

## Unit 8 (Introduction to Trigonometry & its Applications)

### Exercise 8.1 Multiple Choice Questions (MCQs)

#### Question 1:

If  $\cos A = \frac{4}{5}$ , then the value of  $\tan A$  is

- (a)  $\frac{3}{5}$
- (b)  $\frac{3}{4}$
- (c)  $\frac{4}{3}$
- (d)  $\frac{5}{3}$

**Solution:**

(b) Given,  $\cos A = \frac{4}{5}$

$$\therefore \sin A = \sqrt{1 - \cos^2 A}$$

$$= \sqrt{1 - \left(\frac{4}{5}\right)^2} = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$\text{Now, } \tan A = \frac{\sin A}{\cos A} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$$

Hence, the required value of  $\tan A$  is  $3/4$ .

$$\left[ \begin{array}{l} \therefore \sin^2 A + \cos^2 A = 1 \\ \therefore \sin A = \sqrt{1 - \cos^2 A} \end{array} \right]$$

#### Question 2:

If  $\sin A = \frac{1}{2}$  then the value of  $\cot A$  is

- (a)  $\sqrt{3}$
- (b)  $\frac{1}{\sqrt{3}}$
- (c)  $\frac{\sqrt{3}}{1}$
- (d) 1

**Solution:**

(a) Given,  $\sin A = \frac{1}{2}$

$$\therefore \cos A = \sqrt{1 - \sin^2 A} = \sqrt{1 - \left(\frac{1}{2}\right)^2}$$

$$= \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

[ $\because \sin^2 A + \cos^2 = 1 \Rightarrow \cos A = \sqrt{1 - \sin^2 A}$ ]

Now,  $\cot A = \frac{\cos A}{\sin A} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

**Question 3:**

The value of the expression  $\operatorname{cosec}(75^\circ + 0) - \sec(15^\circ - 0) - \tan(55^\circ + 0) + \cot(35^\circ - 0)$  is

- (a) -1                      (b) 0                      (c) 1                      (d)  $\frac{3}{2}$

**Solution:**

(b) Given, expression =  $\operatorname{cosec}(75^\circ + 0) - \sec(15^\circ - 0) - \tan(55^\circ + 0) + \cot(35^\circ - 0)$

=  $\operatorname{cosec}[90^\circ - (15^\circ - 0)] - \sec(15^\circ - 0) - \tan(55^\circ + 0) + \cot(90^\circ - (55^\circ + 0))$

=  $\sec(15^\circ - 0) - \sec(15^\circ - 0) - \tan(55^\circ + 0) + \tan(55^\circ + 0)$

[ $\because \operatorname{cosec}(90^\circ - 0) = \sec 0$  and  $\cot(90^\circ - 0) = \tan 0$ ]

= 0

Hence, the required value of the given expression is 0.

**Question 4:**

If  $\sin \theta = \frac{3}{5}$ , then  $\cos \theta$  is equal to

- (a)  $\frac{b}{\sqrt{b^2 - a^2}}$                       (b)  $\frac{b}{a}$                       (c)  $\frac{\sqrt{b^2 - a^2}}{b}$                       (d)  $\frac{a}{\sqrt{b^2 - a^2}}$

**Solution:**

(c) Given,  $\sin \theta = \frac{a}{b}$                       [ $\because \sin^2 \theta + \cos^2 \theta = 1 \Rightarrow \cos \theta = \sqrt{1 - \sin^2 \theta}$ ]

$\therefore \cos \theta = \sqrt{1 - \sin^2 \theta}$

$$= \sqrt{1 - \left(\frac{a}{b}\right)^2} = \sqrt{1 - \frac{a^2}{b^2}} = \frac{\sqrt{b^2 - a^2}}{b}$$

**Question 5:**

If  $\cos(\alpha + \beta) = 0$ , then  $\sin(\alpha - \beta)$  can be reduced to

- (a)  $\cos \beta$                       (b)  $\cos 2\beta$                       (c)  $\sin \alpha$                       (d)  $\sin 2\alpha$

**Solution:**

(b) Given,  $\cos(\alpha + \beta) = 0 = \cos 90^\circ$                       [ $\because \cos 90^\circ = 0$ ]

$\Rightarrow \alpha + \beta = 90^\circ$                       ... (i)

$\Rightarrow \alpha = 90^\circ - \beta$                       [put the value from Eq. (i)]

Now,  $\sin(\alpha - \beta) = \sin(90^\circ - \beta - \beta)$                       [ $\because \sin(90^\circ - \theta) = \cos \theta$ ]

$= \sin(90^\circ - 2\beta)$

$= \cos 2\beta$

Hence,  $\sin(\alpha - \beta)$  can be reduced to  $\cos 2\beta$ .

**Question 6:**

The value of  $(\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ)$  is

- (a) 0                      (b) 1                      (c) 2                      (d)  $\frac{1}{2}$

**Solution:**

(b)  $\tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \dots \tan 89^\circ$

=  $\tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \dots \tan 44^\circ \cdot \tan 45^\circ \cdot \tan 46^\circ \dots \tan 87^\circ \cdot \tan 88^\circ \cdot \tan 89^\circ$

$$\begin{aligned}
&= \tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \dots \tan 44^\circ \cdot (1) \cdot \tan (90^\circ - 44^\circ) \dots \tan (90^\circ - 3^\circ) \\
&\tan (90^\circ - 2^\circ) \cdot \tan (90^\circ - 1^\circ) \quad (\because \tan 45^\circ = 1) \\
&= \tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \dots \tan 44^\circ (1) \cdot \cot 44^\circ \dots \cot 3^\circ \cdot \cot 2^\circ \cdot \cot 1^\circ \\
&= \tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \dots \tan 44^\circ (1) \cdot \frac{1}{\tan 44^\circ} \dots \frac{1}{\tan 30^\circ} \cdot \frac{1}{\tan 2^\circ} \cdot \frac{1}{\tan 1^\circ} \quad \left[ \because \tan (90^\circ - \theta) = \cot \theta \right] \\
&= 1 \quad \left[ \because \cot \theta = \frac{1}{\tan \theta} \right]
\end{aligned}$$

**Question 7:**

If  $\cos 9\alpha = \sin \alpha$  and  $9\alpha < 90^\circ$ , then the value of  $\tan 5\alpha$  is

- (a)  $\frac{1}{\sqrt{3}}$
- (b)  $\frac{\sqrt{3}}{1}$
- (c) 1
- (d) 0

**Solution:**

**(c)** Given,  $\cos 9\alpha = \sin \alpha$  and  $9\alpha < 90^\circ$  i.e., acute angle.

$$\sin (90^\circ - 9\alpha) = \sin \alpha \quad [\because \cos A = \sin (90^\circ - A)]$$

$$\Rightarrow 90^\circ - 9\alpha = \alpha$$

$$\Rightarrow 10\alpha = 90^\circ$$

$$\Rightarrow \alpha = 9^\circ$$

$$\therefore \tan 5\alpha = \tan (5 \times 9^\circ) = \tan 45^\circ = 1 \quad [\because \tan 45^\circ = 1]$$

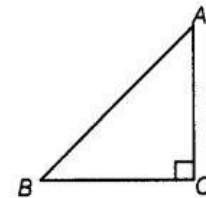
**Question 8:**

If  $\Delta ABC$  is right angled at C, then the value of  $\cos (A + B)$  is

- (a) 0
- (b) 1
- (c)  $\frac{1}{2}$
- (d)  $\frac{\sqrt{3}}{2}$

**Solution:**

**(a)** We know that, in  $\Delta ABC$ , sum of three angles =  $180^\circ$   
i.e.,  $\angle A + \angle B + \angle C = 180^\circ$   
But right angled at C i.e.,  $\angle C = 90^\circ$  [given]  
 $\angle A + \angle B + 90^\circ = 180^\circ$   
 $\Rightarrow A + B = 90^\circ$  [ $\because \angle A = A$ ]  
 $\therefore \cos (A + B) = \cos 90^\circ = 0$



**Question 9:**

If  $\sin A + \sin^2 A = 1$ , then the value of  $(\cos^2 A + \cos^4 A)$  is

- (a) 1
- (b)  $\frac{1}{2}$
- (c) 2
- (d) 3

**Solution:**

**(a)** Given,  $\sin A + \sin^2 A = 1$

$$\Rightarrow \sin A = 1 - \sin^2 A = \cos^2 A \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

On squaring both sides, we get

$$\sin^2 A = \cos^4 A$$

$$\Rightarrow 1 - \cos^2 A = \cos^4 A$$

$$\Rightarrow \cos^2 A + \cos^4 A = 1$$

**Question 10:**

If  $\sin \alpha = \frac{1}{2}$  and  $\cos \beta = \frac{1}{2}$  then the value of  $(\alpha + \beta)$  is

- (a)  $0^\circ$
- (b)  $30^\circ$
- (c)  $60^\circ$
- (d)  $90^\circ$

**Solution:**

$$\begin{aligned}
 \text{(d) Given,} \quad \sin \alpha &= \frac{1}{2} = \sin 30^\circ && \left[ \because \sin 30^\circ = \frac{1}{2} \right] \\
 \Rightarrow \quad \alpha &= 30^\circ \\
 \text{and} \quad \cos \beta &= \frac{1}{2} = \cos 60^\circ && \left[ \because \cos 60^\circ = \frac{1}{2} \right] \\
 \Rightarrow \quad \beta &= 60^\circ \\
 \therefore \quad \alpha + \beta &= 30^\circ + 60^\circ = 90^\circ
 \end{aligned}$$

**Question 11:**

The value of the expression

$$\left( \frac{\sin^2 22^\circ + \sin^2 68^\circ}{\cos^2 22^\circ + \cos^2 68^\circ} + \sin^2 63^\circ + \cos 63^\circ \sin 27^\circ \right) \text{ is}$$

