Question 1:

If $\left(\frac{x}{3}+1,\ y-\frac{2}{3}\right)=\left(\frac{5}{3},\ \frac{1}{3}\right)$, find the values of x and y.

Answer:

It is given that $\left(\frac{x}{3}+1,\ y-\frac{2}{3}\right)=\left(\frac{5}{3},\ \frac{1}{3}\right)$.

Since the ordered pairs are equal, the corresponding elements will also be equal.

Therefore,

$\frac{x}{3}+1=\frac{5}{3}$ and $y-\frac{2}{3}=\frac{1}{3}$.

$$\frac{x}{3}+1=\frac{5}{3}$$
$$\Rightarrow \frac{x}{3}=\frac{5}{3}-1$$
$$\Rightarrow \frac{x}{3}=\frac{2}{3}$$
$$\Rightarrow x=2$$

$$y-\frac{2}{3}=\frac{1}{3}$$
$$\Rightarrow y=\frac{1}{3}+\frac{2}{3}$$
$$\Rightarrow y=1$$

∴ x = 2 and y = 1

Question 2:

If the set A has 3 elements and the set B = {3, 4, 5}, then find the number of elements in (A × B)?

Answer:

It is given that set A has 3 elements and the elements of set B are 3, 4, and 5.

⇒ Number of elements in set B = 3

Number of elements in (A × B)

= (Number of elements in A) × (Number of elements in B)

= 3 × 3 = 9

Thus, the number of elements in (A × B) is 9.

Question 3:

If G = {7, 8} and H = {5, 4, 2}, find G × H and H × G.

Answer:

G = {7, 8} and H = {5, 4, 2}

We know that the Cartesian product P × Q of two non-empty sets P and Q is defined as

P × Q = {(p, q): p ∈ P, q ∈ Q}

∴ G × H = {(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)}

H × G = {(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)}

Question 4:

State whether each of the following statement are true or false. If the statement is false, rewrite the given statement correctly.

(i) If P = {m, n} and Q = {n, m}, then P × Q = {(m, n), (n, m)}.

(ii) If A and B are non-empty sets, then A × B is a non-empty set of ordered pairs (x, y) such that x ∈ A and y ∈ B.

(iii) If A = {1, 2}, B = {3, 4}, then A × (B ∩ Φ) = Φ.

Answer:

(i) False

If P = {m, n} and Q = {n, m}, then

P × Q = {(m, m), (m, n), (n, m), (n, n)}

(ii) True

(iii) True

Question 5:

If A = {–1, 1}, find A × A × A.

Answer:

It is known that for any non-empty set A, A × A × A is defined as

A × A × A = {(a, b, c): a, b, c ∈ A}

It is given that A = {–1, 1}

∴ A × A × A = {(–1, –1, –1), (–1, –1, 1), (–1, 1, –1), (–1, 1, 1),

(1, –1, –1), (1, –1, 1), (1, 1, –1), (1, 1, 1)}

Question 6:

If $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$. Find A and B.

Answer:

It is given that $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$

We know that the Cartesian product of two non-empty sets P and Q is defined as $P \times Q = \{(p, q): p \in P, q \in Q\}$

$\therefore$ A is the set of all first elements and B is the set of all second elements.

Thus, $A = \{a, b\}$ and $B = \{x, y\}$

Question 7:

Let $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$. Verify that

(i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(ii) $A \times C$ is a subset of $B \times D$

Answer:

(i) To verify: $A \times (B \cap C) = (A \times B) \cap (A \times C)$

We have $B \cap C = \{1, 2, 3, 4\} \cap \{5, 6\} = \Phi$

$\therefore$L.H.S. $= A \times (B \cap C) = A \times \Phi = \Phi$

$A \times B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)\}$

$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$

$\therefore$ R.H.S. $= (A \times B) \cap (A \times C) = \Phi$

$\therefore$L.H.S. = R.H.S

Hence, $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(ii) To verify: $A \times C$ is a subset of $B \times D$

$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$

$B \times D = \{(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)\}$

We can observe that all the elements of set $A \times C$ are the elements of set $B \times D$.

Therefore, $A \times C$ is a subset of $B \times D$.

Question 8:

Let A = {1, 2} and B = {3, 4}. Write A × B. How many subsets will A × B have? List them.

Answer:

A = {1, 2} and B = {3, 4}

∴ A × B = {(1, 3), (1, 4), (2, 3), (2, 4)}

⇒ n(A × B) = 4

We know that if C is a set with n(C) = m, then n[P(C)] = 2^m.

