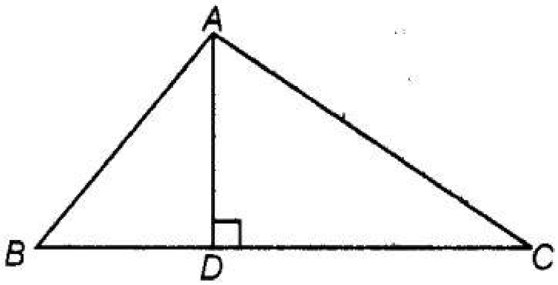
Unit 6 (Triangles)

Exercise 6.1 Multiple Choice Questions (MCQs)

Question 1:

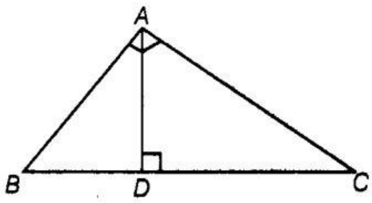
In figure, if ∠BAC =90° and AD⊥BC. Then,



(a) BD.CD = BC² (b) AB.AC = BC² (c) BD.CD=AD² (d) AB.AC =AD²

Solution:

(c) *In* ΔADB *and* ΔADC,



$$\angle D = \angle D = 90^\circ$$

$$\angle DBA = \angle DAC \quad \text{[each equal to } 90^\circ - \angle C]$$

$\therefore$ $$\Delta ADB \sim \Delta ADC \quad \text{[by AAA similarity criterion]}$$

$\therefore$ $$\frac{BD}{AD} = \frac{AD}{CD}$$

$\Rightarrow$ $$BD \cdot CD = AD^2$$

Question 2:

If the lengths of the diagonals of rhombus are 16 cm and 12 cm. Then, the length of the sides of the rhombus is

(a) 9 cm (b) 10 cm (c) 8 cm (d) 20 cm

Solution:

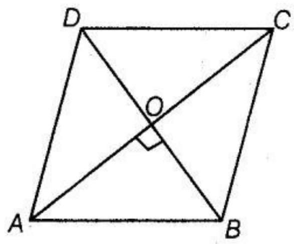
(b) We know that, the diagonals of a rhombus are perpendicular bisector of each other.

Given, AC = 16 cm and BD = 12 cm [let]

∴ AO = 8cm, SO = 6cm

and $\angle AOB = 90°$

In right angled $\angle AOB$,



$$AB^2 = AO^2 + OB^2 \quad \text{[by Pythagoras theorem]}$$

$\Rightarrow$ $AB^2 = 8^2 + 6^2 = 64 + 36 = 100$

$\therefore$ $AB = 10\text{cm}$

Question 3:

If ΔABC ~ ΔEDF and ΔABC is not similar to ΔDEF, then which of the following is not true?

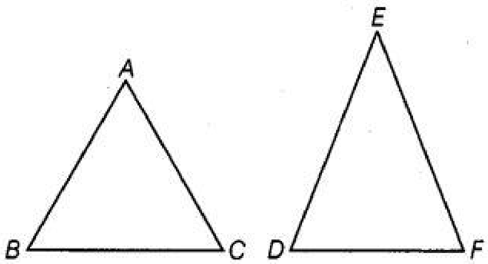
(a) BC · EF = AC · FD (b) AB · EF = AC · DE

(c) BC · DE = AB · EF (d) BC · DE = AB · FD

Solution:

(c) Given, $\Delta ABC \sim \Delta EDF$

$\therefore$ $\frac{AB}{ED} = \frac{BC}{DF} = \frac{AC}{EF}$



Taking first two terms, we get

$$\frac{AB}{ED} = \frac{BC}{DF}$$

$\Rightarrow$ $AB \cdot DF = ED \cdot BC$

or $BC \cdot DE = AB \cdot DF$

So, option (d) is true.

Taking last two terms, we get

$$\frac{BC}{DF} = \frac{AC}{EF}$$

$\Rightarrow$ $BC \cdot EF = AC \cdot DF$

So, option (a) is also true.

Taking first and last terms, we get

$$\frac{AB}{ED} = \frac{AC}{EF}$$

$\Rightarrow$ $AB \cdot EF = ED \cdot AC$

Hence, option (b) is true.

Question 4:

If in two Δ PQR ,$\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$,then

(a)Δ PQR~Δ CAB (b) Δ PQR ~ Δ ABC

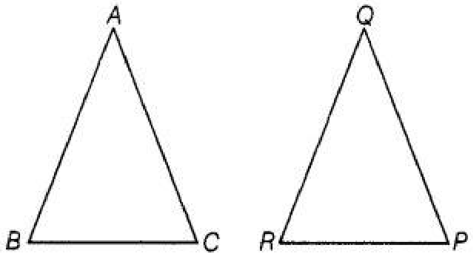
(c)Δ CBA ~ Δ PQR (d) Δ BCA ~ Δ PQR

Solution:

(a) Given, in two Δ ABC and Δ PQR$\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$

which shows that sides of one triangle are proportional to the side of the other triangle, then their corresponding angles are also equal, so by SSS similarity, triangles are similar.

i.e., $\Delta CAB \sim \Delta PQR$



Question 5:

In figure, two line segments AC and BD intersect each other at the point P such that PA = 6 cm, PB = 3 cm, PC = 2.5 cm, PD = 5 cm, ∠APB = 50° and ∠CDP = 30°. Then, ∠PBA is equal to

(a) 50° (b) 30° (c) 60° (d) 100°

Solution:

(d) In ΔAPB and ΔCPD, $\angle APB = \angle CPD = 50°$ [vertically opposite angles]

$$\frac{AP}{PD} = \frac{6}{5} \quad \ldots(i)$$

and $$\frac{BP}{CP} = \frac{3}{2.5} = \frac{6}{5} \quad \ldots(ii)$$

From Eqs. (i) and (ii)

$$\frac{AP}{PD} = \frac{BP}{CP}$$

$\therefore$ $\Delta APB \sim \Delta DPC$ [by SAS similarity criterion]

$\therefore$ $\angle A = \angle D = 30°$ [corresponding angles of similar triangles]

In ΔAPB, $\angle A + \angle B + \angle APB = 180°$ [sum of angles of a triangle = 180°]

$\Rightarrow$ $30° + \angle B + 50° = 180°$

$\therefore$ $\angle B = 180° - (50° + 30°) = 100°$

i.e., $\angle PBA = 100°$

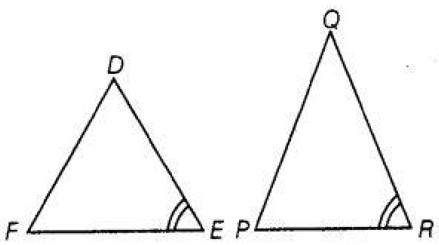
Question 6:

If in two Δ DEF and Δ PQR, ∠D = ∠Q and ∠R = ∠E, then which of the following is not true?

(a) $\frac{EF}{PR} = \frac{DF}{PQ}$ (b) $\frac{DE}{PQ} = \frac{EF}{RP}$ (c) $\frac{DE}{QR} = \frac{DF}{PQ}$ (d) $\frac{EF}{RP} = \frac{DE}{QR}$

Solution:

(b) Given, in ΔDEF, ∠D = ∠Q, ∠R = ∠E



$\therefore$ $\Delta DEF \sim \Delta QRP$ [by AAA similarity criterion]

$\Rightarrow$ $\angle F = \angle P$ [corresponding angles of similar triangles]

$\therefore$ $$\frac{DF}{QP} = \frac{ED}{RQ} = \frac{FE}{PR}$$

Question 7:

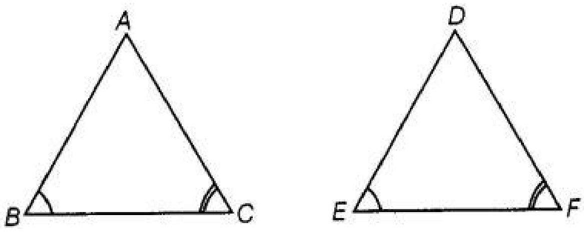
In Δ ABC and ΔDEF, ∠B = ∠E, ∠F = ∠C and AB = 30E Then, the two triangles are

(a) congruent but not similar (b) similar but not congruent

(c) neither congruent nor similar (d) congruent as well as similar

Solution:

(b) In ΔABC and ΔDEF, ∠B = ∠E, ∠F = ∠C and AB = 3DE



We know that, if in two triangles corresponding two angles are same, then they are similar by AAA similarity criterion. Also, ΔA8C and ΔDEF do not satisfy any rule of congruency, (SAS, ASA, SSS), so both are not congruent.

Question 8:

If $\Delta ABC \sim \Delta PQR$ with $\frac{BC}{QR} = \frac{1}{3}$, then $\frac{ar(\Delta PRQ)}{ar(\Delta BCA)}$ is equal to

(a) 9 (b) 3 (c) $\frac{1}{3}$ (d) $\frac{1}{9}$

Solution:

(a) Given, $\Delta ABC \sim \Delta PQR$ and $\frac{BC}{QR} = \frac{1}{3}$

We know that, the ratio of the areas of two similar triangles is equal to square of the ratio of their corresponding sides.

$$\therefore \quad \frac{ar(\Delta PRQ)}{ar(\Delta BCA)} = \frac{(QR)^2}{(BC)^2} = \left(\frac{QR}{BC}\right)^2 = \left(\frac{3}{1}\right)^2 = \frac{9}{1} = 9$$

Question 9:

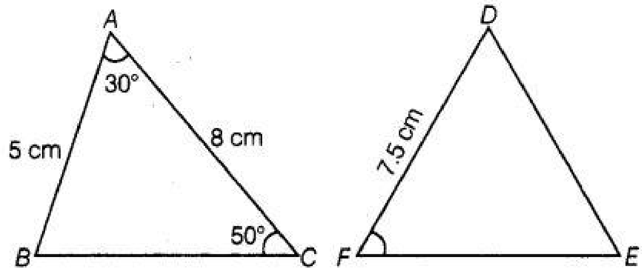
If ΔABC ~ΔDFE, ∠A = 30°, ∠C = 50°, AB = 5 cm, AC = 8 cm and OF = 7.5 cm. Then, which of the following is true?