- (a) 3                                      (b) 2                                      (c) 1                                      (d) 0

**Solution:**

$$\begin{aligned}
 \text{(b) Given expression, } & \frac{\sin^2 22^\circ + \sin^2 68^\circ}{\cos^2 22^\circ + \cos^2 68^\circ} + \sin^2 63^\circ + \cos 63^\circ \sin 27^\circ \\
 &= \frac{\sin^2 22^\circ + \sin^2 (90^\circ - 22^\circ)}{\cos^2 (90^\circ - 68^\circ) + \cos^2 68^\circ} + \sin^2 63^\circ + \cos 63^\circ \sin (90^\circ - 63^\circ) \\
 &= \frac{\sin^2 22^\circ + \cos^2 22^\circ}{\sin^2 68^\circ + \cos^2 68^\circ} + \sin^2 63^\circ + \cos 63^\circ \cdot \cos 63^\circ \quad \left[ \because \sin (90^\circ - \theta) = \cos \theta \right. \\
 & \quad \left. \text{and } \cos (90^\circ - \theta) = \sin \theta \right] \\
 &= \frac{1}{1} + (\sin^2 63^\circ + \cos^2 63^\circ) \quad \left[ \because \sin^2 \theta + \cos^2 \theta = 1 \right] \\
 &= 1 + 1 = 2
 \end{aligned}$$

**Question 12:**

If  $4 \tan \theta = 3$ , then  $\left( \frac{4 \sin \theta - \cos \theta}{4 \sin \theta + \cos \theta} \right)$  is equal to

- (a)  $\frac{2}{3}$   
 (b)  $\frac{1}{3}$   
 (c)  $\frac{1}{2}$   
 (d)  $\frac{3}{4}$

**Solution:**

$$\begin{aligned}
 \text{(c) Given,} \quad 4 \tan \theta &= 3 \\
 \Rightarrow \quad \tan \theta &= \frac{3}{4} \quad \dots \text{(i)} \\
 \therefore \quad \frac{4 \sin \theta - \cos \theta}{4 \sin \theta + \cos \theta} &= \frac{4 \frac{\sin \theta}{\cos \theta} - 1}{4 \frac{\sin \theta}{\cos \theta} + 1} \\
 & \quad \left[ \text{divide by } \cos \theta \text{ in both numerator and denominator} \right] \\
 &= \frac{4 \tan \theta - 1}{4 \tan \theta + 1} \quad \left[ \because \tan \theta = \frac{\sin \theta}{\cos \theta} \right] \\
 &= \frac{4 \left( \frac{3}{4} \right) - 1}{4 \left( \frac{3}{4} \right) + 1} = \frac{3 - 1}{3 + 1} = \frac{2}{4} = \frac{1}{2} \quad \left[ \text{put the value from Eq. (i)} \right]
 \end{aligned}$$

**Question 13:**

If  $\sin \theta - \cos \theta = 0$ , then the value of  $(\sin^4 \theta + \cos^4 \theta)$  is

- (a) 1  
 (b)  $\frac{3}{4}$   
 (c)  $\frac{1}{2}$

(d)  $\frac{1}{4}$

**Solution:**

(c) Given,

$$\sin \theta - \cos \theta = 0$$

$$\Rightarrow \sin \theta = \cos \theta \Rightarrow \frac{\sin \theta}{\cos \theta} = 1$$

$$\Rightarrow \tan \theta = 1 \quad \left[ \because \tan \theta = \frac{\sin \theta}{\cos \theta} \text{ and } \tan 45^\circ = 1 \right]$$

$$\Rightarrow \tan \theta = \tan 45^\circ$$

$$\therefore \theta = 45^\circ$$

Now,  $\sin^4 \theta + \cos^4 \theta = \sin^4 45^\circ + \cos^4 45^\circ$

$$\begin{aligned} &= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4 \quad \left[ \because \sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}} \right] \\ &= \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2} \end{aligned}$$

**Question 14:**

$\sin(45^\circ + \theta) - \cos(45^\circ - \theta)$  is equal to

- (a)  $2 \cos \theta$                       (b) 0                      (c)  $2 \sin \theta$                       (d) 1

**Solution:**

(b)  $\sin(45^\circ + \theta) - \cos(45^\circ - \theta) = \cos[90^\circ - (45^\circ + \theta)] - \cos(45^\circ - \theta)$  [ $\because \cos(90^\circ - \theta) = \sin \theta$ ]  
 $= \cos(45^\circ - \theta) - \cos(45^\circ - \theta)$   
 $= 0$

**Question 15:**

If a pole 6 m high casts a shadow  $2\sqrt{3}$  m long on the ground, then the Sun's elevation is

- (a)  $60^\circ$                       (b)  $45^\circ$                       (c)  $30^\circ$                       (d)  $90^\circ$

**Solution:**

(a) Let  $BC = 6$  m be the height of the pole and  $AB = 2\sqrt{3}$  m be the length of the shadow on the ground. Let the Sun's rays make an angle  $\theta$  on the ground.

Now, in  $\triangle BAC$ ,

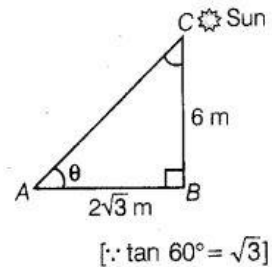
$$\tan \theta = \frac{BC}{AB}$$

$$\Rightarrow \tan \theta = \frac{6}{2\sqrt{3}} = \frac{3}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow \tan \theta = \frac{3\sqrt{3}}{3} = \sqrt{3} = \tan 60^\circ$$

$$\therefore \theta = 60^\circ$$

Hence, the Sun's elevation is  $60^\circ$ .



**Exercise 8.2 Very Short Answer Type Questions**

Write whether **True** or **False** and justify your answer.

**Question 1:**

$$\frac{\tan 47^\circ}{\cot 43^\circ} = 1$$

**Solution:**

True

$$\frac{\tan 47^\circ}{\cot 43^\circ} = \frac{\tan(90^\circ - 43^\circ)}{\cot 43^\circ} = \frac{\cot 43^\circ}{\cot 43^\circ} = 1 \quad \left[ \because \tan(90^\circ - \theta) = \cot \theta \right]$$

**Question 2:**

The value of the expression  $(\cos^2 23^\circ - \sin^2 67^\circ)$  is positive.

**Solution:**

**False**

$$\begin{aligned} \cos^2 23^\circ - \sin^2 67^\circ &= (\cos 23^\circ - \sin 67^\circ)(\cos 23^\circ + \sin 67^\circ) [\because (a^2 - b^2) = (a - b)(a + b)] \\ &= [\cos 23^\circ - \sin (90^\circ - 23^\circ)] (\cos 23^\circ + \sin 67^\circ) \\ &= (\cos 23^\circ - \cos 23^\circ) (\cos 23^\circ + \sin 67^\circ) [\because \sin (90^\circ - \theta) = \cos \theta] \\ &= 0 \cdot (\cos 23^\circ + \sin 67^\circ) = 0 \end{aligned}$$

which may be either positive or negative.

**Question 3:**

The value of the expression  $(\sin 80^\circ - \cos 80^\circ)$  is negative.

**Solution:**

**False**

We know that, sine is increasing when,  $0^\circ < \theta < 90^\circ$  and  $\cos \theta$  is decreasing when,  $0^\circ < \theta < 90^\circ$ .

$$\therefore \sin 80^\circ - \cos 80^\circ > 0 \quad \text{[positive]}$$

**Question 4:**

$$\sqrt{(1 - \cos^2 \theta) \sec^2 \theta} = \tan \theta$$

**Solution:**

**True**

$$\begin{aligned} \sqrt{(1 - \cos^2 \theta) \sec^2 \theta} &= \sqrt{\sin^2 \theta \cdot \sec^2 \theta} && [\because \sin^2 \theta + \cos^2 \theta = 1] \\ &= \sqrt{\sin^2 \theta \cdot \frac{1}{\cos^2 \theta}} = \sqrt{\tan^2 \theta} = \tan \theta && \left[ \because \sec \theta = \frac{1}{\cos \theta}, \tan \theta = \frac{\sin \theta}{\cos \theta} \right] \end{aligned}$$

**Question 5:**

If  $\cos A + \cos^2 A = 1$ , then  $\sin^2 A + \sin^4 A = 1$

**Solution:**

**True**

$$\begin{aligned} \therefore \cos A + \cos^2 A &= 1 \\ \Rightarrow \cos A &= 1 - \cos^2 A = \sin^2 A && [\because \sin^2 A + \cos^2 A = 1] \\ \Rightarrow \cos^2 A &= \sin^4 A \\ \Rightarrow 1 - \sin^2 A &= \sin^4 A \\ \Rightarrow \sin^2 A + \sin^4 A &= 1 && [\because \cos^2 A = 1 - \sin^2 A] \end{aligned}$$

**Question 6:**

$(\tan \theta + 2)(2 \tan \theta + 1) = 5 \tan \theta + \sec^2 \theta$

**Solution:**

**False**

$$\begin{aligned} \text{LHS} &= (\tan \theta + 2)(2 \tan \theta + 1) \\ &= 2 \tan^2 \theta + 4 \tan \theta + \tan \theta + 2 \\ &= 2(\sec^2 \theta - 1) + 5 \tan \theta + 2 && [\because \sec^2 \theta - \tan^2 \theta = 1] \\ &= 2 \sec^2 \theta + 5 \tan \theta = \text{RHS} \end{aligned}$$

**Question 7:**

If the length of the shadow of a tower is increasing, then the angle of elevation of the Sun is also increasing.