Therefore, the set A × B has 2^4 = 16 subsets. These are

Φ, {(1, 3)}, {(1, 4)}, {(2, 3)}, {(2, 4)}, {(1, 3), (1, 4)}, {(1, 3), (2, 3)},

{(1, 3), (2, 4)}, {(1, 4), (2, 3)}, {(1, 4), (2, 4)}, {(2, 3), (2, 4)},

{(1, 3), (1, 4), (2, 3)}, {(1, 3), (1, 4), (2, 4)}, {(1, 3), (2, 3), (2, 4)},

{(1, 4), (2, 3), (2, 4)}, {(1, 3), (1, 4), (2, 3), (2, 4)}

Question 9:

Let A and B be two sets such that n(A) = 3 and n (B) = 2. If (x, 1), (y, 2), (z, 1) are in A × B, find A and B, where x, *y* and z are distinct elements.

Answer:

It is given that n(A) = 3 and n(B) = 2; and (x, 1), (y, 2), (z, 1) are in A × B.

We know that A = Set of first elements of the ordered pair elements of A × B

B = Set of second elements of the ordered pair elements of A × B.

∴ x, y, and z are the elements of A; and 1 and 2 are the elements of B.

Since n(A) = 3 and n(B) = 2, it is clear that A = {x, y, z} and B = {1, 2}.

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Question 10:

The Cartesian product A × A has 9 elements among which are found (–1, 0) and (0, 1). Find the set A and the remaining elements of A × A.

Answer:

We know that if n(A) = p and n(B) = q, then n(A × B) = pq.

∴ n(A × A) = n(A) × n(A)

It is given that n(A × A) = 9

$\therefore$ n(A) × n(A) = 9

$\Rightarrow$ n(A) = 3

The ordered pairs (–1, 0) and (0, 1) are two of the nine elements of A × A.

We know that A × A = {(a, a): a ∈ A}. Therefore, –1, 0, and 1 are elements of A.

Since n(A) = 3, it is clear that A = {–1, 0, 1}.

The remaining elements of set A × A are (–1, –1), (–1, 1), (0, –1), (0, 0),

(1, –1), (1, 0), and (1, 1)

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Question 1:

Let A = {1, 2, 3, … , 14}. Define a relation R from A to A by R = {(x, y): 3x – y = 0, where x, y ∈ A}. Write down its domain, codomain and range.

Answer:

The relation R from A to A is given as

R = {(x, y): 3x – y = 0, where x, y ∈ A}

i.e., R = {(x, y): 3x = y, where x, y ∈ A}

$\therefore$R = {(1, 3), (2, 6), (3, 9), (4, 12)}

The domain of R is the set of all first elements of the ordered pairs in the relation.

$\therefore$Domain of R = {1, 2, 3, 4}

The whole set A is the codomainof the relation R.

$\therefore$Codomain of R = A = {1, 2, 3, …, 14}

The range of R is the set of all second elements of the ordered pairs in the relation.

$\therefore$Range of R = {3, 6, 9, 12}

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Question 2:

Define a relation R on the set N of natural numbers by R = {(x, y): y = x + 5, x is a natural number less than 4; x, y ∈ N}. Depict this relationship using roster form. Write down the domain and the range.

Answer:

R = {(x, y): y = x + 5, x is a natural number less than 4, x, y ∈ N}

The natural numbers less than 4 are 1, 2, and 3.

∴R = {(1, 6), (2, 7), (3, 8)}

The domain of R is the set of all first elements of the ordered pairs in the relation.

∴ Domain of R = {1, 2, 3}

The range of R is the set of all second elements of the ordered pairs in the relation.

∴ Range of R = {6, 7, 8}

Question 3:

A = {1, 2, 3, 5} and B = {4, 6, 9}. Define a relation R from A to B by R = {(x, y): the difference between x and y is odd; x ∈ A, y ∈ B}. Write R in roster form.

Answer:

A = {1, 2, 3, 5} and B = {4, 6, 9}

R = {(x, y): the difference between x and y is odd; x ∈ A, y ∈ B}

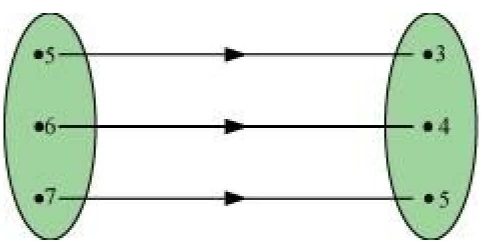
∴R = {(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)}

Question 4:

The given figure shows a relationship between the sets P and Q. write this relation

(i) in set-builder form (ii) in roster form.