(a) DE =12 cm, ∠F =50° (b) DE = 12 cm, ∠F =100°

(c) EF = 12 cm, ∠D = 100° (d) EF = 12 cm,∠D =30°

Solution:

(b) *Given,* AABC ~ ADFE, *then* ∠A = ∠D = 30°, ∠C= ∠E = *50°*



$\therefore$ $\angle B = \angle F = 180° - (30° + 50°) = 100°$

Also, $AB = 5\,cm$, $AC = 8\,cm$ and $DF = 7.5\,cm$

$\therefore$ $\frac{AB}{DF} = \frac{AC}{DE}$

$\Rightarrow$ $\frac{5}{7.5} = \frac{8}{DE}$

$\therefore$ $DE = \frac{8 \times 7.5}{5} = 12\,cm$

Hence, $DE = 12\,cm$, $\angle F = 100°$

Question 10:

If in ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar, when

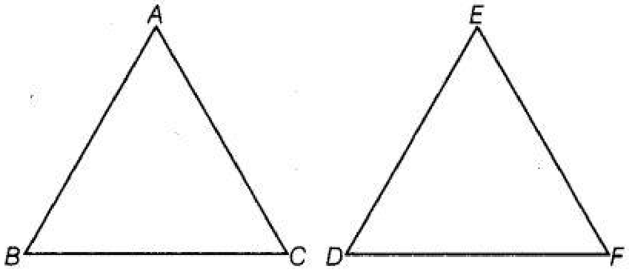
(a) ∠B = ∠E (b) ∠A = ∠D

(c)∠B = ∠D (d) ∠A = ∠F

Solution:

(c) Given, in ΔABC and ΔEDF,

$$\frac{AB}{DE} = \frac{BC}{FD}$$



By converse of basic proportionality theorem,

$$\Delta ABC \sim \Delta EDF$$

Then, $\angle B = \angle D, \angle A = \angle E$

and $\angle C = \angle F$

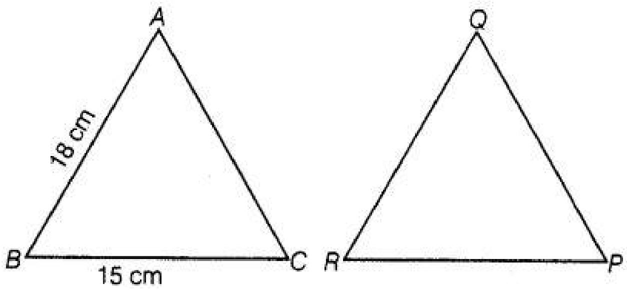
Question 11:

If $\Delta ABC \sim \Delta QRP$, $\dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \dfrac{9}{4}$, $AB = 18$ cm and $BC = 15$ cm, then PR is equal to

(a) 10 cm (b) 12 cm (c) $\frac{20}{3}$cm (d) 8 cm

Solution:

(a) Given, Δ ABC ~Δ QRP, AB = 18cm and BC = 15cm



We know that, the ratio of area of two similar triangles is equal to the ratio of square of their corresponding sides.

∴ $\dfrac{ar(\Delta ABC)}{ar(\Delta QRP)} = \dfrac{(BC)^2}{(RP)^2}$

But given, $\dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \dfrac{9}{4}$ [given]

⇒ $\dfrac{(15)^2}{(RP)^2} = \dfrac{9}{4}$ [∵ $BC = 15$ cm, given]

⇒ $(RP)^2 = \dfrac{225 \times 4}{9} = 100$

∴ $RP = 10$ cm

Question 12:

If S is a point on side PQ of a Δ PQR such that PS = QS = RS, then

(a) $PR \cdot QR = RS^2$ (b) $QS^2 + RS^2 = QR^2$

(c) $PR^2 + QR^2 = PQ^2$ (d) $PS^2 + RS^2 = PR^2$

Solution:

(c) Given, in ΔPQR,

$$PS = QS = RS \quad \ldots(i)$$

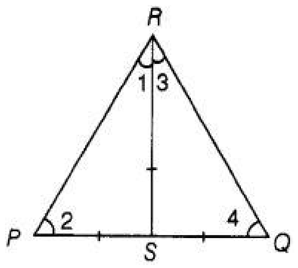
In ΔPSR, $PS = RS$ [from Eq. (i)]

$\Rightarrow$ $\angle 1 = \angle 2$ ...(ii)

Similarly, in ΔRSQ,

$\Rightarrow$ $\angle 3 = \angle 4$ ...(iii)

[corresponding angles of equal sides are equal]



Now, in ΔPQR, sum of angles = 180°

$\Rightarrow$ $\angle P + \angle Q + \angle R = 180°$

$\Rightarrow$ $\angle 2 + \angle 4 + \angle 1 + \angle 3 = 180°$ [using Eqs. (ii) and (iii)]

$\Rightarrow$ $\angle 1 + \angle 3 + \angle 1 + \angle 3 = 180°$

$\Rightarrow$ $2(\angle 1 + \angle 3) = 180°$

$\Rightarrow$ $\angle 1 + \angle 3 = \frac{180°}{2} = 90°$

$\therefore$ $\angle R = 90°$

In ΔPQR, by Pythagoras theorem,

$$PR^2 + QR^2 = PQ^2$$

Exercise 6.2 Very Short Answer Type Questions

Question 1:

Is the triangle with sides 25 cm, 5 cm and 24 cm a right triangle? Give reason for your answer.

Solution:

False

Let a = 25 cm, b = 5 cm and c =24 cm

Now, $b^2 + c^2 = (5)^2 + (24)^2$

$= 25 + 576 = 601 \neq (25)^2$

Hence, given sides do not make a right triangle because it does not satisfy the property of Pythagoras theorem.

Question 2:

It is given that $\Delta DEF \sim \Delta RPQ$. Is it true to say that $\angle D = \angle R$ and $\angle F = \angle P$? Why?

Solution:

False

We know that, if two triangles are similar, then their corresponding angles are equal.

$\therefore$ $\angle D = \angle R$, $\angle E = \angle P$ and $\angle F = Q$

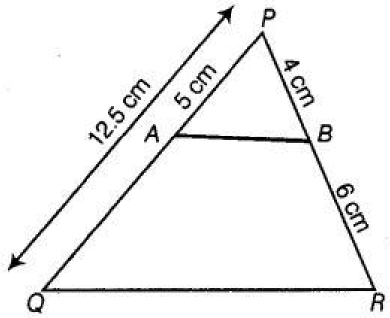
Question 3:

A and B are respectively the points on the sides PQ and PR of a ΔPQR such that PQ = 12.5 cm, PA = 5 cm, BR = 6 cm and PB = 4 cm. Is AB || QR ? Give reason for your answer.

Solution:

False

Given, PQ = 12.5 cm, PA = 5 cm, BR = 6 cm and PB = 4 cm



Then, $QA = QP - PA = 12.5 - 5 = 7.5\text{ cm}$

Now, $\frac{PA}{AQ} = \frac{5}{7.5} = \frac{50}{75} = \frac{2}{3}$ ...(i)

and $\frac{PB}{BR} = \frac{4}{6} = \frac{2}{3}$ ...(ii)

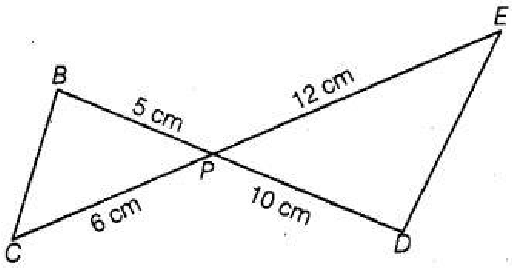
From Eqs. (i) and (ii), $\frac{PA}{AQ} = \frac{PB}{BR}$

By converse of basic proportionality theorem,

$AB \parallel QR$

Question 4:

In figure, BD and CE intersect each other at the point P. Is ΔPBC ~ ΔPDE? Why?



Solution:

True

$\angle BPC = \angle EPD$ [vertically opposite angles]

Now, $\frac{PB}{PD} = \frac{5}{10} = \frac{1}{2}$ ...(i)

and $\frac{PC}{PE} = \frac{6}{12} = \frac{1}{2}$ ...(ii)

From Eqs. (i) and (ii), $\frac{PB}{PD} = \frac{PC}{PE}$

Since, one angle of ΔPBC is equal to one angle of ΔPDE and the sides including these angles are proportional, so both triangles are similar.

Hence, ΔPBC ~ ΔPDE, by SAS similarity criterion.

Question 5:

In ΔPQR and ΔMST, ∠P = 55°, ∠Q =25°, ∠M = 100° and ∠S = 25°. Is ΔQPR ~ ΔTSM? Why?

Solution:

False

We know that, the sum of three angles of a triangle is 180°.

