

$$\Rightarrow \quad \theta=n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{12},\ n\in Z$$

Objective Type Questions

Q30. If sin + cosec =2, then sin^2 + cosec^2 is equal to

(a) 1

(b) 4

(c) 2

(d) None of these

Sol. (c)

$$\sin\theta + \text{cosec}\,\theta = 2$$

$$\Rightarrow \quad \sin\theta + \frac{1}{\sin\theta} = 2$$

$$\Rightarrow \quad \sin^2\theta + 1 = 2\sin\theta \quad \Rightarrow \quad (\sin\theta - 1)^2 = 0 \quad \Rightarrow \quad \sin\theta = 1$$

$$\therefore \quad \sin^2\theta + \text{cosec}^2\theta = 1 + 1 = 2$$

Q31. If $f(x) = \cos^2 x + \sec^2 x$, then '

(a) f(x) <1

(b) f(x) = 1

(c) 2 <f(x) < 1

(d) fx) ≥ 2

Q32. If tan θ = 1/2 and tan ϕ = 1/3, then the value of θ + ϕ is

(a) $\frac{\pi}{6}$ (b) π (c) 0 (d) $\frac{\pi}{4}$

Sol. (d) We have, $\tan\theta = \frac{1}{2}$ and $\tan\phi = \frac{1}{3}$

$$\tan(\theta+\phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\cdot\tan\phi} = \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}} = \frac{\frac{5}{6}}{1-\frac{1}{6}} = 1$$

$$\therefore \quad \theta + \phi = \frac{\pi}{4}$$

Q33. Which of the following is not correct?

(a) sin θ = – 1/5 (b) cos θ = 1 (c) sec θ = -1/2 (d) tan θ = 20

Sol: (c)

We know that, the range of sec θ is R – (-1, 1).

Hence, sec θ cannot be equal to -1/2

Q34. The value of tan 1° tan 2° tan 3° … tan 89° is
(a) 0
(b) 1
(c) 1/2
(d) Not defined

Sol: (b)
tan 1° tan 2° tan 3° … tan 89°
= [tan 1° tan 2° … tan 44°] tan 45°[tan (90° – 44°) tan (90° – 43°)… tan (90° – 1°)]
= [tan 1° tan 2° … tan 44°] [cot 44° cot 43°……. cot 1°]
= 1-1… 1-1 = 1

35. The value of $\frac{1-\tan^2 15°}{1+\tan^2 15°}$ is

(a) 1 (b) $\sqrt{3}$ (c) $\frac{\sqrt{3}}{2}$ (d) 2

Sol. (c)

We know that $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$$\therefore \quad \frac{1-\tan^2 15°}{1+\tan^2 15°} = \cos 30° = \frac{\sqrt{3}}{2}$$

Q36. The value of cos 1° cos 2° cos 3° … cos 179° is
(a) 1/√2
(b) 0
(c) 1
(d) -1

Sol: (b)
Since cos 90° = 0, we have
cos 1° cos 2° cos 3° …cos 90°… cos 179° = 0

Q37. If tan θ = 3 and θ lies in the third quadrant, then the value of sin θ is

(a) $\frac{1}{\sqrt{10}}$ (b) $-\frac{1}{\sqrt{10}}$ (c) $\frac{-3}{\sqrt{10}}$ (d) $\frac{3}{\sqrt{10}}$

Sol. (c)

$$\tan\theta = 3 \Rightarrow \cot\theta = \frac{1}{3}$$

Now, $\operatorname{cosec}^2\theta = 1 + \cot^2\theta = 1 + \frac{1}{9} = \frac{10}{9}$

$\Rightarrow$ $\sin^2\theta = \frac{9}{10} \Rightarrow \sin\theta = -\frac{3}{\sqrt{10}}$ (as θ lies in third quadrant)

Q38. The value of tan 75° – cot 75° is equal to

(a) $2\sqrt{3}$ (b) $2+\sqrt{3}$ (c) $2-\sqrt{3}$ (d) 1

Sol. (a)

$$\tan 75° - \cot 75° = \frac{\sin 75°}{\cos 75°} - \frac{\cos 75°}{\sin 75°} = \frac{2(\sin^2 75° - \cos^2 75°)}{2\sin 75°\cos 75°} = \frac{-2\cos 150°}{\sin 150°}$$

$$= -2\cot 150° = -2\cot(180° - 30°) = 2\cot 30° = 2\sqrt{3}$$

Q39. Which of the following is correct?

(a) sin 1° > sin 1

(b) sin 1° < sin 1

(c) sin l° = sin l

(d) sin l° = π/18° sin 1

Sol: We know that, in first quadrant if θ is increasing, then sin θ is also increasing.

∴sin 1° < sin 1 [∵ 1 radian = 57∘30′]

40. If $\tan\alpha = \dfrac{m}{m+1}$ and $\tan\beta = \dfrac{1}{2m+1}$, then $\alpha+\beta$ is equal to

(a) $\dfrac{\pi}{2}$ (b) $\dfrac{\pi}{3}$ (c) $\dfrac{\pi}{6}$ (d) $\dfrac{\pi}{4}$

Sol. **(d)** Given that, $\tan\alpha = \dfrac{m}{m+1}$ and $\tan\beta = \dfrac{1}{2m+1}$

Now, $$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\cdot\tan\beta} = \frac{\dfrac{m}{m+1} + \dfrac{1}{2m+1}}{1 - \left(\dfrac{m}{m+1}\right)\left(\dfrac{1}{2m+1}\right)}$$

$$= \frac{m(2m+1)+m+1}{(m+1)(2m+1)-m} = \frac{2m^2+2m+1}{2m^2+3m+1-m} = 1$$

$\therefore \quad \alpha+\beta = \dfrac{\pi}{4}$

Q41. The minimum value of 3 cos x + 4 sin x + 8 is

(a) 5

(b) 9

(c) 7

(d) 3

Sol: (d)