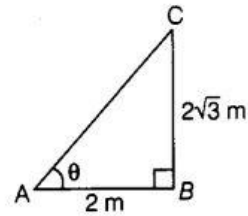
**Solution:**

**False**

To understand the fact of this question, consider the following example

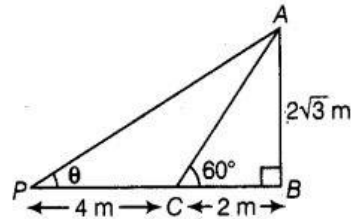
- I. A tower  $2\sqrt{3}$  m high casts a shadow 2 m long on the ground, then the Sun's elevation is  $60^\circ$ .

$$\begin{aligned} \text{In } \triangle ACB, \quad \tan \theta &= \frac{AB}{BC} = \frac{2\sqrt{3}}{2} \\ \Rightarrow \quad \tan \theta &= \sqrt{3} = \tan 60^\circ \\ \therefore \quad \theta &= 60^\circ \end{aligned}$$



- II. A same height of tower casts a shadow 4m more from preceding point, then the Sun's elevation is  $30^\circ$ .

$$\begin{aligned} \text{In } \triangle APB, \quad \tan \theta &= \frac{AB}{PB} = \frac{AB}{PC + CB} \\ \Rightarrow \quad \tan \theta &= \frac{2\sqrt{3}}{4 + 2} = \frac{2\sqrt{3}}{6} \\ \Rightarrow \quad \tan \theta &= \frac{\sqrt{3}}{3} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3}{3\sqrt{3}} \\ \Rightarrow \quad \tan \theta &= \frac{1}{\sqrt{3}} = \tan 30^\circ \\ \therefore \quad \theta &= 30^\circ \end{aligned}$$



Hence, we conclude from above two examples that if the length of the shadow of a tower is increasing, then the angle of elevation of the Sun is decreasing.

#### Alternate Method

False, we know that, if the elevation moves towards the tower, it increases and if its elevation moves away the tower, it decreases. Hence, if the shadow of a tower is increasing, then the angle of elevation of a Sun is not increasing.

#### Question 8:

If a man standing on a platform 3 m above the surface of a lake observes a cloud and its reflection in the lake, then the angle of elevation of the cloud is equal to the angle of depression of its reflection.

#### Solution:

False

From figure, we observe that, a man standing on a platform at point P, 3 m above the surface of a lake observes a cloud at point C. Let the height of the cloud from the surface of the platform is  $h$  and angle of elevation of the cloud is  $\theta_1$ .

Now at same point P a man observes a cloud reflection in the lake at this time the height of reflection of cloud in lake is

$(h + 3)$  because in lake platform height is also added to reflection of cloud.

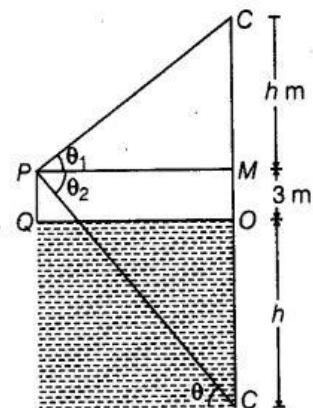
So, angle of depression is different in the lake from the angle of elevation of the cloud above the surface of a lake

$$\begin{aligned} \text{In } \triangle MPC, \quad \tan \theta_1 &= \frac{CM}{PM} = \frac{h}{PM} \\ \Rightarrow \quad \frac{\tan \theta_1}{h} &= \frac{1}{PM} \quad \dots(i) \\ \text{In } \triangle CPM, \quad \tan \theta_2 &= \frac{CM}{PM} = \frac{OC + OM}{PM} = \frac{h + 3}{PM} \\ \Rightarrow \quad \frac{\tan \theta_2}{h + 3} &= \frac{1}{PM} \quad \dots(ii) \end{aligned}$$

From Eqs. (i) and (ii),

$$\begin{aligned} \frac{\tan \theta_1}{h} &= \frac{\tan \theta_2}{h + 3} \\ \Rightarrow \quad \tan \theta_2 &= \left( \frac{h + 3}{h} \right) \tan \theta_1 \end{aligned}$$

Hence,  $\theta_1 \neq \theta_2$



#### Alternate Method

False, we know that, if P is a point above the lake at a distance  $d$ , then the reflection of the point in the lake would be at the same distance. Also, the angle of elevation and depression

from the surface of the lake is same.

Here, the man is standing on a platform 3 m above the surface, so its angle of elevation to the cloud and angle of depression to the reflection of the cloud is not same.

**Question 9:**

The value of  $2 \sin \theta$  can be  $a + \frac{1}{a}$  where  $a$  is a positive number and  $a \neq 1$ .

**Solution:**

**False**

Given,  $a$  is a positive number and  $a \neq 1$ , then  $AM > GM$

$$\Rightarrow \frac{a + \frac{1}{a}}{2} > \sqrt{a \cdot \frac{1}{a}} \Rightarrow \left(a + \frac{1}{a}\right) > 2$$

[since, AM and GM of two number's  $a$  and  $b$  are  $\frac{(a+b)}{2}$  and  $\sqrt{ab}$ , respectively]

$$\Rightarrow 2 \sin \theta > 2 \quad \left[ \because 2 \sin \theta = a + \frac{1}{a} \right]$$
$$\Rightarrow \sin \theta > 1 \quad \left[ \because -1 \leq \sin \theta \leq 1 \right]$$

Which is not possible.

Hence, the value of  $2 \sin \theta$  can not be  $a + \frac{1}{a}$

**Question 10:**

$\cos \theta = \frac{a^2 + b^2}{2ab}$  where  $a$  and  $b$  are two distinct numbers such that  $ab > 0$ .

**Solution:**

**False**

Given,  $a$  and  $b$  are two distinct numbers such that  $ab > 0$ .

Using,

$$AM > GM$$

[since, AM and GM of two number  $a$  and  $b$  are  $\frac{a+b}{2}$  and  $\sqrt{ab}$ , respectively]

$$\Rightarrow \frac{a^2 + b^2}{2} > \sqrt{a^2 \cdot b^2}$$
$$\Rightarrow a^2 + b^2 > 2ab$$
$$\Rightarrow \frac{a^2 + b^2}{2ab} > 1 \quad \left[ \because \cos \theta = \frac{a^2 + b^2}{2ab} \right]$$
$$\Rightarrow \cos \theta > 1 \quad \left[ \because -1 \leq \cos \theta \leq 1 \right]$$

which is not possible.

Hence,  $\cos \theta \neq \frac{a^2 + b^2}{2ab}$

**Question 11:**

The angle of elevation of the top of a tower is  $30^\circ$ . If the height of the tower is doubled, then the angle of elevation of its top will also be doubled.

**Solution:**

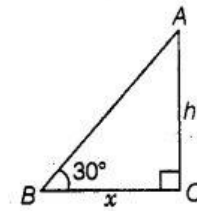
**False**



**Case I** Let the height of the tower is  $h$  and  $BC = x$  m  
In  $\triangle ABC$ ,

$$\tan 30^\circ = \frac{AC}{BC} = \frac{h}{x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x} \quad \dots(i)$$



**Case II** By condition, the height of the tower is doubled. i.e.,  $PR = 2h$ .

In  $\triangle PQR$ ,  $\tan \theta = \frac{PR}{QR} = \frac{2h}{x}$

$$\Rightarrow \tan \theta = \frac{2}{x} \times \frac{x}{\sqrt{3}} \quad \left[ \because h = \frac{x}{\sqrt{3}}, \text{ from Eq. (i)} \right]$$

$$\Rightarrow \tan \theta = \frac{2}{\sqrt{3}} = 1.15$$

$$\therefore \theta = \tan^{-1}(1.15) < 60^\circ$$

Hence, the required angle is not doubled.

### Question 12:

If the height of a tower and the distance of the point of observation from its foot, both are increased by 10%, then the angle of elevation of its top remains unchanged.

**Solution:**

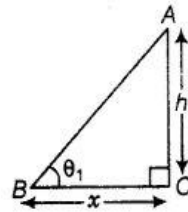
True

**Case I** Let the height of a tower be  $h$  and the distance of the point of observation from its foot is  $x$ .

In  $\triangle ABC$ ,

$$\tan \theta_1 = \frac{AC}{BC} = \frac{h}{x}$$

$$\Rightarrow \theta_1 = \tan^{-1}\left(\frac{h}{x}\right) \quad \dots(i)$$



**Case II** Now, the height of a tower increased by 10% =  $h + 10\%$  of  $h = h + h \times \frac{10}{100} = \frac{11h}{10}$

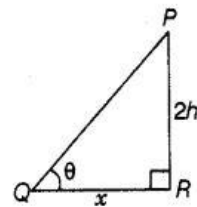
and the distance of the point of observation from its foot =  $x + 10\%$  of  $x$

$$= x + x \times \frac{10}{100} = \frac{11x}{10}$$

In  $\triangle PQR$ ,  $\tan \theta_2 = \frac{PR}{QR} = \frac{\left(\frac{11h}{10}\right)}{\left(\frac{11x}{10}\right)}$

$$\Rightarrow \tan \theta_2 = \frac{h}{x}$$

$$\Rightarrow \theta_2 = \tan^{-1}\left(\frac{h}{x}\right) \quad \dots(ii)$$



From Eqs. (i) and (ii),

$$\theta_1 = \theta_2$$

Hence, the required angle of elevation of its top remains unchanged

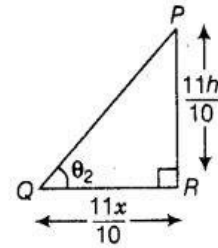
### Exercise 8.3 Short Answer Type Questions

**Question 1:**

$$\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = 2 \operatorname{cosec} \theta$$