In ΔPQR, $\angle P + \angle Q + \angle R = 180°$

$\Rightarrow 55° + 25° + \angle R = 180°$

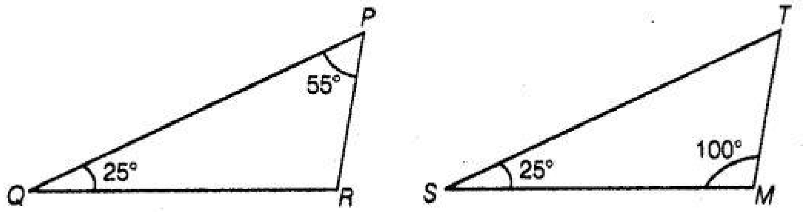
$\Rightarrow \angle R = 180° - (55° + 25°) = 180° - 80° = 100°$

In ΔTSM, $\angle T + \angle S + \angle M = 180°$

$\Rightarrow \angle T + \angle 25° + 100° = 180°$

$\Rightarrow \angle T = 180° - (25° + 100°)$

$= 180° - 125° = 55°$



In ΔPQR and A TSM, and

∠P = ∠T, ∠Q = ∠S,

and ∠R = ∠M

ΔPQR ~ ΔTSM [since, all corresponding angles are equal]

Hence, Δ QPR is not similar to ΔTSM, since correct correspondence is P ↔ T, Q < r→ S and R ↔ M

Question 6:

Is the following statement true? Why? “Two quadrilaterals are similar, if their corresponding angles are equal”.

Solution:

False

Two quadrilaterals are similar, if their corresponding angles are equal and corresponding sides must also be proportional.

Question 7:

Two sides and the perimeter of one triangle are respectively three times the corresponding sides and the perimeter of the other triangle. Are the two triangles similar? Why?

Solution:

True

Here, the corresponding two sides and the perimeters of two triangles are proportional, then third side of both triangles will also in proportion.

Question 8:

If in two right triangles, one of the acute angles of one triangle is equal to an acute angle of the other triangle. Can you say

that two triangles will be similar? Why?

Solution:

True

Let two right angled triangles be ΔABC and ΔPQR.

Question 9:

The ratio of the corresponding altitudes of two similar triangles is $\frac{3}{5}$. Is it correct to say that ratio of their areas is $\frac{6}{5}$? Why?

Solution:

False

By the property of area of two similar triangles,

$$\left(\frac{\text{Area}_1}{\text{Area}_2}\right)=\left(\frac{\text{Altitude}_1}{\text{Altitude}_2}\right)^2$$

$$\Rightarrow \left(\frac{\text{Area}_1}{\text{Area}_2}\right)=\left(\frac{3}{5}\right)^2 \qquad \left[\because \frac{\text{altitude}_1}{\text{altitude}_2}=\frac{3}{5}, \text{given}\right]$$

$$=\frac{9}{25}\neq\frac{6}{5}$$

So, given statement is not correct,

Question 10:

D is a point on side QR of ΔPQR such that PD ⊥ QR. Will it be correct to say that ΔPQD ~ ΔRPD? Why?

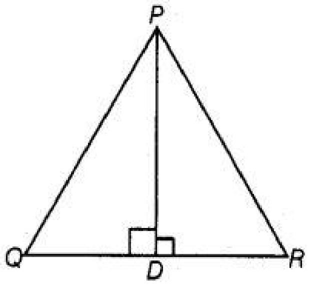
Solution:

False

In ΔPQD and ΔRPD,

PD = PD [common side]

∠PDQ = ∠PDR [each 90°]



Here, no other sides or angles are equal, so we can say that ∠PQD is not similar to ΔRPD.

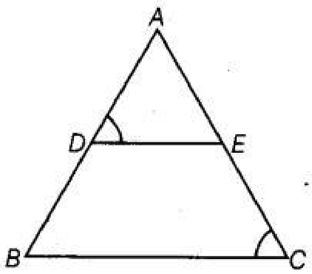
But, if ∠P = 90°,

then ∠DPQ = ∠PRD

[each equal to 90° – ∠0 and by ASA similarity criterion, ΔPQD ~ΔRPD]

Question 11:

In figure, if ∠D = ∠C, then it is true that ΔADE ~ ΔACB? Why?



Solution:

True

In ΔADE and ΔACB,

∠A = ∠A [common angle]

∠D = ∠C [given]

ΔADE ~ ΔACB [by AAA similarity criterion]

Question 12:

Is it true to say that, if in two triangles, an angle of one triangle is equal to an angle of another triangle and two sides of one triangle are proportional to the two sides of the other triangle, then the triangles are similar? Give reason for your answer.

Solution:

False

Because, according to SAS similarity criterion, if one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

Here, one angle and two sides of two triangles are equal but these sides not including equal angle, so given statement is not correct.

Exercise 6.3 Short Answer Type Questions

Question 1:

In a ΔPQR, $PR^2 – PQ^2 = QR^2$ and M is a point on side PR such that QM ⊥ PR.

Prove that $QM^2 = PM \times MR$.

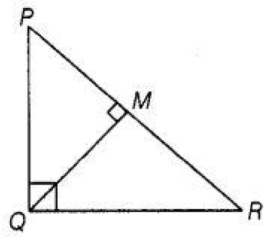
Solution:

Given In Δ PQR, $PR^2 - PQ^2 = QR^2$ and $QM \perp PR$

To prove $QM^2 = PM \times MR$

Proof Since, $PR^2 - PQ^2 = QR^2$

$\Rightarrow$ $PR^2 = PQ^2 + QR^2$



So, ΔPQR is right angled triangle at Q.

In ΔQMR and ΔPMQ, $\angle M = \angle M$ [each 90°]

$\angle MQR = \angle QPM$ [each equal to $90° - \angle R$]

$\therefore$ $\Delta QMR \sim \Delta PMQ$ [by AAA similarity criterion]

Now, using property of area of similar triangles, we get

$$\frac{ar(\Delta QMR)}{ar(\Delta PMQ)} = \frac{(QM)^2}{(PM)^2}$$

$\Rightarrow$ $$\frac{\frac{1}{2} \times RM \times QM}{\frac{1}{2} \times PM \times QM} = \frac{(QM)^2}{(PM)^2}$$ [$\because$ area of triangle $= \frac{1}{2} \times$ base $\times$ height]

$\Rightarrow$ $QM^2 = PM \times RM$ **Hence proved.**

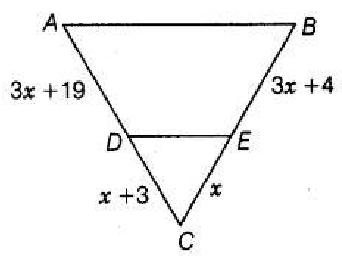
Question 2:

Find the value of x for which DE || AB in given figure.

Solution:

Given, $DE \parallel AB$

$\therefore$ $\frac{CD}{AD} = \frac{CE}{BE}$ [by basic proportionality theorem]



$\Rightarrow$ $\frac{x+3}{3x+19} = \frac{x}{3x+4}$

$\Rightarrow$ $(x+3)(3x+4) = x(3x+19)$

$\Rightarrow$ $3x^2 + 4x + 9x + 12 = 3x^2 + 19x$

$\Rightarrow$ $19x - 13x = 12$

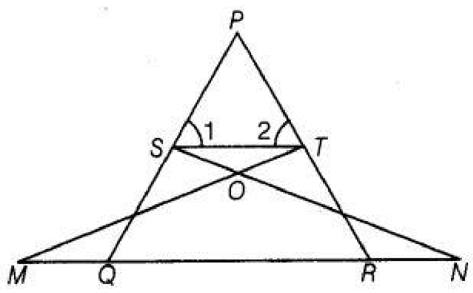
$\Rightarrow$ $6x = 12$

$\therefore$ $x = \frac{12}{6} = 2$

Hence, the required value of x is 2.

Question 3:

In figure, if $\angle 1 = \angle 2$ and $\Delta NSQ = \Delta MTR$, then prove that $\Delta PTS \sim \Delta PRQ$.



Solution:

Given $\Delta NSQ \cong \Delta MTR$ and $\angle 1 = \angle 2$

To prove $\Delta PTS \sim \Delta PRQ$

Proof Since, $\Delta NSQ \cong \Delta MTR$

So, $SQ = TR$ ...(i)

Also, $\angle 1 = \angle 2 \Rightarrow PT = PS$ ...(ii)

[since, sides opposite to equal angles are also equal]

From Eqs. (i) and (ii), $\frac{PS}{SQ} = \frac{PT}{TR}$

$\Rightarrow$ $ST \parallel QR$ [by converse of basic proportionality theorem]

$\therefore$ $\angle 1 = \angle PQR$

and $\angle 2 = \angle PRQ$

In ΔPTS and ΔPRQ,

$\angle P = \angle P$ [common angles]

$\angle 1 = \angle PQR$

$\angle 2 = \angle PRQ$

$\therefore$ $\Delta PTS \sim \Delta PRQ$ [by AAA similarity criterion]

Hence proved.

Question 4:

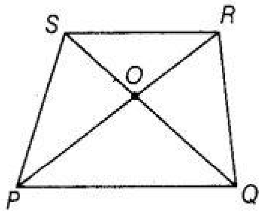
Diagonals of a trapezium PQRS intersect each other at the point 0, PQ || RS and PQ = 3 RS. Find the ratio of the areas of

Δ POQ and Δ ROS.