What is its domain and range?



Answer:

According to the given figure, P = {5, 6, 7}, Q = {3, 4, 5}

(i) R = {(x, y): y = x – 2; x ∈ P} or R = {(x, y): y = x – 2 for x = 5, 6, 7}

(ii) R = {(5, 3), (6, 4), (7, 5)}

Domain of R = {5, 6, 7}

Range of R = {3, 4, 5}

Question 5:

Let A = {1, 2, 3, 4, 6}. Let R be the relation on A defined by

{(a, b): a, b ∈ A, b is exactly divisible by a}.

(i) Write R in roster form

(ii) Find the domain of R

(iii) Find the range of R.

Answer:

A = {1, 2, 3, 4, 6}, R = {(a, b): a, b ∈ A, b is exactly divisible by a}

(i) R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)}

(ii) Domain of R = {1, 2, 3, 4, 6}

(iii) Range of R = {1, 2, 3, 4, 6}

Question 6:

Determine the domain and range of the relation R defined by R = {(x, x + 5): x ∈ {0, 1, 2, 3, 4, 5}}.

Answer:

R = {(x, x + 5): x ∈ {0, 1, 2, 3, 4, 5}}

∴ R = {(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)}

∴Domain of R = {0, 1, 2, 3, 4, 5}

Range of R = {5, 6, 7, 8, 9, 10}

Question 7:

Write the relation R = {(x, x^3): x is a prime number less than 10} in roster form.

Answer:

R = {(x, x^3): x is a prime number less than 10}

The prime numbers less than 10 are 2, 3, 5, and 7.

∴R = {(2, 8), (3, 27), (5, 125), (7, 343)}

Question 8:

Let A = {x, y, z} and B = {1, 2}. Find the number of relations from A to B.

Answer:

It is given that A = {x, y, z} and B = {1, 2}.

∴ A × B = {(x, 1), (x, 2), (y, 1), (y, 2), (z, 1), (z, 2)}

Since n(A × B) = 6, the number of subsets of A × B is 2^6.

Therefore, the number of relations from A to B is 2^6.

Question 9:

Let R be the relation on Z defined by R = {(a, b): a, b ∈ Z, a – b is an integer}. Find the domain and range of R.

Answer:

R = {(a, b): a, b ∈ Z, a – b is an integer}

It is known that the difference between any two integers is always an integer.

∴Domain of R = Z

Range of R = Z

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Question 1:

Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.

(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}

(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}

(iii) {(1, 3), (1, 5), (2, 5)}

Answer:

(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}

Since 2, 5, 8, 11, 14, and 17 are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain = {2, 5, 8, 11, 14, 17} and range = {1}

(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}

Since 2, 4, 6, 8, 10, 12, and 14 are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain = {2, 4, 6, 8, 10, 12, 14} and range = {1, 2, 3, 4, 5, 6, 7}

(iii) {(1, 3), (1, 5), (2, 5)}

Since the same first element i.e., 1 corresponds to two different images i.e., 3 and 5, this relation is not a function.

Question 2:

Find the domain and range of the following real function:

(i) f(x) = –|x| (ii) $f(x)=\sqrt{9-x^2}$

Answer:

(i) f(x) = –|x|, x ∈ R

We know that |x| = $\begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

$\therefore f(x) = -|x| = \begin{cases} -x, & x \geq 0 \\ x, & x < 0 \end{cases}$

Since f(x) is defined for x ∈ R, the domain of f is R.

It can be observed that the range of f(x) = –|x| is all real numbers except positive real numbers.

∴The range of f is (–∞, 0].

(ii) $f(x)=\sqrt{9-x^2}$

Since $\sqrt{9-x^2}$ is defined for all real numbers that are greater than or equal to –3 and less than or equal to 3, the domain of f(x) is {x : –3 ≤ x ≤ 3} or [–3, 3].

For any value of x such that –3 ≤ x ≤ 3, the value of f(x) will lie between 0 and 3.

∴The range of f(x) is {x: 0 ≤ x ≤ 3} or [0, 3].

Question 3:

A function f is defined by f(x) = 2x – 5. Write down the values of

(i) f(0), (ii) f(7), (iii) f(–3)

Answer:

The given function is f(x) = 2x – 5.