**Solution:**

$$\begin{aligned} \text{LHS} &= \frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = \frac{\sin^2 \theta + (1 + \cos \theta)^2}{\sin \theta (1 + \cos \theta)} \\ &= \frac{\sin^2 \theta + 1 + \cos^2 \theta + 2 \cos \theta}{\sin \theta (1 + \cos \theta)} \quad [ \because (a + b)^2 = a^2 + b^2 + 2ab ] \\ &= \frac{1 + 1 + 2 \cos \theta}{\sin \theta (1 + \cos \theta)} \quad [ \because \sin^2 \theta + \cos^2 \theta = 1 ] \\ &= \frac{2(1 + \cos \theta)}{\sin \theta (1 + \cos \theta)} = \frac{2}{\sin \theta} \\ &= 2 \operatorname{cosec} \theta = \text{RHS} \end{aligned}$$



$$\left[ \because \operatorname{cosec} \theta = \frac{1}{\sin \theta} \right]$$

**Question 2:**

$$\frac{\tan A}{1 + \sec A} - \frac{\tan A}{1 - \sec A} = 2 \operatorname{cosec} A$$

**Solution:**

$$\begin{aligned} \text{LHS} &= \frac{\tan A}{1 + \sec A} - \frac{\tan A}{1 - \sec A} = \frac{\tan A (1 - \sec A - 1 - \sec A)}{(1 + \sec A)(1 - \sec A)} \\ &= \frac{\tan A (-2 \sec A)}{(1 - \sec^2 A)} = \frac{2 \tan A \cdot \sec A}{(\sec^2 A - 1)} \quad [ \because (a + b)(a - b) = a^2 - b^2 ] \\ &= \frac{2 \tan A \cdot \sec A}{\tan^2 A} \quad [ \because \sec^2 A - \tan^2 A = 1 ] \quad \left[ \because \sec \theta = \frac{1}{\cos \theta} \text{ and } \tan \theta = \frac{\sin \theta}{\cos \theta} \right] \\ &= \frac{2 \sec A}{\tan A} = \frac{2}{\sin A} = 2 \operatorname{cosec} A = \text{RHS} \quad \left[ \because \operatorname{cosec} \theta = \frac{1}{\sin \theta} \right] \end{aligned}$$

**Question 3:**

If  $\tan A = \frac{3}{4}$ , then  $\sin A \cos A = \frac{12}{25}$

**Solution:**

Given,  $\tan A = \frac{3}{4} = \frac{P}{B} = \frac{\text{Perpendicular}}{\text{Base}}$

Let  $P = 3k$  and  $B = 4k$

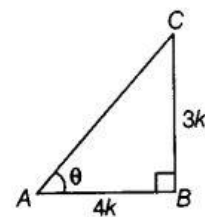
By Pythagoras theorem,

$$\begin{aligned} H^2 &= P^2 + B^2 = (3k)^2 + (4k)^2 \\ &= 9k^2 + 16k^2 = 25k^2 \end{aligned}$$

$\Rightarrow H = 5k$  [since, side cannot be negative]

$\therefore \sin A = \frac{P}{H} = \frac{3k}{5k} = \frac{3}{5}$  and  $\cos A = \frac{B}{H} = \frac{4k}{5k} = \frac{4}{5}$

Now,  $\sin A \cos A = \frac{3}{5} \cdot \frac{4}{5} = \frac{12}{25}$



Hence proved.

**Question 4:**

$$(\sin \alpha + \cos \alpha)(\tan \alpha + \cot \beta) = \sec \alpha + \operatorname{cosec} \beta$$

**Solution:**

$$\begin{aligned}
\text{LHS} &= (\sin \alpha + \cos \alpha) (\tan \alpha + \cot \alpha) \\
&= (\sin \alpha + \cos \alpha) \left( \frac{\sin \alpha}{\cos \alpha} + \frac{\cos \alpha}{\sin \alpha} \right) \\
&= (\sin \alpha + \cos \alpha) \left( \frac{\sin^2 \alpha + \cos^2 \alpha}{\sin \alpha \cdot \cos \alpha} \right) \\
&= (\sin \alpha + \cos \alpha) \cdot \frac{1}{(\sin \alpha \cdot \cos \alpha)} \\
&= \frac{1}{\cos \alpha} + \frac{1}{\sin \alpha} \\
&= \sec \alpha + \csc \alpha = \text{RHS}
\end{aligned}$$

$$\left[ \because \tan \theta = \frac{\sin \theta}{\cos \theta} \text{ and } \cot \theta = \frac{\cos \theta}{\sin \theta} \right]$$

$$[\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$\left[ \because \sec \theta = \frac{1}{\cos \theta} \text{ and } \csc \theta = \frac{1}{\sin \theta} \right]$$

### Question 5:

$$(\sqrt{3} + 1) (3 - \cot 30^\circ) = \tan^3 60^\circ - 2\sin 60^\circ$$

### Solution:

$$\text{RHS} = \tan^3 60^\circ - 2\sin 60^\circ = (\sqrt{3})^3 - 2 \cdot \frac{\sqrt{3}}{2} = 3\sqrt{3} - \sqrt{3} = 2\sqrt{3}$$

$$\text{LHS} = (\sqrt{3} + 1) (3 - \cot 30^\circ) = (\sqrt{3} + 1) (3 - \sqrt{3})$$

$$[\because \tan 60^\circ = \sqrt{3} \text{ and } \cot 30^\circ = \sqrt{3}]$$

$$= \sqrt{3} (\sqrt{3})^2 - 1 = \sqrt{3} (3 - 1) = 2\sqrt{3}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

### Question 6:

$$1 + \frac{\cot^2 \alpha}{1 + \csc \alpha} = \csc \alpha$$

### Solution:

$$\text{LHS} = 1 + \frac{\cot^2 \alpha}{1 + \csc \alpha} = 1 + \frac{\cos^2 \alpha / \sin^2 \alpha}{1 + 1/\sin \alpha}$$

$$\left[ \because \cot \theta = \frac{\cos \theta}{\sin \theta} \text{ and } \csc \theta = \frac{1}{\sin \theta} \right]$$

$$= 1 + \frac{\cos^2 \alpha}{\sin \alpha (1 + \sin \alpha)} = \frac{\sin \alpha (1 + \sin \alpha) + \cos^2 \alpha}{\sin \alpha (1 + \sin \alpha)}$$

$$= \frac{\sin \alpha + (\sin^2 \alpha + \cos^2 \alpha)}{\sin \alpha (1 + \sin \alpha)}$$

$$[\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$= \frac{(\sin \alpha + 1)}{\sin \alpha (\sin \alpha + 1)} = \frac{1}{\sin \alpha}$$

$$\left[ \because \csc \theta = \frac{1}{\sin \theta} \right]$$

$$= \csc \alpha = \text{RHS}$$

### Question 7:

$$\tan \theta + \tan (90^\circ - \theta) = \sec \theta \sec (90^\circ - \theta)$$

### Solution:

$$\text{LHS} = \tan \theta + \tan (90^\circ - \theta)$$

$$[\because \tan (90^\circ - \theta) = \cot \theta]$$

$$= \tan \theta + \cot \theta = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$$

$$= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}$$

$$\left[ \because \tan \theta = \frac{\sin \theta}{\cos \theta} \text{ and } \cot \theta = \frac{\cos \theta}{\sin \theta} \right]$$

$$= \frac{1}{\sin \theta \cos \theta}$$

$$[\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$= \sec \theta \csc \theta$$

$$\left[ \because \sec \theta = \frac{1}{\cos \theta} \text{ and } \csc \theta = \frac{1}{\sin \theta} \right]$$

$$= \sec \theta \sec (90^\circ - \theta) = \text{RHS}$$

$$[\because \sec (90^\circ - \theta) = \csc \theta]$$

### Question 8:

Find the angle of elevation of the sun when the shadow of a pole  $h$  m high is  $\sqrt{3}h$  m long.

**Solution:**

Let the angle of elevation of the sun is  $\theta$ .

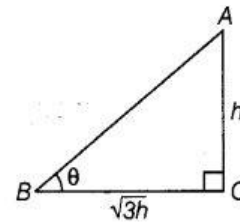
Given, height of pole =  $h$

Now, in  $\triangle ABC$ ,

$$\tan \theta = \frac{AB}{BC} = \frac{h}{\sqrt{3}h}$$

$$\Rightarrow \tan \theta = \frac{1}{\sqrt{3}} = \tan 30^\circ \Rightarrow \theta = 30^\circ$$

Hence, the angle of elevation of the Sun is  $30^\circ$ .



**Question 9:**

If  $\sqrt{3} \tan \theta = 1$ , then find the value of  $\sin^2 \theta - \cos^2 \theta$

**Solution:**

Given that,  $\sqrt{3} \tan \theta = 1$

$$\Rightarrow \tan \theta = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

$$\Rightarrow \theta = 30^\circ$$

$$\text{Now, } \sin^2 \theta - \cos^2 \theta = \sin^2 30^\circ - \cos^2 30^\circ$$

$$= \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \frac{1}{4} - \frac{3}{4} = \frac{1-3}{4} = -\frac{2}{4} = -\frac{1}{2}$$

**Question 10:**

A ladder 15 m long just reaches the top of a vertical wall. If the ladder makes an angle of  $60^\circ$  with the wall, then find the height of the wall.

**Solution:**

Given that, the height of the ladder = 15 m

Let the height of the vertical wall =  $h$

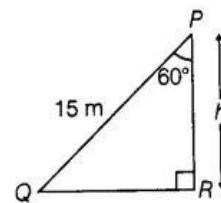
and the ladder makes an angle of elevation  $60^\circ$  with the wall i.e.  $\theta = 60^\circ$

$$\text{In } \triangle QPR, \quad \cos 60^\circ = \frac{PR}{PQ} = \frac{h}{15}$$

$$\Rightarrow \frac{1}{2} = \frac{h}{15}$$

$$\Rightarrow h = \frac{15}{2} \text{ m.}$$

Hence, the required height of the wall  $\frac{15}{2}$  m.



