Unit 4 (Quadatric Euation)

Exercise 4.1 Multiple Choice Questions (MCQs)

Question 1:

Which of the following is a quadratic equation?

(a) $x^2 + 2x + 1 = (4 - x)^2 + 3$

(b) $-2x^2 = (5 - x)(2x - \frac{2}{5})$

(c) $(k + 1)x^2 + -\frac{3}{2}x = 7$, where k = -1

(d) $x^3 - x^2 = (x - 1)^3$

Solution:

(a) Given that, $x^2 + 2x + 1 = (4 - x)^2 + 3$

$\Rightarrow x^2 + 2x + 1 = 16 + x^2 - 8x + 3$

$\Rightarrow 10x - 18 = 0$

which is not of the form $ax^2 + bx + c$, $a \neq 0$. Thus, the equation is not a quadratic equation.

(b) Given that, $-2x^2 = (5 - x)\left(2x - \frac{2}{5}\right)$

$\Rightarrow -2x^2 = 10x - 2x^2 - 2 + \frac{2x}{5}$

$\Rightarrow 50x + 2x - 10 = 0$

$\Rightarrow 52x - 10 = 0$

which is also not a quadratic equation.

(c) Given that, $x^2 (k + 1) + \frac{3}{2}x = 7$

Given, $k = -1$

$\Rightarrow x^2 (-1 + 1) + \frac{3}{2}x = 7$

$\Rightarrow 3x - 14 = 0$

which is also not a quadratic equation.

(d) Given that, $x^3 - x^2 = (x - 1)^3$

$\Rightarrow x^3 - x^2 = x^3 - 3x^2(1) + 3x(1)^2 - (1)^3$

$[\because (a - b)^3 = a^3 - b^3 + 3ab^2 - 3a^2b]$

$\Rightarrow x^3 - x^2 = x^3 - 3x^2 + 3x - 1$

$\Rightarrow -x^2 + 3x^2 - 3x + 1 = 0$

$\Rightarrow 2x^2 - 3x + 1 = 0$

which represents a quadratic equation because it has the quadratic form $ax^2 + bx + c = 0$, $a \neq 0$.

Question 2:

Which of the following is not a quadratic equation?

(a) $2 (x -1)^2 = 4x^2 – 2x +1$

(b) $2x – x^2 = x^2 + 5$

(c) $(-\sqrt{2}X +\sqrt{3})^2 = 3x^2 – 5x$

(d) $(x^2 + 2x)^2 = x^4 + 3 + 4x^2$

Solution:

(d)

(a) Given that, $2(x-1)^2 = 4x^2 - 2x + 1$

$\Rightarrow 2(x^2 + 1 - 2x) = 4x^2 - 2x + 1$

$\Rightarrow 2x^2 + 2 - 4x = 4x^2 - 2x + 1$

$\Rightarrow 2x^2 + 2x - 1 = 0$

which represents a quadratic equation because it has the quadratic form $ax^2 + bx + c = 0, a \neq 0$.

(b) Given that, $2x - x^2 = x^2 + 5$

$\Rightarrow 2x^2 - 2x + 5 = 0$

which also represents a quadratic equation because it has the quadratic form $ax^2 + bx + c = 0, a \neq 0$.

(c) Given that, $(\sqrt{2} \cdot x + \sqrt{3})^2 = 3x^2 - 5x$

$\Rightarrow 2 \cdot x^2 + 3 + 2\sqrt{6} \cdot x = 3x^2 - 5x$

$\Rightarrow x^2 - (5 + 2\sqrt{6})\, x - 3 = 0$

which also represents a quadratic equation because it has the quadratic form $ax^2 + bx + c = 0, \; a \neq 0$.

(d) Given that, $(x^2 + 2x)^2 = x^4 + 3 + 4x^2$

$\Rightarrow x^4 + 4x^2 + 4x^3 = x^4 + 3 + 4x^2$

$\Rightarrow 4x^3 - 3 = 0$

which is not of the form $ax^2 + bx + c, a \neq 0$. Thus, the equation is not quadratic. This is a cubic equation.

Question 3:

Which of the following equations has 2 as a root?

(a) $x^2-4x + 5=0$

(b) $x^2 + 3x-12 =0$

(c) $2x^2 – 7x + 6 = 0$

(d) $3x^2 – 6x – 2 = 0$

Solution:

(c)

(a) Substituting $x = 2$ in $x^2 - 4x + 5$, we get $(2)^2 - 4(2) + 5$

$$= 4 - 8 + 5 = 1 \neq 0.$$

So, $x = 2$ is not a root of $x^2 - 4x + 5 = 0$.

(b) Substituting $x = 2$ in $x^2 + 3x - 12$, we get

$$(2)^2 + 3(2) - 12$$

$$= 4 + 6 - 12 = -2 \neq 0$$

So, $x = 2$ is not a root of $x^2 + 3x - 12 = 0$.

(c) Substituting $x = 2$ in $2x^2 - 7x + 6$, we get

$$2(2)^2 - 7(2) + 6 = 2(4) - 14 + 6$$

$$= 8 - 14 + 6 = 14 - 14 = 0$$

So, $x = 2$ is root of the equation $2x^2 - 7x + 6 = 0$.