Solution:

Given PQRS is a trapezium in which PQ || PS and PQ = 3 RS

$\Rightarrow$ $\frac{PQ}{RS} = \frac{3}{1}$ ...(i)



In ΔPOQ and ΔROS, $\angle SOR = \angle QOP$ [vertically opposite angles]

$\angle SRP = \angle RPQ$ [alternate angles]

$\therefore$ $\Delta POQ \sim \Delta ROS$ [by AAA similarity criterion]

By property of area of similar triangle,

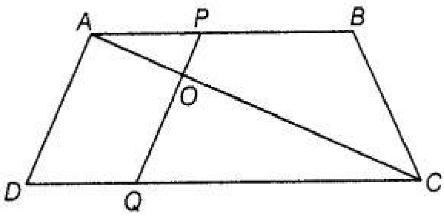
$$\frac{\text{ar}(\Delta POQ)}{\text{ar}(\Delta SOR)} = \frac{(PQ)^2}{(RS)^2} = \left(\frac{PQ}{RS}\right)^2 = \left(\frac{3}{1}\right)^2 \quad \text{[from Eq. (i)]}$$

$$\Rightarrow \frac{\text{ar}(\Delta POQ)}{\text{ar}(\Delta SOR)} = \frac{9}{1}$$

Hence, the required ratio is 9 :1.

Question 5:

In figure, if AB || DC and AC, PQ intersect each other at the point 0. Prove that OA . CQ = 0C . AP.



Solution:

Given AC and PQ intersect each other at the point O and AB || DC

Prove that OA . CQ = 0C . AP.

Proof In ΔAOP and ΔCOQ, $\angle AOP = \angle COQ$ [vertically opposite angles]

$\angle APO = \angle CQO$

[since, $AB \parallel DC$ and PQ is transversal, so alternate angles]

$\therefore$ $\Delta AOP \sim \Delta COQ$ [by AAA similarity criterion]

Then, $\frac{OA}{OC} = \frac{AP}{CQ}$ [since, corresponding sides are proportional]

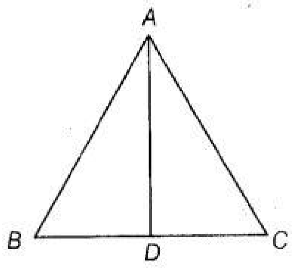
$\Rightarrow$ $OA \cdot CQ = OC \cdot AP$ **Hence proved.**

Question 6:

Find the altitude of an equilateral triangle of side 8 cm.

Solution:

Let ABC be an equilateral triangle of side 8 cm i.e., AB = BC = CA = 8 cm. Draw altitude AD which is perpendicular to BC. Then, D is the mid-point of BC.



$\therefore$ $BD = CD = \frac{1}{2} BC = \frac{8}{2} = 4\,\text{cm}$

Now, $AB^2 = AD^2 + BD^2$ [by Pythagoras theorem]

$\Rightarrow$ $(8)^2 = AD^2 + (4)^2$

$\Rightarrow$ $64 = AD^2 + 16$

$\Rightarrow$ $AD^2 = 64 - 16 = 48$

$\Rightarrow$ $AD = \sqrt{48} = 4\sqrt{3}\,\text{cm}.$

Hence, altitude of an equilateral triangle is $4\sqrt{3}$ cm.

Question 7:

If ΔABC ~ ΔDEF, AB = 4 cm, DE = 6, EF = 9 cm and FD = 12 cm, then find the perimeter of Δ ABC.

Solution:

Given AB = 4cm, DE = 6cm and EF = 9cm and FD = 12 cm

Also, $\Delta ABC \sim \Delta DEF$

$\therefore$ $\frac{AB}{ED} = \frac{BC}{EF} = \frac{AC}{DF}$

$\Rightarrow$ $\frac{4}{6} = \frac{BC}{9} = \frac{AC}{12}$

On taking first two terms, we get

$$\frac{4}{6} = \frac{BC}{9}$$

$\Rightarrow$ $BC = \frac{4 \times 9}{6} = 6\,\text{cm}$

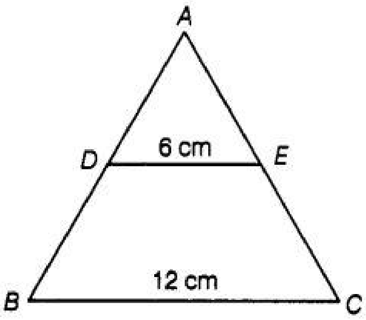
$= AC = \frac{6 \times 12}{9} = 8\,\text{cm}$

Now, perimeter of $\Delta ABC = AB + BC + AC$

$= 4 + 6 + 8 = 18\,\text{cm}$

Question 8:

In figure, if DE || BC, then find the ratio of ar (Δ ADE) and ar (DECB).



Solution:

Given, $DE \parallel BC$, $DE = 6$ cm and $BC = 12$ cm

In ΔABC and ΔADE,

$\angle ABC = \angle ADE$ [corresponding angle]

$\angle ACB = \angle AED$ [corresponding angle]

and $\angle A = \angle A$ [common side]

$\therefore$ $\Delta ABC \sim \Delta AED$ [by AAA similarity criterion]

Then, $$\frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{(DE)^2}{(BC)^2} = \frac{(6)^2}{(12)^2} = \left(\frac{1}{2}\right)^2$$

$\Rightarrow$ $$\frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Let ar $(\Delta ADE) = k$, then ar $(\Delta ABC) = 4k$

Now, ar $(DECB)$ = ar (ABC) − ar(ADE) = $4k - k = 3k$

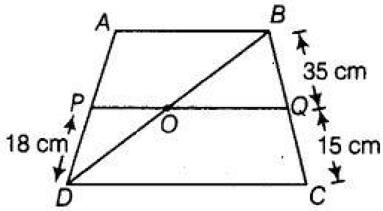
$\therefore$ Required ratio = ar (ADE) : ar $(DECB)$ = $k : 3k = 1 : 3$

Question 9:

ABCD is a trapezium in which AB || DC and P,Q are points on AD and BC respectively, such that PQ || DC, if PD = 18 cm, BQ = 35 cm and QC = 15 cm, find AD.

Solution:

Given, a trapezium ABCD in which AB || DC. P and Q are points on AD and BC, respectively such that PQ || DC. Thus, AB || PQ || DC.



Join BD.

In ΔABD, $PO \parallel AB$ [$\because PQ \parallel AB$]

By basic proportionality theorem, $$\frac{DP}{AP} = \frac{DO}{OB} \quad \ldots(i)$$

In ΔBDC, $OQ \parallel DC$ [$\because PQ \parallel DC$]

By basic proportionality theorem,

$$\frac{BQ}{QC} = \frac{OB}{OD}$$

$\Rightarrow$ $$\frac{QC}{BQ} = \frac{OD}{OB} \quad \ldots(ii)$$

From Eqs. (i) and (ii), $$\frac{DP}{AP} = \frac{QC}{BQ}$$

$\Rightarrow$ $$\frac{18}{AP} = \frac{15}{35}$$

$\Rightarrow$ $$AP = \frac{18 \times 35}{15} = 42$$

$\therefore$ $AD = AP + DP = 42 + 18 = 60$ cm

Question 10:

Corresponding sides of two similar triangles are in the ratio of 2 : 3. If the area of the smaller triangle is 48 cm^2, then find the area of the larger triangle.

Solution:

Given, ratio of corresponding sides of two similar triangles = 2:3 o$\frac{2}{3}$

Area of smaller triangle = 48 cm^2

By the property of area of two similar triangle,

Ratio of area of both riangles = (Ratio of their corresponding sides)^2

i.e., $$\frac{\text{ar (smaller triangle)}}{\text{ar(larger triangle)}} = \left(\frac{2}{3}\right)^2$$

$\Rightarrow$ $$\frac{48}{\text{ar(larger triangle)}} = \frac{4}{9}$$

$\Rightarrow$ $$\text{ar (larger triangle)} = \frac{48 \times 9}{4} = 12 \times 9 = 108 \text{ cm}^2$$

Question 11:

In a Δ PQR, N is a point on PR, such that QN $\perp$ PR. If PN . NR = QN^2, then prove that $\angle PQR = 90°$.

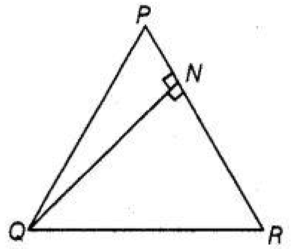
Solution:

Given ΔPQR, N is a point on PR, such that QN $\perp$ PR

and $PN \cdot NR = QN^2$

To prove $\angle PQR = 90°$

Proof We have, $PN \cdot NR = QN^2$



$\Rightarrow$ $PN \cdot NR = QN \cdot QN$

$\Rightarrow$ $\frac{PN}{QN} = \frac{QN}{NR}$ ...(i)

In ΔQNP and ΔRNQ, $\frac{PN}{QN} = \frac{QN}{NR}$

and $\angle PNQ = \angle RNQ$ [each equal to 90°]

$\therefore$ $\Delta QNP \sim \Delta RNQ$ [by SAS similarity criterion]

Then, ΔQNP and ΔRNQ are equiangulars.

i.e., $\angle PQN = \angle QRN$

$\angle RQN = \angle QPN$

On adding both sides, we get

$$\angle PQN + \angle RQN = \angle QRN + \angle QPN$$

$\Rightarrow$ $\angle PQR = \angle QRN + \angle QPN$ ...(ii)

We know that, sum of angles of a triangle = 180°

In ΔPQR, $\angle PQR + \angle QPR + \angle QRP = 180°$

$\Rightarrow$ $\angle PQR + \angle QPN + \angle QRN = 180°$ [$\because \angle QPR = \angle QPN$ and $\angle QRP = \angle QRN$]

$\Rightarrow$ $\angle PQR + \angle PQR = 180°$ [using Eq. (ii)]

$\Rightarrow$ $2\angle PQR = 180°$

$\Rightarrow$ $\angle PQR = \frac{180°}{2} = 90°$

$\therefore$ $\angle PQR = 90°$ **Hence proved.**

Question 12:

Areas of two similar triangles are 36 cm^2 and 100 cm^2. If the length of a side of the larger triangle is 20 cm. Find the length of the corresponding side of the smaller triangle.