Therefore,

(i) f(0) = 2 × 0 – 5 = 0 – 5 = –5

(ii) f(7) = 2 × 7 – 5 = 14 – 5 = 9

(iii) f(–3) = 2 × (–3) – 5 = – 6 – 5 = –11

Question 4:

The function ‘t’ which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C)=\frac{9C}{5}+32$.

Find (i) t (0) (ii) t (28) (iii) t (–10) (iv) The value of C, when t(C) = 212

Answer:

The given function is $t(C)=\frac{9C}{5}+32$.

Therefore,

(i) $t(0)=\frac{9\times 0}{5}+32=0+32=32$

(ii) $t(28)=\frac{9\times 28}{5}+32=\frac{252+160}{5}=\frac{412}{5}$

(iii) $t(-10)=\frac{9\times(-10)}{5}+32=9\times(-2)+32=-18+32=14$

(iv) It is given that t(C) = 212

$$\therefore 212=\frac{9C}{5}+32$$
$$\Rightarrow \frac{9C}{5}=212-32$$
$$\Rightarrow \frac{9C}{5}=180$$
$$\Rightarrow 9C=180\times 5$$
$$\Rightarrow C=\frac{180\times 5}{9}=100$$

Thus, the value of t, when t(C) = 212, is 100.

Question 5:

Find the range of each of the following functions.

(i) f(x) = 2 – 3x, x ∈ R, x > 0.

(ii) f(x) = x^2 + 2, x, is a real number.

(iii) $f(x) = x$, x is a real number

Answer:

(i) $f(x) = 2 - 3x$, $x \in R$, $x > 0$

The values of f(x) for various values of real numbers $x > 0$ can be written in the tabular form as

x	0.01	0.1	0.9	1	2	2.5	4	5	…
f(x)	1.97	1.7	–0.7	–1	–4	–5.5	–10	–13	…

Thus, it can be clearly observed that the range of f is the set of all real numbers less than 2.

i.e., range of f = $(-\infty, 2)$

Alter:

Let $x > 0$

$\Rightarrow 3x > 0$

$\Rightarrow 2 - 3x < 2$

$\Rightarrow f(x) < 2$

$\therefore$Range of f = $(-\infty, 2)$

(ii) $f(x) = x^2 + 2$, x, is a real number

The values of f(x) for various values of real numbers x can be written in the tabular form as

x	0	±0.3	±0.8	±1	±2	±3		…
f(x)	2	2.09	2.64	3	6	11		…..

Thus, it can be clearly observed that the range of f is the set of all real numbers greater than 2.

i.e., range of f = $[2, \infty)$

Alter:

Let x be any real number.

Accordingly,

$x^2 \geq 0$

$\Rightarrow x^2 + 2 \geq 0 + 2$

$\Rightarrow x^2 + 2 \geq 2$

$\Rightarrow f(x) \geq 2$

$\therefore$ Range of f = $[2, \infty)$

(iii) f(x) = x, x is a real number

It is clear that the range of f is the set of all real numbers.

$\therefore$ Range of f = R

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Question 1:

The relation f is defined by $f(x)=\begin{cases} x^2, & 0 \le x \le 3 \\ 3x, & 3 \le x \le 10 \end{cases}$

The relation g is defined by $g(x)=\begin{cases} x^2, & 0 \le x \le 2 \\ 3x, & 2 \le x \le 10 \end{cases}$

Show that f is a function and g is not a function.

Answer:

The relation f is defined as $f(x)=\begin{cases} x^2, & 0 \le x \le 3 \\ 3x, & 3 \le x \le 10 \end{cases}$

It is observed that for

$0 \le x < 3$, $f(x) = x^2$

$3 < x \le 10$, $f(x) = 3x$

Also, at x = 3, $f(x) = 3^2 = 9$ or $f(x) = 3 \times 3 = 9$

i.e., at x = 3, f(x) = 9

Therefore, for $0 \le x \le 10$, the images of f(x) are unique.

Thus, the given relation is a function.

The relation g is defined as $g(x)=\begin{cases} x^2, & 0 \le x \le 2 \\ 3x, & 2 \le x \le 10 \end{cases}$

It can be observed that for x = 2, $g(x) = 2^2 = 4$ and $g(x) = 3 \times 2 = 6$

Hence, element 2 of the domain of the relation g corresponds to two different images i.e., 4 and 6. Hence, this relation is not a function.