**Question 11:**

Simplify  $(1 + \tan^2 \theta) (1 - \sin \theta) (1 + \sin \theta)$

**Solution:**

$$(1 + \tan^2 \theta) (1 - \sin \theta) (1 + \sin \theta) = (1 + \tan^2 \theta) (1 - \sin^2 \theta) \quad [ \because (a - b)(a + b) = a^2 - b^2 ]$$

$$= \sec^2 \theta \cdot \cos^2 \theta$$

$$[ \because 1 + \tan^2 \theta = \sec^2 \theta \text{ and } \cos^2 \theta + \sin^2 \theta = 1 ]$$

$$= \frac{1}{\cos^2 \theta} \cdot \cos^2 \theta = 1$$

$$[ \because \sec \theta = \frac{1}{\cos \theta} ]$$

**Question 12:**

If  $2 \sin^2 \theta - \cos^2 \theta = 2$ , then find the value of  $\theta$ .

**Solution:**

$$\begin{aligned}
\text{Given,} & \quad 2\sin^2 \theta - \cos^2 \theta = 2 \\
\Rightarrow & \quad 2\sin^2 \theta - (1 - \sin^2 \theta) = 2 & [\because \sin^2 \theta + \cos^2 \theta = 1] \\
\Rightarrow & \quad 2\sin^2 \theta + \sin^2 \theta - 1 = 2 \\
\Rightarrow & \quad 3\sin^2 \theta = 3 \\
\Rightarrow & \quad \sin^2 \theta = 1 & [\because \sin 90^\circ = 1] \\
\Rightarrow & \quad \sin \theta = 1 = \sin 90^\circ \\
\therefore & \quad \theta = 90^\circ
\end{aligned}$$

**Question 13:**

Show that  $\frac{\cos^2(45^\circ + \theta) + \cos^2(45^\circ - \theta)}{\tan(60^\circ + \theta) \tan(30^\circ - \theta)} = 1$

**Solution:**

$$\begin{aligned}
\text{LHS} &= \frac{\cos^2(45^\circ + \theta) + \cos^2(45^\circ - \theta)}{\tan(60^\circ + \theta) \cdot \tan(30^\circ - \theta)} \\
&= \frac{\cos^2(45^\circ + \theta) + [\sin\{90^\circ - (45^\circ - \theta)\}]^2}{\tan(60^\circ + \theta) \cdot \cot\{90^\circ - (30^\circ - \theta)\}} & [\because \sin(90^\circ - \theta) = \cos \theta \text{ and } \cot(90^\circ - \theta) = \tan \theta] \\
&= \frac{\cos^2(45^\circ + \theta) + \sin^2(45^\circ + \theta)}{\tan(60^\circ + \theta) \cdot \cot(60^\circ + \theta)} & [\because \sin^2 \theta + \cos^2 \theta = 1] \\
&= \frac{1}{\tan(60^\circ + \theta) \cdot \frac{1}{\tan(60^\circ + \theta)}} = 1 = \text{RHS} & [\because \cot \theta = 1 / \tan \theta]
\end{aligned}$$

**Question 14:**

An observer 1.5 m tall is 20.5 m away from a tower 22 m high. Determine the angle of elevation of the top of the tower from the eye of the observer.

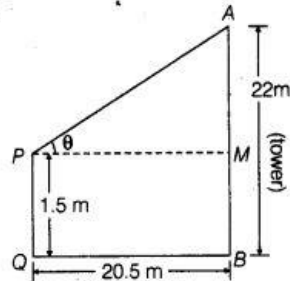
**Solution:**

Let the angle of elevation of the top of the tower from the eye of the observer is  $\theta$

Given that,  $AB = 22 \text{ m}$ ,  $PQ = 1.5 \text{ m} = MB$   
and  $QB = PM = 20.5 \text{ m}$   
 $\Rightarrow AM = AB - MB$   
 $= 22 - 1.5 = 20.5 \text{ m}$

Now, in  $\triangle APM$ ,  $\tan \theta = \frac{AM}{PM} = \frac{20.5}{20.5} = 1$

$\Rightarrow \tan \theta = \tan 45^\circ$   
 $\therefore \theta = 45^\circ$



which may be either positive or negative. Hence, required angle of elevation of the top of the tower from the eye of the observer is  $45^\circ$ .

**Question 15:**

Show that  $\tan^4 \theta + \tan^2 \theta = \sec^4 \theta - \sec^2 \theta$ .

**Solution:**

LHS =  $\tan^4 \theta + \tan^2 \theta = \tan^2 \theta (\tan^2 \theta + 1)$

=  $\tan^2 \theta \cdot \sec^2 \theta$

$[\because \sec^2 \theta = \tan^2 \theta + 1]$

=  $(\sec^2 \theta - 1) \cdot \sec^2 \theta$

$[\because \tan^2 \theta = \sec^2 \theta - 1]$

=  $\sec^4 \theta - \sec^2 \theta = \text{RHS}$

**Exercise 8.4 Long Answer Type Questions**

**Question 1:**

If  $\operatorname{cosec} \theta + \cot \theta = p$ , then prove that  $\cos \theta = \frac{p^2 - 1}{p^2 + 1}$

**Solution:**

Given,  $\operatorname{cosec} \theta + \cot \theta = p$

$$\Rightarrow \frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta} = p \quad \left[ \because \operatorname{cosec} \theta = \frac{1}{\sin \theta} \text{ and } \cot \theta = \frac{\cos \theta}{\sin \theta} \right]$$

$$\Rightarrow \frac{1 + \cos \theta}{\sin \theta} = \frac{p}{1}$$

$$\Rightarrow \frac{(1 + \cos \theta)^2}{\sin^2 \theta} = \frac{p^2}{1} \quad \text{[take square on both sides]}$$

$$\Rightarrow \frac{1 + \cos^2 \theta + 2 \cos \theta}{\sin^2 \theta} = \frac{p^2}{1}$$

Using componendo and dividendo rule, we get

$$\frac{(1 + \cos^2 \theta + 2 \cos \theta) - \sin^2 \theta}{(1 + \cos^2 \theta + 2 \cos \theta) + \sin^2 \theta} = \frac{p^2 - 1}{p^2 + 1}$$

$$\Rightarrow \frac{1 + \cos^2 \theta + 2 \cos \theta - (1 - \cos^2 \theta)}{1 + 2 \cos \theta + (\cos^2 \theta + \sin^2 \theta)} = \frac{p^2 - 1}{p^2 + 1} \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$\Rightarrow \frac{2 \cos^2 \theta + 2 \cos \theta}{2 + 2 \cos \theta} = \frac{p^2 - 1}{p^2 + 1}$$

$$\Rightarrow \frac{2 \cos \theta (\cos \theta + 1)}{2 (\cos \theta + 1)} = \frac{p^2 - 1}{p^2 + 1}$$

$$\therefore \cos \theta = \frac{p^2 - 1}{p^2 + 1} \quad \text{Hence proved.}$$

### Question 2:

Prove that  $\sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta} = \tan \theta + \cot \theta$ .

**Solution:**

$$\text{LHS} = \sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta}$$

$$= \sqrt{\frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta}} \quad \left[ \because \sec \theta = \frac{1}{\cos \theta} \text{ and } \operatorname{cosec} \theta = \frac{1}{\sin \theta} \right]$$

$$= \sqrt{\frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta \cdot \cos^2 \theta}} = \sqrt{\frac{1}{\sin^2 \theta \cdot \cos^2 \theta}} \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$= \frac{1}{\sin \theta \cdot \cos \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cdot \cos \theta} \quad [\because 1 = \sin^2 \theta + \cos^2 \theta]$$

$$= \frac{\sin^2 \theta}{\sin \theta \cdot \cos \theta} + \frac{\cos^2 \theta}{\sin \theta \cdot \cos \theta}$$

$$= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \quad \left[ \because \tan \theta = \frac{\sin \theta}{\cos \theta} \text{ and } \cot \theta = \frac{\cos \theta}{\sin \theta} \right]$$

$$= \tan \theta + \cot \theta = \text{RHS}$$

### Question 3:

The angle of elevation of the top of a tower from certain point is  $30^\circ$ . If the observer moves 20 m towards the tower, the angle of elevation of the top increases by  $15^\circ$ . Find the height of the tower.

**Solution:**

Let the height of the tower be  $h$ .

also,  $SR = x$  m,  $\angle PSR = \theta$

Given that,  $QS = 20$  m

and  $\angle PQR = 30^\circ$

Now, in  $\triangle PSR$ ,

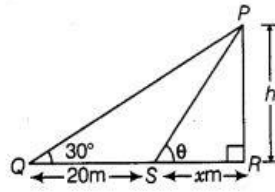
$$\tan \theta = \frac{PR}{SR} = \frac{h}{x}$$

$\Rightarrow$

$$\tan \theta = \frac{h}{x}$$

$\Rightarrow$

$$x = \frac{h}{\tan \theta} \quad \dots(i)$$



Now, in  $\triangle PQR$ ,

$$\tan 30^\circ = \frac{PR}{QR} = \frac{PR}{QS + SR}$$

$\Rightarrow$

$$\tan 30^\circ = \frac{h}{20 + x}$$

$\Rightarrow$

$$20 + x = \frac{h}{\tan 30^\circ} = \frac{h}{1/\sqrt{3}}$$

$\Rightarrow$

$$20 + x = h\sqrt{3}$$

$\Rightarrow$

$$20 + \frac{h}{\tan \theta} = h\sqrt{3} \quad \dots(ii) \text{ [from Eq. (i)]}$$

Since, after moving 20 m towards the tower the angle of elevation of the top increases by  $15^\circ$ .

i.e.,

$$\angle PSR = \theta = \angle PQR + 15^\circ$$

$\Rightarrow$

$$\theta = 30^\circ + 15^\circ = 45^\circ$$

$\therefore$

$$20 + \frac{h}{\tan 45^\circ} = h\sqrt{3} \quad \text{[from Eq. (i)]}$$

$\Rightarrow$

$$20 + \frac{h}{1} = h\sqrt{3}$$

$\Rightarrow$

$$20 = h\sqrt{3} - h$$

$\Rightarrow$

$$h(\sqrt{3} - 1) = 20$$

$\therefore$

$$h = \frac{20}{\sqrt{3} - 1} \cdot \frac{\sqrt{3} + 1}{\sqrt{3} + 1} \quad \text{[by rationalisation]}$$

$\Rightarrow$

$$= \frac{20(\sqrt{3} + 1)}{3 - 1} = \frac{20(\sqrt{3} + 1)}{2}$$

$\Rightarrow$

$$= 10(\sqrt{3} + 1) \text{ m}$$

Hence, the required height of tower is  $10(\sqrt{3} + 1)$  m.