Solution:

Given, area of smaller triangle = 36 cm^2 and area of larger triangle = 100 cm^2

Also, length of a side of the larger triangle = 20 cm

Let length of the corresponding side of the smaller triangle = x cm

By property of area of similar triangle,

$$\frac{\text{ar (larger triangle)}}{\text{ar (smaller triangle)}} = \frac{(\text{Side of larger triangle})^2}{\text{Side of smaller triangle}^2}$$

$\Rightarrow$ $$\frac{100}{36} = \frac{(20)^2}{x^2} \Rightarrow x^2 = \frac{(20)^2 \times 36}{100}$$

$\Rightarrow$ $$x^2 = \frac{400 \times 36}{100} = 144$$

$\therefore$ $$x = \sqrt{144} = 12 \text{ cm}$$

Hence, the length of corresponding side of the smaller triangle is 12 cm.

Question 13:

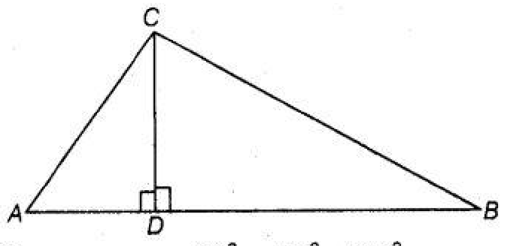
In given figure, if ∠ACB = ∠CDA, AC = 8 cm and AD = 3 cm, then find BD.

Solution:

Given, AC = 8 cm, AD = 3cm and ∠ACB = ∠CDA

From figure, ∠CDA = 90°

∠ACB = ∠CDA = 90°



In right angled ΔADC, $AC^2 = AD^2 + CD^2$

$\Rightarrow$ $(8)^2 = (3)^2 + (CD)^2$

$\Rightarrow$ $64 - 9 = CD^2$

$\Rightarrow$ $CD = \sqrt{55}$ cm

In ΔCDB and ΔADC, $\angle BDC = \angle ADC$ [each 90°]

$\angle DBC = \angle DCA$ [each equal to 90° – ∠ A]

$\therefore$ $\Delta CDB \sim \Delta ADC$

Then, $\frac{CD}{BD} = \frac{AD}{CD}$

$\Rightarrow$ $CD^2 = AD \times BD$

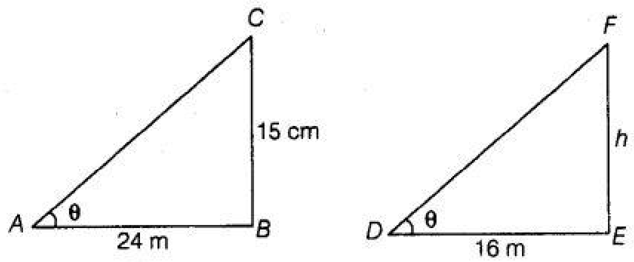
$\therefore$ $BD = \frac{CD^2}{AD} = \frac{(\sqrt{55})^2}{3} = \frac{55}{3}$ cm

Question 14:

A 15 high tower casts a shadow 24 Long at a certain time and at the same time, a telephone pole casts a shadow 16 long. Find the height of the telephone pole.

Solution:

Let BC = 15 m be the tower and its shadow AB is 24 m. At that time ∠CAB = 8, Again, let EF = h be a telephone pole and its shadow DE = 16 m. At the same time ∠EDF = 8 Here, ΔASC and ΔDEF both are right angled triangles.



In ΔABC and ΔDEF, $\angle CAB = \angle EDF = \theta$

$\angle B = \angle E$ [each 90°]

$\therefore$ $\Delta ABC \sim \Delta DEF$ [by AAA similarity criterion]

Then, $\frac{AB}{DE} = \frac{BC}{EF}$

$\Rightarrow$ $\frac{24}{16} = \frac{15}{h}$

$\therefore$ $h = \frac{15 \times 16}{24} = 10$

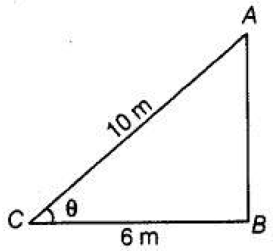
Hence, the height of the telephone pole is 10 m.

Question 15:

Foot of a 10 m long ladder leaning against a vertical wall is 6 m away from the base of the wall. Find the height of the point on the wall where the top of the ladder reaches.

Solution:

Let AB be a vertical wall and AC = 10 m is a ladder. The top of the ladder reaches to A and distance of ladder from the base of the wall BC is 6 m.



In right angled ΔABC, $AC^2 = AB^2 + BC^2$ [by Pythagoras theorem]

$\Rightarrow$ $(10)^2 = AB^2 + (6)^2$

$\Rightarrow$ $100 = AB^2 + 36$

$\Rightarrow$ $AB^2 = 100 - 36 = 64$

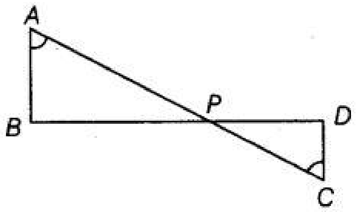
$\therefore$ $AB = \sqrt{64} = 8\,\text{cm}$

Hence, the height of the point on the wall where the top of the ladder reaches is 8 cm.

Exercise 6.4 Long Answer Type Questions

Question 1:

In given figure, if ∠A = ∠C, AB = 6 cm, BP = 15 cm, AP = 12 cm and CP = 4 cm, then find the lengths of PD and CD.



Solution:

Given, ∠A = ∠C, AS = 6cm, BP = 15cm, AP = 12 cm and CP = 4cm

In ΔAPB and ΔCPD, ∠A = ∠C

[given]

∠APS = ∠CPD [vertically opposite angles]

$\therefore$ $\Delta APD \sim \Delta CPD$ [by AAA similarity criterion]

$\Rightarrow$ $\frac{AP}{CP} = \frac{PB}{PD} = \frac{AB}{CD}$

$\Rightarrow$ $\frac{12}{4} = \frac{15}{PD} = \frac{6}{CD}$

On taking first two terms, we get

$\frac{12}{4} = \frac{15}{PD}$

$\Rightarrow$ $PD = \frac{15 \times 4}{12} = 5\,\text{cm}$

On taking first and last term, we get

$\frac{12}{4} = \frac{6}{CD}$

$\Rightarrow$ $CD = \frac{6 \times 4}{12} = 2\,\text{cm}$

Hence, length of PD = 5 cm and length of CD = 2 cm

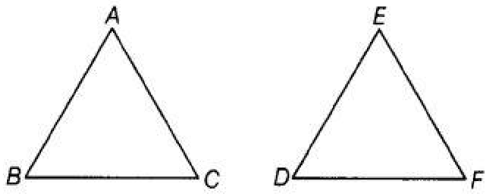
Question 2:

It is given that ΔABC ~ ΔEDF such that AB = 5 cm, AC = 7 cm, DF = 15 cm and DE = 12 cm. Find the lengths of the remaining sides of the triangles,

Solution:

Given, ΔABC ~ ΔEDF, so the corresponding sides of ΔASC and ΔEDF are in the same ratio.

i.e., $$\frac{AB}{ED} = \frac{AC}{EF} = \frac{BC}{DF} \quad \ldots(i)$$



Also, $$AB = 5\,cm,\ AC = 7\,cm$$
$$DF = 15\,cm \text{ and } DE = 12\,cm$$

On putting these values in Eq. (i), we get
$$\frac{5}{12} = \frac{7}{EF} = \frac{BC}{15}$$

On taking first and second terms, we get
$$\frac{5}{12} = \frac{7}{EF}$$

$$\Rightarrow \quad EF = \frac{7 \times 12}{5} = 16.8\,cm$$

On taking first and third terms, we get
$$\frac{5}{12} = \frac{BC}{15}$$

$$\Rightarrow \quad BC = \frac{5 \times 15}{12} = 625\,cm$$

Hence, lengths of the remaining sides of the triangles are EF = 16.8 cm and SC = 625 cm.

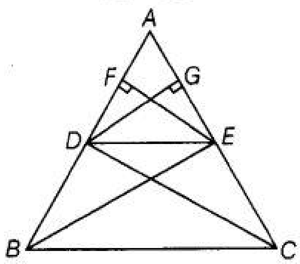
Question 3:

Prove that, if a line is drawn parallel to one side of a triangle to intersect the other two sides, then the two sides are divided in the same ratio.

Solution:

Let a ΔABC in which a line DE parallel to SC intersects AB at D and AC at E.To prove DE divides the two sides in the same ratio.

i.e., $$\frac{AD}{DB} = \frac{AE}{EC}$$



Construction Join *BE*, *CD* and draw $EF \perp AB$ and $DG \perp AC$.