Question 2:

If $f(x) = x^2$, find $\dfrac{f(1.1)-f(1)}{(1.1-1)}$.

Answer:

$$f(x) = x^2$$

$$\therefore \frac{f(1.1)-f(1)}{(1.1-1)} = \frac{(1.1)^2-(1)^2}{(1.1-1)} = \frac{1.21-1}{0.1} = \frac{0.21}{0.1} = 2.1$$

Question 3:

Find the domain of the function $f(x) = \dfrac{x^2+2x+1}{x^2-8x+12}$

Answer:

The given function is $f(x) = \dfrac{x^2+2x+1}{x^2-8x+12}$.

$$f(x) = \frac{x^2+2x+1}{x^2-8x+12} = \frac{x^2+2x+1}{(x-6)(x-2)}$$

It can be seen that function f is defined for all real numbers except at x = 6 and x = 2.

Hence, the domain of f is R – {2, 6}.

Question 4:

Find the domain and the range of the real function f defined by $f(x) = \sqrt{(x-1)}$.

Answer:

The given real function is $f(x) = \sqrt{x-1}$.

It can be seen that $\sqrt{x-1}$ is defined for $(x - 1) \geq 0$.

i.e.,

$f(x) = \sqrt{(x-1)}$ is defined for $x \geq 1$.

Therefore, the domain of f is the set of all real numbers greater than or equal to 1 i.e., the domain of f= $[1, \infty)$.

As $x \geq 1 \Rightarrow (x - 1) \geq 0 \Rightarrow$

$$\sqrt{x-1} \geq 0$$

Therefore, the range of f is the set of all real numbers greater than or equal to 0 i.e., the range of f = $[0, \infty)$.

Question 5:

Find the domain and the range of the real function f defined by f (x) = |x – 1|.

Answer:

The given real function is f (x) = |x – 1|.

It is clear that |x – 1| is defined for all real numbers.

$\therefore$ Domain of f = R

Also, for x $\in$ R, |x – 1| assumes all real numbers.

Hence, the range of f is the set of all non-negative real numbers.

Question 6:

Let $f = \left\{\left(x, \frac{x^2}{1+x^2}\right) : x \in \mathbf{R}\right\}$ be a function from R into R. Determine the range of f.

Answer:

$$f = \left\{\left(x, \frac{x^2}{1+x^2}\right) : x \in \mathbf{R}\right\}$$

$$= \left\{(0, 0), \left(\pm 0.5, \frac{1}{5}\right), \left(\pm 1, \frac{1}{2}\right), \left(\pm 1.5, \frac{9}{13}\right), \left(\pm 2, \frac{4}{5}\right), \left(3, \frac{9}{10}\right), \left(4, \frac{16}{17}\right), \ldots\right\}$$

The range of f is the set of all second elements. It can be observed that all these elements are greater than or equal to 0 but less than 1.

[Denominator is greater numerator]

Thus, range of f = [0, 1)

Question 7:

Let f, g: R → R be defined, respectively by f(x) = x + 1, g(x) = 2x – 3. Find f + g, f – g and $\frac{f}{g}$.

Answer:

f, g: R → R is defined as f(x) = x + 1, g(x) = 2x – 3

(f + g) (x) = f(x) + g(x) = (x + 1) + (2x – 3) = 3x – 2

$\therefore (f + g)(x) = 3x - 2$

$(f - g)(x) = f(x) - g(x) = (x + 1) - (2x - 3) = x + 1 - 2x + 3 = -x + 4$

$\therefore (f - g)(x) = -x + 4$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)},\ g(x) \neq 0,\ x \in \mathbf{R}$$

$$\therefore \left(\frac{f}{g}\right)(x) = \frac{x+1}{2x-3},\ 2x - 3 \neq 0 \text{ or } 2x \neq 3$$

$$\therefore \left(\frac{f}{g}\right)(x) = \frac{x+1}{2x-3},\ x \neq \frac{3}{2}$$

Question 8:

Let f = {(1, 1), (2, 3), (0, –1), (–1, –3)} be a function from Z to Z defined by f(x) = ax + b, for some integers a, b. Determine a, b.