#### Question 4:

If  $1 + \sin^2 \theta = 3 \sin \theta \cos \theta$ , then prove that  $\tan \theta = 1$  or  $\frac{1}{2}$

**Solution:**

Given,  $1 + \sin^2 \theta = 3 \sin \theta \cos \theta$

On dividing by  $\sin^2 \theta$  on both sides, we get

$$\frac{1}{\sin^2 \theta} + 1 = 3 \cdot \cot \theta$$

$\Rightarrow$

$$\operatorname{cosec}^2 \theta + 1 = 3 \cdot \cot \theta$$

$\Rightarrow$

$$1 + \cot^2 \theta + 1 = 3 \cdot \cot \theta$$

$\Rightarrow$

$$\cot^2 \theta - 3 \cot \theta + 2 = 0$$

$\Rightarrow$

$$\cot^2 \theta - 2 \cot \theta - \cot \theta + 2 = 0$$

[by splitting the middle term]

$\Rightarrow$

$$\cot \theta (\cot \theta - 2) - 1 (\cot \theta - 2) = 0$$

$\Rightarrow$

$$(\cot \theta - 2)(\cot \theta - 1) = 0 \Rightarrow \cot \theta = 1 \text{ or } 2$$

$\Rightarrow$

$$\tan \theta = 1 \text{ or } \frac{1}{2}$$

$$\left[ \therefore \tan \theta = \frac{1}{\cot \theta} \right]$$

Hence proved.

#### Question 5:

If  $\sin \theta + 2 \cos \theta = 1$ , then prove that  $2 \sin \theta - \cos \theta = 2$ .

**Solution:**

Given,  $\sin \theta + 2 \cos \theta = 1$

On squaring both sides, we get

$$\begin{aligned} & (\sin \theta + 2 \cos \theta)^2 = 1 \\ \Rightarrow & \sin^2 \theta + 4 \cos^2 \theta + 4 \sin \theta \cdot \cos \theta = 1 \\ \Rightarrow & (1 - \cos^2 \theta) + 4(1 - \sin^2 \theta) + 4 \sin \theta \cdot \cos \theta = 1 & [\because \sin^2 \theta + \cos^2 \theta = 1] \\ \Rightarrow & -\cos^2 \theta - 4\sin^2 \theta + 4 \sin \theta \cdot \cos \theta = -4 \\ \Rightarrow & 4 \sin^2 \theta + \cos^2 \theta - 4 \sin \theta \cdot \cos \theta = 4 \\ \Rightarrow & (2 \sin \theta - \cos \theta)^2 = 4 & [\because a^2 + b^2 - 2ab = (a - b)^2] \\ \Rightarrow & 2 \sin \theta - \cos \theta = 2 & \text{Hence proved.} \end{aligned}$$

**Question 6:**

The angle of elevation of the top of a tower from two points distant  $s$  and  $t$  from its foot are complementary. Prove that the height of the tower is  $\sqrt{st}$ .

**Solution:**

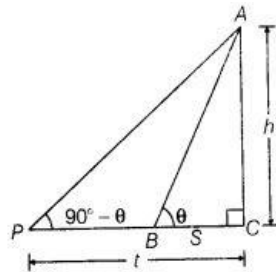
Let the height of the tower is  $h$ .

and  $\angle ABC = \theta$

Given that,  $BC = s, PC = t$

and angle of elevation on both positions are complementary.

i.e.,  $\angle APC = 90^\circ - \theta$



[if two angles are complementary to each other, then the sum of both angles is equal to  $90^\circ$ .]

Now in  $\triangle ABC$ ,  $\tan \theta = \frac{AC}{BC} = \frac{h}{s}$  ... (i)

and in  $\triangle APC$   $\tan(90^\circ - \theta) = \frac{AC}{PC}$   $[\because \tan(90^\circ - \theta) = \cot \theta]$

$$\begin{aligned} \Rightarrow & \cot \theta = \frac{h}{t} \\ \Rightarrow & \frac{1}{\tan \theta} = \frac{h}{t} & [\because \cot \theta = \frac{1}{\tan \theta}] \dots (ii) \end{aligned}$$

On multiplying Eqs. (i) and (ii), we get  $\tan \theta \cdot \frac{1}{\tan \theta} = \frac{h}{s} \cdot \frac{h}{t}$

$$\begin{aligned} \Rightarrow & \frac{h^2}{st} = 1 \\ \Rightarrow & h^2 = st \\ \Rightarrow & h = \sqrt{st} \end{aligned}$$

So, the required height of the tower is  $\sqrt{st}$ .  
Hence proved.

**Question 7:**

The shadow of a tower standing on a level plane is found to be 50 m longer when Sun's elevation is  $30^\circ$  than when it is  $60^\circ$ . Find the height of the tower.

**Solution:**

Let the height of the tower be  $h$  and  $RQ = x$  m

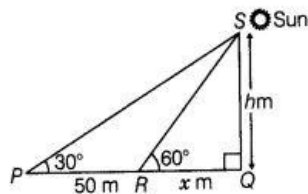
Given that,  $PR = 50$  m  
and  $\angle SPQ = 30^\circ, \angle SRQ = 60^\circ$

Now, in  $\triangle SRQ$ ,  $\tan 60^\circ = \frac{SQ}{RQ}$

$$\Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}} \dots (i)$$



and in  $\triangle SPQ$ ,  $\tan 30^\circ = \frac{SQ}{PQ} = \frac{SQ}{PR + RQ} = \frac{h}{50 + x}$



$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{50 + x}$$

$$\Rightarrow \sqrt{3} \cdot h = 50 + x$$

$$\Rightarrow \sqrt{3} \cdot h = 50 + \frac{h}{\sqrt{3}} \quad \text{[from Eq. (i)]}$$

$$\Rightarrow (\sqrt{3} - \frac{1}{\sqrt{3}})h = 50$$

$$\Rightarrow \frac{(3-1)}{\sqrt{3}}h = 50$$

$$\therefore h = \frac{50\sqrt{3}}{2}$$

$$h = 25\sqrt{3} \text{ m}$$

Hence, the required height of tower is  $25\sqrt{3}$  m.

### Question 8:

A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff of height  $h$ . At a point on the plane, the angles of elevation of the bottom and the top of the flag staff are  $\alpha$  and  $\beta$  respectively. Prove that the height of the tower is  $\left(\frac{h \tan \alpha}{\tan \beta - \tan \alpha}\right)$

### Solution:

Let the height of the tower be  $H$  and  $OR = x$

Given that, height of flag staff =  $h = FP$  and  $\angle PRO = \alpha$ ,  $\angle FRO = \beta$

Now, in  $\triangle PRO$ ,  $\tan \alpha = \frac{PO}{RO} = \frac{H}{x}$

$$\Rightarrow x = \frac{H}{\tan \alpha} \quad \dots(i)$$

and in  $\triangle FRO$ ,  $\tan \beta = \frac{FO}{RO} = \frac{FP + PO}{RO}$

$$\Rightarrow \tan \beta = \frac{h + H}{x} \quad \dots(ii)$$

From Eqs. (i) and (ii),

$$\frac{H}{\tan \alpha} = \frac{h + H}{\tan \beta}$$

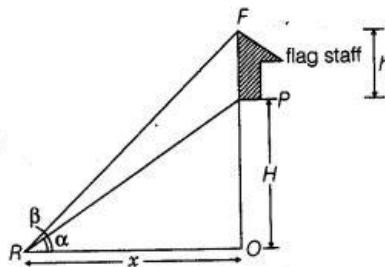
$$\Rightarrow H \tan \beta = h \tan \alpha + H \tan \alpha$$

$$\Rightarrow H \tan \beta - H \tan \alpha = h \tan \alpha$$

$$\Rightarrow H (\tan \beta - \tan \alpha) = h \tan \alpha \Rightarrow H = \frac{h \tan \alpha}{\tan \beta - \tan \alpha}$$

Hence the required height of tower is  $\frac{h \tan \alpha}{\tan \beta - \tan \alpha}$

Hence proved.



### Question 9:

If  $\tan \theta + \sec \theta = l$ , then prove that  $\sec \theta = \frac{l^2 + 1}{2l}$ .

### Solution:

Given,  $\tan \theta + \sec \theta = l \quad \dots(i)$

[multiply by  $(\sec \theta - \tan \theta)$  on numerator and denominator LHS]

$$\Rightarrow \frac{(\tan \theta + \sec \theta)(\sec \theta - \tan \theta)}{(\sec \theta - \tan \theta)} = l \Rightarrow \frac{(\sec^2 \theta - \tan^2 \theta)}{(\sec \theta - \tan \theta)} = l$$

$$\Rightarrow \frac{1}{\sec \theta - \tan \theta} = l \quad [\because \sec^2 \theta - \tan^2 \theta = 1]$$

$$\Rightarrow \sec \theta - \tan \theta = \frac{1}{l} \quad \dots(ii)$$

On adding Eqs. (i) and (ii), we get

$$\Rightarrow 2 \sec \theta = l + \frac{1}{l}$$

$$\Rightarrow \sec \theta = \frac{l^2 + 1}{2l} \quad \text{Hence proved.}$$

**Question 10:**

If  $\sin\theta + \cos\theta = p$  and  $\sec\theta + \operatorname{cosec}\theta = q$ , then prove that  $q(p^2 - 1) = 2p$ .