Proof Here, $$\frac{ar\,(\Delta ADE)}{ar\,(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times DB \times EF} \quad \left[\because \text{area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}\right]$$

$$= \frac{AD}{DB} \quad \ldots(i)$$

similarly, $$\frac{ar\,(\Delta ADE)}{ar\,(\Delta DEC)} = \frac{\frac{1}{2} \times AE \times GD}{\frac{1}{2} \times EC \times GD} = \frac{AE}{EC} \quad \ldots(ii)$$

Now, since, Δ*BDE* and Δ*DEC* lie between the same parallel *DE* and *BC* and on the same base *DE*.

So, $$ar\,(\Delta BDE) = ar\,(\Delta DEC) \quad \ldots(iii)$$

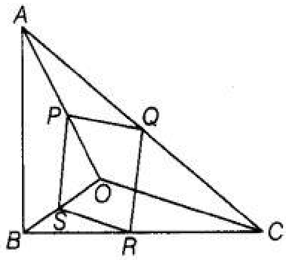
From Eqs. (i), (ii) and (iii),
$$\frac{AD}{DB} = \frac{AE}{EC}$$ **Hence proved.**

Question 4:

In the given figure, if PQRS is a parallelogram and AB || PS, then prove that 0C || SR.

Solution:

Given PQRS is a parallelogram, so PQ || SR and PS || QR. Also, AB || PS



To prove $OC \parallel SR$

Proof in ΔOPS and ΔOAB,

$$PS \parallel AB$$

$$\angle POS = \angle AOB \quad \text{[common angle]}$$

$$\angle OSP = \angle OBA \quad \text{[corresponding angles]}$$

$\therefore$ $$\Delta OPS \sim \Delta OAB \quad \text{[by AAA similarity criterion]}$$

Then, $$\frac{PS}{AB} = \frac{OS}{OB} \quad \ldots\text{(i)}$$

In ΔCQR and ΔCAB,

$$QR \parallel PS \parallel AB$$

$$\angle QCR = \angle ACB \quad \text{[common angle]}$$

$$\angle CRQ = \angle CBA \quad \text{[corresponding angles]}$$

$\therefore$ $$\Delta CQR \sim \Delta CAB$$

Then, $$\frac{QR}{AB} = \frac{CR}{CB}$$

$\Rightarrow$ $$\frac{PS}{AB} = \frac{CR}{CB} \quad \ldots\text{(ii)}$$

[since, PQRS is a parallelogram, so $PS \equiv QR$]

From Eqs. (i) and (ii),

$$\frac{OS}{OB} = \frac{CR}{CB} \text{ or } \frac{OB}{OS} = \frac{CB}{CR}$$

On subtracting from both sides, we get

$$\frac{OB}{OS} - 1 = \frac{CB}{CR} - 1$$

$\Rightarrow$ $$\frac{OB - OS}{OS} = \frac{CB - CR}{CR}$$

$\Rightarrow$ $$\frac{BS}{OS} = \frac{BR}{CR}$$

By converse of basic proportionality theorem,

$$SR \parallel OC$$

Hence proved.

Question 5:

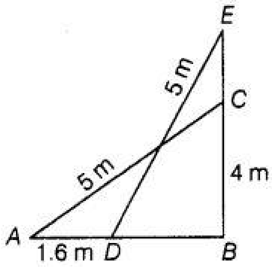
A 5m long ladder is placed leaning towards a vertical wall such that it reaches the wall at a point 4 m high. If the foot of the ladder is moved 1.6 m towards the wall, then find the distance by which the top of the ladder would slide upwards on the wall.

Solution:

Let AC be the ladder of length 5 m and BC = 4 m be the height of the wall, which ladder is placed. If the foot of the ladder is moved 1.6 m towards the wall i.e, AD = 1.6 m, then the ladder is slide upward i.e. ,CE = x m.

In right angled ΔABC,

$$AC^2 = AB^2 + BC^2 \quad \text{[by Pythagoras theorem]}$$
$\Rightarrow$ $(5)^2 = (AB)^2 + (4)^2$
$\Rightarrow$ $AB^2 = 25 - 16 = 9 \Rightarrow AB = 3\,m$
$\therefore$ $DB = AB - AD = 3 - 1.6 = 1.4\,m$



In right angled ΔEBD, $ED^2 = EB^2 + BD^2$ [by Pythagoras theorem]
$\Rightarrow$ $(5)^2 = (EB)^2 + (1.4)^2$ [$\because BD = 1.4\,m$]
$\Rightarrow$ $25 = (EB)^2 + 1.96$
$\Rightarrow$ $(EB)^2 = 25 - 1.96 = 23.04$
$\Rightarrow$ $EB = \sqrt{23.04} = 4.8$
Now, $EC = EB - BC = 4.8 - 4 = 0.8$

Hence, the top of the ladder would slide upwards on the wall at distance 0.8 m.

Question 6:

For going to a city B from city A there is a route via city C such that AC ⊥ CB, AC = 2x km and CB = 2(x+ 7) km. It is proposed to construct a 26 km highway which directly connects the two cities A and B. Find how much distance will be saved in reaching city B from city A after the construction of the highway.

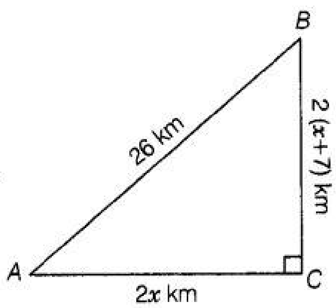
Solution:

Given, AC ⊥ CB, km,CB = 2(x + 7) km and AB = 26 km

On drawing the figure, we get the right angled Δ ACB right angled at C.

Now, In ΔACB, by Pythagoras theorem,

$AB^2 = AC^2 + BC^2$
$\Rightarrow$ $(26)^2 = (2x)^2 + \{2(x+7)\}^2$
$\Rightarrow$ $676 = 4x^2 + 4(x^2 + 49 + 14x)$
$\Rightarrow$ $676 = 4x^2 + 4x^2 + 196 + 56x$
$\Rightarrow$ $676 = 8x^2 + 56x + 196$
$\Rightarrow$ $8x^2 + 56x - 480 = 0$
On dividing by 8, we get $x^2 + 7x - 60 = 0$
$\Rightarrow$ $x^2 + 12x - 5x - 60 = 0$
$\Rightarrow$ $x(x+12) - 5(x+12) = 0$
$\Rightarrow$ $(x+12)(x-5) = 0$
$\therefore$ $x = -12,\ x = 5$



Since, distance cannot be negative.
$\therefore$ $x = 5$ [$\because x \neq -12$]
Now, $AC = 2x = 10\,km$
and $BC = 2(x+7) = 2(5+7) = 24\,km$
The distance covered to reach city B from city A via city C
$= AC + BC$
$= 10 + 24$
$= 34\,km$
Distance covered to reach city B from city A after the construction of the highway
$= BA = 26\,km$

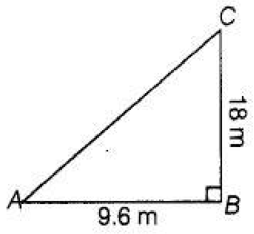
Hence, the required saved distance is 34 – 26 i.e., 8 km.

Question 7:

A flag pole 18 m high casts a shadow 9.6 m long. Find the distance of the top of the pole from the far end of the shadow.

Solution:

Let BC = 18 m be the flag pole and its shadow be AB = 9.6 m. The distance of the top of the pole, C from the far end i.e., A of the shadow is AC.



In right angled ΔABC, $AC^2 = AB^2 + BC^2$ [by Pythagoras theorem]

$\Rightarrow AC^2 = (9.6)^2 + (18)^2$

$AC^2 = 92.16 + 324$

$\Rightarrow AC^2 = 416.16$

$\therefore AC = \sqrt{416.16} = 20.4\text{m}$

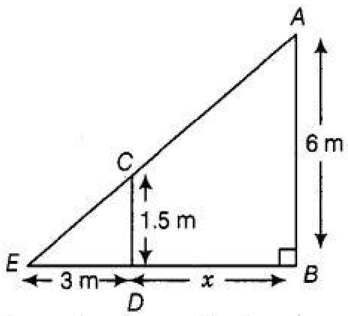
Hence, the required distance is 20.4 m.

Question 8:

A street light bulb is fixed on a pole 6 m above the level of the street. If a woman of height 1.5 m casts a shadow of 3 m, then find how far she is away from the base of the pole.

Solution:

Let A be the position of the street bulb fixed on a pole AB = 6 m and CD = 1.5 m be the height of a woman and her shadow be ED = 3 m. Let distance between pole and woman be x m.



Here, woman and pole both are standing vertically.

So, $CD \parallel AB$

In ΔCDE and ΔABE, $\angle E = \angle E$ [common angle]

$\angle ABE = \angle CDE$ [each equal to 90°]

$\therefore \Delta CDE \sim \Delta ABE$ [by AAA similarity criterion]

Then, $\dfrac{ED}{EB} = \dfrac{CD}{AB}$

$\Rightarrow \dfrac{3}{3+x} = \dfrac{1.5}{6}$

$\Rightarrow 3 \times 6 = 1.5(3 + x)$

$\Rightarrow 18 = 1.5 \times 3 + 1.5x$

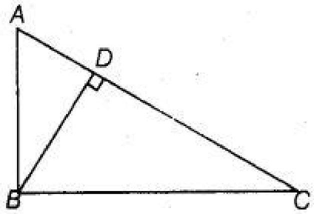
$\Rightarrow 1.5x = 18 - 4.5$

$\therefore x = \dfrac{13.5}{1.5} = 9\text{m}$

Hence, she is at the distance of 9 m from the base of the pole.