3 cos x + 4sin x + 8 = 5 (3/5 cos x + 4/5sin x) + 8

= 5(sin α cos x + cos α sin x) + 8

= 5 sin(α + x) + 8, where tan α = 3/4

Q42. The value of tan 3A – tan 2A – tan A is

(a) tan 3A . tan 2A . tan A

(b) -tan 3A .tan 2A . tan A

(c) tan A . tan 2A – tan 2A . tan 3A – tan 3A . tan A

(d) None of these

Sol: (a)

3A= A+ 2A

=> tan 3A = tan (A + 2A)

=> tan 3 A = tanA + tan2A/ 1 – tan A . tan 2A

=> tan A + tan 2A = tan 3A – tan 3A• tan 2A . tan A

=> tan 3 A – tan 2A – tan A = tan 3A . tan 2A . tan A

Q43. The value of sin (45° +)- cos (45° –) is

(a) 2 cos

(b) 2 sin

(c) 1

(d) 0

Sol: (d)

sin (45° +) – cos (45° –) = sin (45° +) – sin (90° – (45° –))

= sin (45° +) – sin (45°+) = 0

Q44. The value of (π/4+) cot (π/4-) is

(a) -1

(b) 0

(c) 1

(d) Not defined

Sol. (c)

$$\cot\left(\frac{\pi}{4}+\theta\right)\cot\left(\frac{\pi}{4}-\theta\right)=\cot\left(\frac{\pi}{4}+\theta\right)\cot\left(\frac{\pi}{2}-\left(\frac{\pi}{4}-\theta\right)\right)$$

$$=\cot\left(\frac{\pi}{4}+\theta\right)\tan\left(\frac{\pi}{4}+\theta\right)=1$$

45. $\cos 2\theta \cos 2\phi + \sin^2(\theta-\phi) - \sin^2(\theta+\phi)$ is equal to

(a) $\sin 2(\theta+\phi)$ (b) $\cos 2(\theta+\phi)$ (c) $\sin 2(\theta-\phi)$ (d) $\cos 2(\theta-\phi)$

Sol. (b)

$$\cos 2\theta \cos 2\phi + \sin^2(\theta-\phi) - \sin^2(\theta+\phi)$$

$$= \cos 2\theta \cos 2\phi + \sin(\theta-\phi+\theta+\phi)\sin(\theta-\phi-\theta-\phi)$$

$$= \cos 2\theta \cos 2\phi - \sin 2\theta \sin 2\phi = \cos(2\theta+2\phi) = \cos 2(\theta+\phi)$$

Q46. The value of cos 12° + cos 84° + cos 156° + cos 132° is

(a) 1/2

(b) 1

(c) -1/2

(d) 1/8

Sol. (c)

$$\cos 12° + \cos 84° + \cos 156° + \cos 132°$$
$$= (\cos 12° + \cos 132°) + (\cos 84° + \cos 156°)$$
$$= 2 \cos 72° \cos 60° + 2 \cos 120° \cos 36°$$
$$= \cos 72° - \cos 36° = \sin 18° - \cos 36°$$
$$= \left(\frac{\sqrt{5}-1}{4}\right) - \left(\frac{\sqrt{5}+1}{4}\right) = \frac{-1}{2}$$

Q47. If tan A = 1/2 and tan B = 1/3 then tan (2A + B) is equal to

(a) 1

(b) 2

(c) 3

(d) 4

Sol. (c)

We have, $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$

Now, $\tan(2A + B) = \frac{\tan 2A + \tan B}{1 - \tan 2A \cdot \tan B}$

Also, $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A} = \frac{2 \cdot \frac{1}{2}}{1 - \frac{1}{4}} = \frac{4}{3}$

$\therefore \quad \tan(2A + B) = \frac{\frac{4}{3} + \frac{1}{3}}{1 - \frac{4}{3} \cdot \frac{1}{3}} = \frac{\frac{5}{3}}{\frac{5}{9}} = 3$

48. The value of $\sin\frac{\pi}{10}\sin\frac{13\pi}{10}$ is

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) $-\frac{1}{4}$ (d) 1

Sol. (c)

$$\sin\frac{\pi}{10}\sin\frac{13\pi}{10}=\sin\frac{\pi}{10}\sin\left(\pi+\frac{3\pi}{10}\right)=-\sin\frac{\pi}{10}\sin\frac{3\pi}{10}$$

$$=-\sin 18^\circ\sin 54^\circ=-\sin 18^\circ\cos 36^\circ$$

$$=-\left(\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4}\right)=\frac{1-5}{16}=-\frac{1}{4}$$

Q49. The value of sin 50° – sin 70° + sin 10° is equal to

(a) 1

(b) 0

(c) 1

(d) 2

Sol. (b)

$\sin 50^\circ - \sin 70^\circ + \sin 10^\circ$

$$= 2\cos\left(\frac{50^\circ+70^\circ}{2}\right)\sin\left(\frac{50^\circ-70^\circ}{2}\right)+\sin 10^\circ$$

$$=-2\cos 60^\circ\sin 10^\circ+\sin 10^\circ=-2\cdot\frac{1}{2}\sin 10^\circ+\sin 10^\circ=0$$

Q50. If sin + cos =1, then the value of sin 2 is

(a) 1

(b) 1

(c) 0

(d) -1