Answer:

f = {(1, 1), (2, 3), (0, –1), (–1, –3)}

f(x) = ax + b

$(1, 1) \in f$

$\Rightarrow f(1) = 1$

$\Rightarrow a \times 1 + b = 1$

$\Rightarrow a + b = 1$

$(0, -1) \in f$

$\Rightarrow f(0) = -1$

$\Rightarrow a \times 0 + b = -1$

$\Rightarrow b = -1$

On substituting b = –1 in a + b = 1, we obtain $a + (-1) = 1 \Rightarrow a = 1 + 1 = 2$.

Thus, the respective values of a and b are 2 and –1.

Question 9:

Let R be a relation from N to N defined by $R = \{(a, b): a, b \in N \text{ and } a = b^2\}$. Are the following true?

(i) $(a, a) \in R$, for all $a \in N$

(ii) $(a, b) \in R$, implies $(b, a) \in R$

(iii) $(a, b) \in R$, $(b, c) \in R$ implies $(a, c) \in R$.

Justify your answer in each case.

Answer:

$R = \{(a, b): a, b \in N \text{ and } a = b^2\}$

(i) It can be seen that $2 \in N$;however, $2 \neq 2^2 = 4$.

Therefore, the statement "$(a, a) \in R$, for all $a \in N$" is not true.

(ii) It can be seen that $(9, 3) \in N$ because $9, 3 \in N$ and $9 = 3^2$.

Now, $3 \neq 9^2 = 81$; therefore, $(3, 9) \notin N$

Therefore, the statement "$(a, b) \in R$, implies $(b, a) \in R$" is not true.

(iii) It can be seen that $(16, 4) \in R$, $(4, 2) \in R$ because $16, 4, 2 \in N$ and $16 = 4^2$ and $4 = 2^2$.

Now, $16 \neq 2^2 = 4$; therefore, $(16, 2) \notin N$

Therefore, the statement "$(a, b) \in R$, $(b, c) \in R$ implies $(a, c) \in R$" is not true.

Question 10:

Let $A = \{1, 2, 3, 4\}$, $B = \{1, 5, 9, 11, 15, 16\}$ and $f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$. Are the following true?

(i) f is a relation from A to B (ii) f is a function from A to B.

Justify your answer in each case.

Answer:

$A = \{1, 2, 3, 4\}$ and $B = \{1, 5, 9, 11, 15, 16\}$

$\therefore A \times B = \{(1, 1), (1, 5), (1, 9), (1, 11), (1, 15), (1, 16), (2, 1), (2, 5), (2, 9), (2, 11), (2, 15), (2, 16), (3, 1), (3, 5), (3, 9), (3, 11), (3, 15), (3, 16), (4, 1), (4, 5), (4, 9), (4, 11), (4, 15), (4, 16)\}$

It is given that $f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$

(i) A relation from a non-empty set A to a non-empty set B is a subset of the Cartesian product $A \times B$.

It is observed that f is a subset of $A \times B$.

Thus, f is a relation from A to B.

(ii) Since the same first element i.e., 2 corresponds to two different images i.e., 9 and 11, relation f is not a function.

Question 11:

Let f be the subset of $Z \times Z$ defined by $f = \{(ab, a + b): a, b \in Z\}$. Is f a function from Z to Z: justify your answer.

Answer:

The relation f is defined as $f = \{(ab, a + b): a, b \in Z\}$

We know that a relation f from a set A to a set B is said to be a function if every element of set A has unique images in set B.

Since $2, 6, -2, -6 \in Z$, $(2 \times 6, 2 + 6), (-2 \times -6, -2 + (-6)) \in f$

i.e., $(12, 8), (12, -8) \in f$

It can be seen that the same first element i.e., 12 corresponds to two different images i.e., 8 and –8. Thus, relation f is not a function.

Question 12:

Let $A = \{9, 10, 11, 12, 13\}$ and let $f: A \to N$ be defined by f(n) = the highest prime factor of n. Find the range of f.

Answer:

$A = \{9, 10, 11, 12, 13\}$

$f: A \to N$ is defined as

f(n) = The highest prime factor of n

Prime factor of 9 = 3

Prime factors of 10 = 2, 5

Prime factor of 11 = 11

Prime factors of 12 = 2, 3

Prime factor of 13 = 13

$\therefore$ f(9) = The highest prime factor of 9 = 3

f(10) = The highest prime factor of 10 = 5

f(11) = The highest prime factor of 11 = 11

f(12) = The highest prime factor of 12 = 3

f(13) = The highest prime factor of 13 = 13

The range of f is the set of all f(n), where $n \in A$.

$\therefore$ Range of $f = \{3, 5, 11, 13\}$