Question 9:

In given figure, ABC is a triangle right angled at B and BD ⊥ AC. If AD = 4 cm and CD = 5 cm, then find BD and AB.



Solution:

Given, ΔABC in which $\angle B = 90°$ and $BD \perp AC$

Also, $AD = 4\text{ cm}$ and $CD = 5\text{ cm}$

In ΔADB and ΔCDB, $\angle ADB = \angle CDB$ [each equal to 90°]

and $\angle BAD = \angle DBC$ [each equal to $90° - \angle C$]

$\therefore$ $\Delta DBA \sim \Delta DCB$ [by AAA similarity criterion]

Then, $\dfrac{DB}{DA} = \dfrac{DC}{DB}$

$\Rightarrow$ $DB^2 = DA \times DC$

$\Rightarrow$ $DB^2 = 4 \times 5$

$\Rightarrow$ $DB = 2\sqrt{5}\text{ cm}$

In right angled ΔBDC, $BC^2 = BD^2 + CD^2$ [by Pythagoras theorem]

$\Rightarrow$ $BC^2 = (2\sqrt{5})^2 + (5)^2$

$= 20 + 25 = 45$

$\Rightarrow$ $BC = \sqrt{45} = 3\sqrt{5}$

Again, $\Delta DBA \sim \Delta DCB$,

$\therefore$ $\dfrac{DB}{DC} = \dfrac{BA}{BC}$

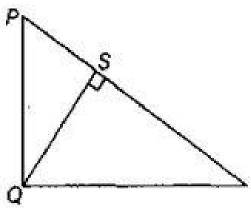
$\Rightarrow$ $\dfrac{2\sqrt{5}}{5} = \dfrac{BA}{3\sqrt{5}}$

$\therefore$ $BA = \dfrac{2\sqrt{5} \times 3\sqrt{5}}{5} = 6\text{ cm}$

Hence, $BD = 2\sqrt{5}$ cm and $AB = 6$ cm

Question 10:

In given figure PQR is a right triangle, right angled at Q and QS ⊥ PR. If PQ = 6 cm and PS = 4 cm, then find QS, RS and QR.



Solution:

Given, ΔPQR in which ∠Q = 90°, QS ⊥ PR and PQ = 6 cm, PS = 4 cm In ΔSQP and ΔSRQ,

$\angle PSQ = \angle RSQ$ [each equal to 90°]

$\angle SPQ = \angle SQR$ [each equal to $90° - \angle R$]

$\therefore$ $\Delta SQP \sim \Delta SRQ$

Then, $\dfrac{SQ}{PS} = \dfrac{SR}{SQ}$

$\Rightarrow$ $SQ^2 = PS \times SR$ ...(i)

In right angled ΔPSQ, $PQ^2 = PS^2 + QS^2$ [by Pythagoras theorem]

$\Rightarrow (6)^2 = (4)^2 + QS^2$

$\Rightarrow 36 = 16 + QS^2$

$\Rightarrow QS^2 = 36 - 16 = 20$

$\therefore QS = \sqrt{20} = 2\sqrt{5}$ cm

On putting the value of QS in Eq. (i), we get

$$(2\sqrt{5})^2 = 4 \times SR$$

$\Rightarrow SR = \frac{4 \times 5}{4} = 5$ cm

In right angled ΔQSR, $QR^2 = QS^2 + SR^2$

$\Rightarrow QR^2 = (2\sqrt{5})^2 + (5)^2$

$\Rightarrow QR^2 = 20 + 25$

$\therefore QR = \sqrt{45} = 3\sqrt{5}$ cm

Hence, $QS = 2\sqrt{5}$ cm, $RS = 5$ cm and $QR = 3\sqrt{5}$ cm

Question 11:

In ΔPQR, PD ⊥ QR such that D lies on QR, if PQ = a, PR = b, QD = c and DR = d, then prove that (a + b)(a -b) = (c + d) (c -d).

Solution:

Given In A PQR, PD 1 QR, PQ = a, PR = b,QD = c and DR =d

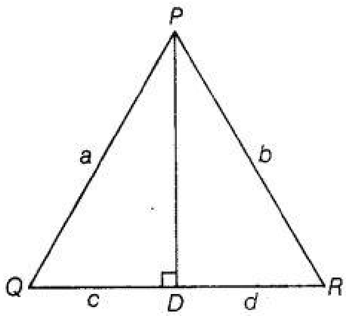
To prove (a + b) (a-b) = (c + d)(c-d)

Proof In right angled ΔPDQ,

$PQ^2 = PD^2 + QD^2$ [by Pythagoras theorem]

$\Rightarrow a^2 = PD^2 + c^2$

$\Rightarrow PD^2 = a^2 - c^2$ ...(i)



In right angled ΔPDR, $PR^2 = PD^2 + DR^2$ [by Pythagoras theorem]

$\Rightarrow b^2 = PD^2 + d^2$

$\Rightarrow PD^2 = b^2 - d^2$ ...(ii)

From Eqs. (i) and (ii),

$$a^2 - c^2 = b^2 - d^2$$

$\Rightarrow a^2 - b^2 = c^2 - d^2$

$\Rightarrow (a - b)(a + b) = (c - d)(c + d)$ **Hence proved.**

Question 12:

In a quadrilateral ΔBCD, ∠A+ ∠D = 90°. Prove that $AC^2 + BD^2 = AD^2 + BC^2$.

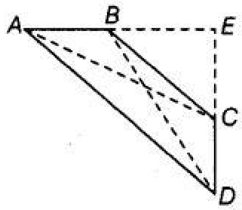
Solution:

Given Quadrilateral ΔBCD, in which ∠A+ ∠D = 90°

To prove $AC^2 + BD^2 = AD^2 + BC^2$

Construct Produce AB and CD to meet at E.

Also, join AC and BD.



Proof In ΔAED, $\angle A + \angle D = 90°$ [given]

$\therefore$ $\angle E = 180° - (\angle A + \angle D) = 90°$

[$\because$ sum of angles of a triangle = 180°]

Then, by Pythagoras theorem, $AD^2 = AE^2 + DE^2$

In ΔBEC, by Pythagoras theorem, $BC^2 = BE^2 + EF^2$

On adding both equations, we get

$$AD^2 + BC^2 = AE^2 + DE^2 + BE^2 + CE^2 \quad \text{...(i)}$$

In ΔAEC, by Pythagoras theorem,

$$AC^2 = AE^2 + CE^2$$

and in ΔBED, by Pythagoras theorem,

$$BD^2 = BE^2 + DE^2$$

On adding both equations, we get

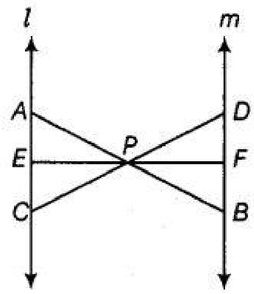
$$AC^2 + BD^2 = AE^2 + CE^2 + BE^2 + DE^2 \quad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$AC^2 + BD^2 = AD^2 + BC^2$$ **Hence proved.**

Question 13:

In given figure, $l \parallel m$ and line segments AB, CD and EF are concurrent at point P. Prove that $\frac{AE}{BF} = \frac{AC}{BD} = \frac{CE}{FD}$.



Solution:

Given $l \parallel m$ and line segments AB, CD and EF are concurrent at point P.

To prove $\frac{AE}{BF} = \frac{AC}{BD} = \frac{CE}{FD}$

Proof In ΔAPC and ΔBPD,

$\angle APC = \angle BPD$ [vertically opposite angles]

$\angle PAC = \angle PBD$ [alternate angles]

$\therefore$ $\Delta APC \sim \Delta BPD$ [by AAA similarity criterion]

Then, $\frac{AP}{PB} = \frac{AC}{BD} = \frac{PC}{PD}$ ...(i)

In ΔAPE and ΔBPF,

$\angle APE = \angle BPF$ [vertically opposite angles]

$\angle PAE = \angle PBF$ [alternate angles]

$\therefore$ $\Delta APE \sim \Delta BPF$ [by AAA similarity criterion]

Then, $\frac{AP}{PB} = \frac{AE}{BF} = \frac{PE}{PF}$ ...(ii)

In ΔPEC and ΔPFD,

$\angle EPC = \angle FPD$ [vertically opposite angles]

$\angle PCE = \angle PDF$ [alternate angles]

$\therefore$ $\Delta PEC \sim \Delta PFD$ [by AAA similarity criterion]

Then, $\frac{PE}{PF} = \frac{PC}{PD} = \frac{EC}{FD}$ ...(iii)

From Eqs. (i), (ii) and (iii),

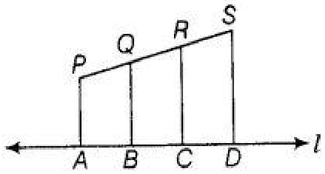
$$\frac{AP}{PB} = \frac{AC}{BD} = \frac{AE}{BF} = \frac{PE}{PF} = \frac{EC}{FD}$$

$\therefore$ $\frac{AE}{BF} = \frac{AC}{BD} = \frac{CE}{FD}$ **Hence proved.**

Question 14:

In figure, PA, QB, RC and SD are all perpendiculars to a line i, AB = 6 cm, BC = 9 cm, CD =

12 cm and SP = 36 cm. Find PQ, QR and RS.



Solution:

Given, AS = 6 cm, BC = 9 cm, CD = 12 cm and SP = 36 cm

Also, PA, QB, RC and SD are all perpendiculars to line l.