**Solution:**

Given that,  $\sin\theta + \cos\theta = p$  ... (i)  
 and  $\sec\theta + \operatorname{cosec}\theta = q$

$$\Rightarrow \frac{1}{\cos\theta} + \frac{1}{\sin\theta} = q \quad \left[ \because \sec\theta = \frac{1}{\cos\theta} \text{ and } \operatorname{cosec}\theta = \frac{1}{\sin\theta} \right]$$

$$\Rightarrow \frac{\sin\theta + \cos\theta}{\sin\theta \cdot \cos\theta} = q$$

$$\Rightarrow \frac{p}{\sin\theta \cdot \cos\theta} = q \quad [\text{from Eq. (i)}]$$

$$\Rightarrow \sin\theta \cdot \cos\theta = \frac{p}{q} \quad [\text{from Eq. (i)}] \dots \text{(ii)}$$

On squaring both sides, we get

$$\Rightarrow (\sin\theta + \cos\theta)^2 = p^2$$

$$\Rightarrow (\sin^2\theta + \cos^2\theta) + 2\sin\theta \cdot \cos\theta = p^2 \quad [ \because (a + b)^2 = a^2 + 2ab + b^2 ]$$

$$\Rightarrow 1 + 2\sin\theta \cdot \cos\theta = p^2 \quad [ \because \sin^2\theta + \cos^2\theta = 1 ]$$

$$\Rightarrow 1 + 2 \cdot \frac{p}{q} = p^2 \quad [\text{from Eq. (ii)}]$$

$$\Rightarrow q + 2p = p^2q \Rightarrow 2p = p^2q - q$$

$$\Rightarrow q(p^2 - 1) = 2p \quad \text{Hence proved.}$$

**Question 11:**

If  $a \sin\theta + b \cos\theta = c$ , then prove that  $a \cos\theta - b \sin\theta = \sqrt{a^2 + b^2 - c^2}$ .

**Solution:**

Given that,  $a \sin\theta + b \cos\theta = c$

On squaring both sides,

$$(a \cdot \sin\theta + \cos\theta \cdot b)^2 = c^2$$

$$\Rightarrow a^2 \sin^2\theta + b^2 \cos^2\theta + 2ab \sin\theta \cdot \cos\theta = c^2 \quad [ \because (x + y)^2 = x^2 + 2xy + y^2 ]$$

$$\Rightarrow a^2(1 - \cos^2\theta) + b^2(1 - \sin^2\theta) + 2ab \sin\theta \cdot \cos\theta = c^2 \quad [ \because \sin^2\theta + \cos^2\theta = 1 ]$$

$$\Rightarrow a^2 - a^2 \cos^2\theta + b^2 - b^2 \sin^2\theta + 2ab \sin\theta \cdot \cos\theta = c^2$$

$$\Rightarrow a^2 + b^2 - c^2 = a^2 \cos^2\theta + b^2 \sin^2\theta - 2ab \sin\theta \cdot \cos\theta$$

$$\Rightarrow (a^2 + b^2 - c^2) = (a \cos\theta - b \sin\theta)^2 \quad [ \because a^2 + b^2 - 2ab = (a - b)^2 ]$$

$$\Rightarrow (a \cos\theta - b \sin\theta)^2 = a^2 + b^2 - c^2$$

$$\Rightarrow a \cos\theta - b \sin\theta = \sqrt{a^2 + b^2 - c^2} \quad \text{Hence proved.}$$

**Question 12:**

Prove that  $\frac{1 + \sec\theta - \tan\theta}{1 + \sec\theta + \tan\theta} = \frac{1 - \sin\theta}{\cos\theta}$

**Solution:**

$$\begin{aligned}
\text{LHS} &= \frac{1 + \sec \theta - \tan \theta}{1 + \sec \theta + \tan \theta} \\
&= \frac{1 + 1/\cos \theta - \sin \theta/\cos \theta}{1 + 1/\cos \theta + \sin \theta/\cos \theta} \quad \left[ \because \sec \theta = \frac{1}{\cos \theta} \text{ and } \tan \theta = \frac{\sin \theta}{\cos \theta} \right] \\
&= \frac{\cos \theta + 1 - \sin \theta}{\cos \theta + 1 + \sin \theta} = \frac{(\cos \theta + 1) - \sin \theta}{(\cos \theta + 1) + \sin \theta} = \frac{2\cos^2 \frac{\theta}{2} - 2\sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2}}{2\cos^2 \frac{\theta}{2} + 2\sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2}} \\
& \quad \left[ \because 1 + \cos \theta = 2\cos^2 \frac{\theta}{2} \text{ and } \sin \theta = 2\sin \frac{\theta}{2} \cos \frac{\theta}{2} \right] \\
&= \frac{2\cos^2 \frac{\theta}{2} - 2\sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2}}{2\cos^2 \frac{\theta}{2} + 2\sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2}} = \frac{2\cos \frac{\theta}{2} \left( \cos \frac{\theta}{2} - \sin \frac{\theta}{2} \right)}{2\cos \frac{\theta}{2} \left( \cos \frac{\theta}{2} + \sin \frac{\theta}{2} \right)} \\
&= \frac{\cos \frac{\theta}{2} - \sin \frac{\theta}{2}}{\cos \frac{\theta}{2} + \sin \frac{\theta}{2}} \times \frac{\left( \cos \frac{\theta}{2} - \sin \frac{\theta}{2} \right)}{\left( \cos \frac{\theta}{2} - \sin \frac{\theta}{2} \right)} \quad \text{[by rationalisation]} \\
&= \frac{\left( \cos \frac{\theta}{2} - \sin \frac{\theta}{2} \right)^2}{\left( \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} \right)} \quad \left[ \because (a-b)^2 = a^2 + b^2 - 2ab \text{ and } (a-b)(a+b) = (a^2 - b^2) \right] \\
&= \frac{\left( \cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} \right) - \left( 2\sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2} \right)}{\cos \theta} \quad \left[ \because \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} = \cos \theta \right] \\
&= \frac{1 - \sin \theta}{\cos \theta} \quad \left[ \because \sin^2 \frac{\theta}{2} + \cos^2 \frac{\theta}{2} = 1 \right] \\
&= \text{RHS} \quad \text{Hence proved.}
\end{aligned}$$

### Question 13:

The angle of elevation of the top of a tower 30 m high from the foot of another tower in the same plane is  $60^\circ$  and the angle of elevation of the top of the second tower from the foot of the first tower is  $30^\circ$ . Find the distance between the two towers and also the height of the tower.

#### Solution:

Let distance between the two towers =  $AB = x$  m

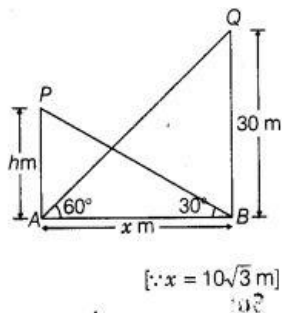
and height of the other tower =  $PA = h$  m

Given that, height of the tower =  $QB = 30$  m and  $\angle QAB = 60^\circ$ ,  $\angle PBA = 30^\circ$

$$\begin{aligned}
\text{Now, in } \triangle QAB, \quad \tan 60^\circ &= \frac{QB}{AB} = \frac{30}{x} \\
\Rightarrow \quad \sqrt{3} &= \frac{30}{x} \\
\therefore \quad x &= \frac{30}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3} \text{ m}
\end{aligned}$$

and in  $\triangle PBA$ ,

$$\begin{aligned}
\tan 30^\circ &= \frac{PA}{AB} = \frac{h}{x} \\
\Rightarrow \quad \frac{1}{\sqrt{3}} &= \frac{h}{10\sqrt{3}} \\
\Rightarrow \quad h &= 10 \text{ m}
\end{aligned}$$



Hence, the required distance and height are  $10\sqrt{3}$  m and 10 m, respectively.

### Question 14:

From the top of a tower  $h$  m high, angles of depression of two objects, which are in line with the foot of the tower are  $\alpha$  and  $\beta$  ( $\beta > \alpha$ ). Find the distance between the two objects.

#### Solution:

Let the distance between two objects is  $x$  m,

and  $CD = y$  m.

Given that,  $\angle BAX = \alpha = \angle ABD$ , [alternate angle]

$\angle CAY = \beta = \angle ACD$  [alternate angle]

Now, in  $\triangle ACD$ ,

$$\tan \beta = \frac{AD}{CD} = \frac{h}{y}$$

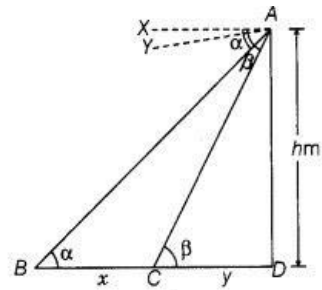
$$\Rightarrow y = \frac{h}{\tan \beta} \quad \dots(i)$$

and in  $\triangle ABD$ ,

$$\tan \alpha = \frac{AD}{BD} \Rightarrow \frac{AD}{BC + CD}$$

$$\Rightarrow \tan \alpha = \frac{h}{x + y} \Rightarrow x + y = \frac{h}{\tan \alpha}$$

$$\Rightarrow y = \frac{h}{\tan \alpha} - x \quad \dots(ii)$$



From Eqs. (i) and (ii),

$$\frac{h}{\tan \beta} = \frac{h}{\tan \alpha} - x$$

$$\therefore x = \frac{h}{\tan \alpha} - \frac{h}{\tan \beta}$$

$$= h \left( \frac{1}{\tan \alpha} - \frac{1}{\tan \beta} \right) = h (\cot \alpha - \cot \beta) \quad \left[ \because \cot \theta = \frac{1}{\tan \theta} \right]$$

which is the required distance between the two objects.