PA || QS|| SC || SD

By basic proportionality theorem,

$$PQ : QR : RS = AB : BC : CD$$
$$= 6 : 9 : 12$$

Let $PQ = 6x, QR = 9x$ and $RS = 12x$

Since, length of $PS = 36$ km

$\therefore$ $PQ + QR + RS = 36$

$\Rightarrow$ $6x + 9x + 12x = 36$

$\Rightarrow$ $27x = 36$

$\therefore$ $x = \frac{36}{27} = \frac{4}{3}$

Now, $PQ = 6x = 6 \times \frac{4}{3} = 8$ cm

$QR = 9x = 9 \times \frac{4}{3} = 12$ cm

and $RS = 12x = 12 \times \frac{4}{3} = 16$ cm

Question 15:

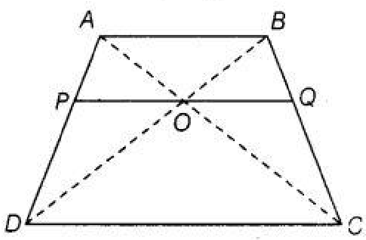
0 is the point of intersection of the diagonals AC and BD of a trapezium ABCD with AB || DC. Through 0, a line segment PQ is drawn parallel to AB meeting AD in P and BC in Q, prove that PO = QO.

Solution:

Given ABCD is a trapezium. Diagonals AC and BD are intersect at 0.

PQ||AB||DC.

To prove $PO = QO$



Proof In ΔABD and ΔPOD,

$PO \parallel AB$ [$\because PQ \parallel AB$]

$\angle D = \angle D$ [common angle]

$\angle ABD = \angle POD$ [corresponding angles]

$\therefore$ $\Delta ABD \sim \Delta POD$ [by AAA similarity criterion]

Then, $\frac{OP}{AB} = \frac{PD}{AD}$ ...(i)

In ΔABC and ΔOQC,

$OQ \parallel AB$ [$\because OQ \parallel AB$]

$\angle C = \angle C$ [common angle]

$\angle BAC = \angle QOC$ [corresponding angle]

$\therefore$ $\Delta ABC \sim \Delta OQC$ [by AAA similarity criterion]

Then, $\frac{OQ}{AB} = \frac{QC}{BC}$ ...(ii)

Now, in ΔADC, $OP \parallel DC$

$\therefore \quad \frac{AP}{PP} = \frac{OA}{OC}$ [by basic proportionality theorem]...(iii)

In ΔABC, $OQ \parallel AB$

$\therefore \quad \frac{BQ}{QC} = \frac{OA}{OC}$ [by basic proportionality theorem]...(iv)

From Eqs. (iii) and (iv),

$$\frac{AP}{PD} = \frac{BQ}{QC}$$

Adding 1 on both sides, we get

$$\frac{AP}{PD} + 1 = \frac{BQ}{QC} + 1$$

$$\Rightarrow \quad \frac{AP + PD}{PD} = \frac{BQ + QC}{QC}$$

$$\Rightarrow \quad \frac{AD}{PD} = \frac{BC}{QC}$$

$$\Rightarrow \quad \frac{PD}{AD} = \frac{QC}{BC}$$

$\Rightarrow \quad \frac{OP}{AB} = \frac{OQ}{BC}$ [from Eqs. (i) and (ii)]

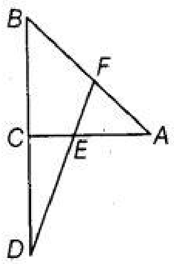
$\Rightarrow \quad \frac{OP}{AB} = \frac{OQ}{AB}$ [from Eq. (ii)]

$\Rightarrow \quad OP = OQ$ **Hence proved.**

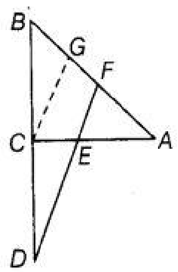
Question 16:

In figure, line segment DF intersects the side AC of a ΔABC at the point E such that E is the mid-point of CA and

∠AEF = ∠AFE. Prove that $\frac{BD}{CD} = \frac{BF}{CF}$.



Solution:



Given ΔABC, E is the mid-point of CA and $\angle AEF = \angle AFE$

To prove $\frac{BD}{CD} = \frac{BF}{CE}$

Construction Take a point G on AB such that $CG \parallel EF$.

Proof Since, E is the mid-point of CA.

$\therefore \quad CE = AE$ ...(i)

In ΔACG, $CG \parallel EF$ and E is mid-point of CA.

So, $\quad CE = GF$ ...(ii)

[by mid-point theorem]

Now, in ΔBCG and ΔBDF, $CG \parallel EF$

$\therefore \quad \frac{BC}{CD} = \frac{BG}{GF}$ [by basic proportionality theorem]

$$\Rightarrow \quad \frac{BC}{CD} = \frac{BF - GF}{GF} \Rightarrow \frac{BC}{CD} = \frac{BF}{GF} - 1$$

$\Rightarrow \quad \frac{BC}{CD} + 1 = \frac{BF}{CE}$ [from Eq. (ii)]

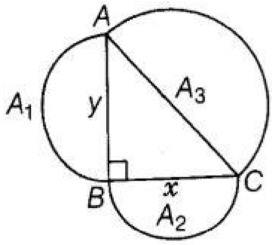
$\Rightarrow \quad \frac{BC + CD}{CD} = \frac{BF}{CE} \Rightarrow \frac{BD}{CD} = \frac{BF}{CE}$ **Hence proved.**

Question 17:

Prove that the area of the semi-circle drawn on the hypotenuse of a right angled triangle is equal to the sum of the areas of the semi-circles drawn on the other two sides of the triangle.

Solution:

Let ABC be a right triangle, right angled at B and AB = y, BC = x.
Three semi-circles are drawn on the sides AB, BC and AC, respectively with diameters AB, BC and AC, respectively.
Again, let area of circles with diameters AB, BC and AC are respectively A_1, A_2 and A_3.



To prove $A_3 = A_1 + A_2$

Proof In ΔABC, by Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad AC^2 = y^2 + x^2$$

$$\Rightarrow \quad AC = \sqrt{y^2 + x^2}$$

We know that, area of a semi-circle with radius, $r = \dfrac{\pi r^2}{2}$

$\therefore$ Area of semi-circle drawn on AC, $A_3 = \dfrac{\pi}{2}\left(\dfrac{AC}{2}\right)^2 = \dfrac{\pi}{2}\left(\dfrac{\sqrt{y^2 + x^2}}{2}\right)^2$

$$\Rightarrow \quad A_3 = \frac{\pi(y^2 + x^2)}{8} \quad \text{...(i)}$$

Now, area of semi-circle drawn on AB, $A_1 = \dfrac{\pi}{2}\left(\dfrac{AB}{2}\right)^2$

$$\Rightarrow \quad A_1 = \frac{\pi}{2}\left(\frac{y}{2}\right)^2 \Rightarrow A_1 = \frac{\pi y^2}{8} \quad \text{...(ii)}$$

and area of semi-circle drawn on BC, $A_2 = \dfrac{\pi}{2}\left(\dfrac{BC}{2}\right)^2 = \dfrac{\pi}{2}\left(\dfrac{x}{2}\right)^2$

$$\Rightarrow \quad A_2 = \frac{\pi x^2}{8}$$

On adding Eqs. (ii) and (iii), we get $A_1 + A_2 = \dfrac{\pi y^2}{8} + \dfrac{\pi x^2}{8}$

$$= \frac{\pi(y^2 + x^2)}{8} = A_3 \quad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad A_1 + A_2 = A_3$$

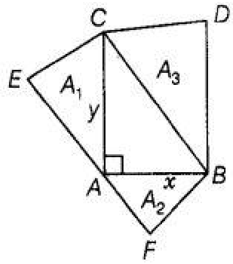
Hence proved.

Question 18:

Prove that the area of the equilateral triangle drawn on the hypotenuse of a right angled triangle is equal to the sum of the areas of the equilateral triangle drawn on the other two sides of the triangle.

Solution:

Let a right triangle BAC in which ∠A is right angle and AC = y, AB = x.
Three equilateral triangles ΔAEC, Δ AFB and ΔCBD are drawn on the three sides of ΔABC.
Again let area of triangles made on AC, AS and BC are A_1, A_2 and A_3, respectively.
To prove $A_3 = A_1 + A_2$



Proof In ΔCAB, by Pythagoras theorem,

$$BC^2 = AC^2 + AB^2$$

$$\Rightarrow \quad BC^2 = y^2 + x^2$$

$$\Rightarrow \quad BC = \sqrt{y^2 + x^2}$$

We know that, area of an equilateral triangle $= \dfrac{\sqrt{3}}{4}(\text{Side})^2$

$\therefore$ Area of equilateral ΔAEC, $A_1 = \dfrac{\sqrt{3}}{4}(AC)^2$

$$\Rightarrow \quad A_1 = \frac{\sqrt{3}}{4}y^2 \quad \text{....(i)}$$

and area of equilateral ΔAFB, $A_2 = \dfrac{\sqrt{3}}{4}(AB)^2 = \dfrac{\sqrt{3}}{4}\sqrt{(y^2 + x^2)}$

$$= \frac{\sqrt{3}}{4}(y^2 + x^2) = \frac{\sqrt{3}}{4}y^2 + \frac{\sqrt{3}}{4}x^2$$

$$= A_1 + A_2 \quad \text{[from Eqs. (i) and (ii)]}$$

Hence proved.