**Hence proved.**

### Question 15:

A ladder against a vertical wall at an inclination  $\alpha$  to the horizontal. Its foot is pulled away from the wall through a distance  $p$ , so that its upper end slides a distance  $q$  down the wall and then the ladder makes an angle  $\beta$  to the horizontal. Show that  $\frac{p}{q} = \frac{\cos \beta - \cos \alpha}{\sin \alpha - \sin \beta}$

**Solution:**

Let  $OQ = x$  and  $OA = y$

Given that,  $BQ = q$ ,  $SA = p$  and  $AB = SQ =$  Length of ladder

Also,  $\angle BAO = \alpha$  and  $\angle QSO = \beta$

Now, in  $\triangle BAO$ ,

$$\cos \alpha = \frac{OA}{AB}$$

$$\Rightarrow \cos \alpha = \frac{y}{AB}$$

$$\Rightarrow y = AB \cos \alpha = OA$$

$$\text{and } \sin \alpha = \frac{OB}{AB}$$

$$\Rightarrow OB = BA \sin \alpha$$

Now, in  $\triangle QSO$

$$\cos \beta = \frac{OS}{SQ}$$

$$\Rightarrow OS = SQ \cos \beta = AB \cos \beta \quad [\because AB = SQ] \dots(iii)$$

$$\text{and } \sin \beta = \frac{OQ}{SQ}$$

$$\Rightarrow OQ = SQ \sin \beta = AB \sin \beta \quad [\because AB = SQ] \dots(iv)$$

$$\text{Now, } SA = OS - AO$$

$$p = AB \cos \beta - AB \cos \alpha \quad \dots(v)$$

$$\text{and } BQ = BO - QO$$

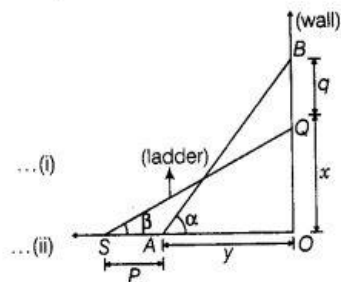
$$q = BA \sin \alpha - AB \sin \beta \quad \dots(vi)$$

$$\Rightarrow q = AB (\sin \alpha - \sin \beta)$$

Eq. (v) divided by Eq. (vi), we get

$$\frac{p}{q} = \frac{AB (\cos \beta - \cos \alpha)}{AB (\sin \alpha - \sin \beta)} = \frac{\cos \beta - \cos \alpha}{\sin \alpha - \sin \beta}$$

$$\Rightarrow \frac{p}{q} = \frac{\cos \beta - \cos \alpha}{\sin \alpha - \sin \beta} \quad \text{Hence proved.}$$



### Question 16:

The angle of elevation of the top of a vertical tower from a point on the ground is  $60^\circ$ . From another point 10 m vertically above the first, its angle of elevation is  $45^\circ$ . Find the height of the tower.

**Solution:**

Let the height the vertical tower,  $OT = H$

and  
 Given that,  
 and  
 Now, in  $\Delta TPO$ ,

$$OP = AB = x \text{ m}$$

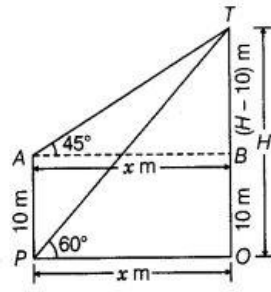
$$AP = 10 \text{ m}$$

$$\angle TPO = 60^\circ, \angle TAB = 45^\circ$$

$$\tan 60^\circ = \frac{OT}{OP} = \frac{H}{x}$$

$$\Rightarrow \sqrt{3} = \frac{H}{x}$$

$$\Rightarrow x = \frac{H}{\sqrt{3}} \quad \dots(i)$$



and in  $\Delta TAB$ ,

$$\tan 45^\circ = \frac{TB}{AB} = \frac{H-10}{x}$$

$$\Rightarrow 1 = \frac{H-10}{x} \Rightarrow x = H-10$$

$$\Rightarrow \frac{H}{\sqrt{3}} = H-10 \quad \text{[from Eq. (i)]}$$

$$\Rightarrow H - \frac{H}{\sqrt{3}} = 10 \Rightarrow H \left(1 - \frac{1}{\sqrt{3}}\right) = 10$$

$$\Rightarrow H \left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) = 10$$

$$\therefore H = \frac{10\sqrt{3}}{\sqrt{3}-1} \cdot \frac{\sqrt{3}+1}{\sqrt{3}+1} \quad \text{[by rationalisation]}$$

$$= \frac{10\sqrt{3}(\sqrt{3}+1)}{3-1} = \frac{10\sqrt{3}(\sqrt{3}+1)}{2}$$

$$\Rightarrow = 5\sqrt{3}(\sqrt{3}+1) = 5(\sqrt{3}+3) \text{ m.}$$

Hence, the required height of the tower is  $5(\sqrt{3}+3)$  m.

**Question 17:**

A window of a house is  $h$  m above the ground. From the window, the angles of elevation and depression of the top and the bottom of another house situated on the opposite side of the lane are found to be  $\alpha$  and  $\beta$ , respectively. Prove that the height of the other house is  $h(1 + \tan \alpha \cot \beta)$  m.

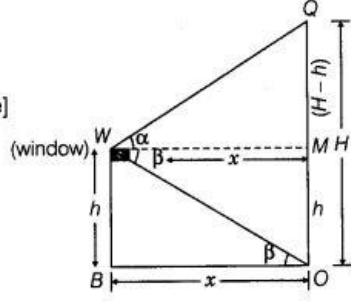
**Solution:**

Let the height of the other house =  $OQ = H$   
 and  $OB = MW = x$  m  
 Given that, height of the first house =  $WB = h = MO$   
 and  $\angle QWM = \alpha, \angle OWM = \beta = \angle WOB$

[alternate angle]

Now, in  $\Delta WOB$ ,  $\tan \beta = \frac{WB}{OB} = \frac{h}{x}$

$$\Rightarrow x = \frac{h}{\tan \beta} \quad \dots(i)$$



And in  $\Delta QWM$ ,  $\tan \alpha = \frac{QM}{WM} = \frac{OQ - MO}{WM}$

$$\Rightarrow \tan \alpha = \frac{H-h}{x}$$

$$\Rightarrow x = \frac{H-h}{\tan \alpha} \quad \dots(ii)$$

From Eqs. (i) and (ii),

$$\frac{h}{\tan \beta} = \frac{H-h}{\tan \alpha}$$

$$\Rightarrow h \tan \alpha = (H-h) \tan \beta$$

$$\Rightarrow h \tan \alpha = H \tan \beta - h \tan \beta$$

$$\Rightarrow H \tan \beta = h(\tan \alpha + \tan \beta)$$

$$\therefore H = h \left( \frac{\tan \alpha + \tan \beta}{\tan \beta} \right)$$

$$= h \left( 1 + \tan \alpha \cdot \frac{1}{\tan \beta} \right) = h(1 + \tan \alpha \cdot \cot \beta) \quad \left[ \because \cot \theta = \frac{1}{\tan \theta} \right]$$

Hence, the required height of the other house is  $h(1 + \tan \alpha \cdot \cot \beta)$  **Hence proved.**

**Question 18:**

The lower window of a house is at a height of 2 m above the ground and its upper window is 4 m vertically above the lower window. At certain instant the angles of elevation of a balloon from these windows are observed to be  $60^\circ$  and  $30^\circ$ , respectively. Find the height of the balloon above the ground.

**Solution:**

Let the height of the balloon from above the ground is H.

A and  $OP = w_2R = w_1Q = x$

Given that, height of lower window from above the ground =  $w_2P = 2 \text{ m} = OR$

Height of upper window from above the lower window =  $w_1w_2 = 4 \text{ m} = QR$

$$\begin{aligned} \therefore BQ &= OB - (QR + RO) \\ &= H - (4 + 2) \\ &= H - 6 \end{aligned}$$

and  $\angle Bw_1Q = 30^\circ$

$\Rightarrow \angle Bw_2R = 60^\circ$

Now, in  $\Delta Bw_2R$ ,

$$\tan 60^\circ = \frac{BR}{w_2R} = \frac{BQ + QR}{x}$$

$\Rightarrow \sqrt{3} = \frac{(H-6) + 4}{x}$

$\Rightarrow x = \frac{H-2}{\sqrt{3}} \dots (i)$

and in  $\Delta Bw_1Q$ ,

$$\tan 30^\circ = \frac{BQ}{w_1Q}$$

$$\tan 30^\circ = \frac{H-6}{x} = \frac{1}{\sqrt{3}}$$

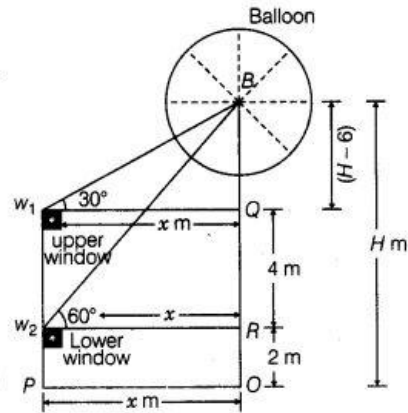
$\Rightarrow x = \sqrt{3}(H-6) \dots (ii)$

From Eqs. (i) and (ii),

$$\sqrt{3}(H-6) = \frac{(H-2)}{\sqrt{3}}$$

$$3(H-6) = H-2 = 3H-18 = H-2$$

$\Rightarrow 2H = 16 \Rightarrow H = 8$



So, the required height is 8 m.

Hence, the required height of the balloon from above the ground is 8 m.