(d) Substituting $x = 2$ in $3x^2 - 6x - 2$, we get

$$3(2)^2 - 6(2) - 2$$

$$= 12 - 12 - 2 = -2 \neq 0$$

So, $x = 2$ is not a root of $3x^2 - 6x - 2 = 0$.

Question 4:

If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of k is

(a) 2 (b) -2 (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

Solution:

(a) Since, $\frac{1}{2}$ is a root of the quadratic equation $x^2 + kx - \frac{5}{4} = 0$.

Then, $\left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0$

$\Rightarrow \frac{1}{4} + \frac{k}{2} - \frac{5}{4} = 0 \Rightarrow \frac{1 + 2k - 5}{4} = 0$

$\Rightarrow 2k - 4 = 0$

$\Rightarrow 2k = 4 \Rightarrow k = 2$

Question 5:

Which of the following equations has the sum of its roots as 3?

(a) $2x^2 - 3x + 6 = 0$

(b) $-x^2 + 3x - 3 = 0$

(c) $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + 1 = 0$

(d) $3x^2 - 3x + 3 = 0$

Solution:

(b)

(a) Given that, $2x^2 - 3x + 6 = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 2, b = -3 \text{ and } c = 6$$

$\therefore$ Sum of the roots $= \frac{-b}{a} = \frac{-(-3)}{2} = \frac{3}{2}$

So, sum of the roots of the quadratic equation $2x^2 - 3x + 6 = 0$ is not 3, so it is not the answer.

(b) Given that, $-x^2 + 3x - 3 = 0$

On compare with $ax^2 + bx + c = 0$, we get

$$a = -1, b = 3 \text{ and } c = -3$$

$\therefore$ Sum of the roots $= \frac{-b}{a} = \frac{-(3)}{-1} = 3$

So, sum of the roots of the quadratic equation $-x^2 + 3x - 3 = 0$ is 3, so it is the answer.

(c) Given that, $\sqrt{2}\, x^2 - \frac{3}{\sqrt{2}} x + 1 = 0$

$\Rightarrow$ $2x^2 - 3x + \sqrt{2} = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 2, b = -3 \text{ and } c = \sqrt{2}$$

$\therefore$ Sum of the roots $= \frac{-b}{a} = \frac{-(-3)}{2} = \frac{3}{2}$

So, sum of the roots of the quadratic equation $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + 1 = 0$ is not 3, so it is not the answer.

(d) Given that, $3x^2 - 3x + 3 = 0$

$\Rightarrow$ $x^2 - x + 1 = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = -1 \text{ and } c = 1$$

$\therefore$ Sum of the roots $= \frac{-b}{a} = \frac{-(-1)}{1} = 1$

So, sum of the roots of the quadratic equation $3x^2 - 3x + 3 = 0$ is not 3, so it is not the answer.

Question 6:

Value(s) of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is/are

(a) 0 (b) 4 (c) 8 (d) 0, 8

Solution:

(d)

Given equation is $2x^2 - kx + k- 0$

On comparing with $ax^2 + bx + c = 0$, we get

a = 2, b= – k and c = k

For equal roots, the discriminant must be zero.

i.e., $D = b^2 - 4ac = 0$

$\Rightarrow$ $(-k)^2 - 4(2)k = 0$

$\Rightarrow$ $k^2 - 8k = 0$

$\Rightarrow$ $k(k - 8) = 0$

$\therefore$ $k = 0, 8$

Hence, the required values of k are 0 and 8.

Question 7:

Which constant must be added and subtracted to solve the quadratic equation $9x^2 + \frac{3}{4}x - \sqrt{2} = 0$ by the method of completing the square?

(a) $\frac{1}{8}$ (b) $\frac{1}{64}$ (c) $\frac{1}{4}$ (d) $\frac{9}{64}$

Solution:

(b) Given equation is $9x^2 + \frac{3}{4}x - \sqrt{2} = 0$.

$$(3x)^2 + \frac{1}{4}(3x) - \sqrt{2} = 0$$

On putting $3x = y$, we have $y^2 + \frac{1}{4}y - \sqrt{2} = 0$

$$y^2 + \frac{1}{4}y + \left(\frac{1}{8}\right)^2 - \left(\frac{1}{8}\right)^2 - \sqrt{2} = 0$$

$$\left(y + \frac{1}{8}\right)^2 = \frac{1}{64} + \sqrt{2}$$

$$\left(y + \frac{1}{8}\right)^2 = \frac{1 + 64 \cdot \sqrt{2}}{64}$$

Thus, $\frac{1}{64}$ must be added and subtracted to solve the given equation.

Question 8:

The quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has

(a) two distinct real roots

(b) two equal real roots

(c) no real roots

(d) more than 2 real roots

Solution:

(c) Given equation is $2x^2 - \sqrt{5}x + 1 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 2, b = -\sqrt{5} \text{ and } c = 1$$

$\therefore$ Discriminant, $D = b^2 - 4ac = (-\sqrt{5})^2 - 4 \times (2) \times (1) = 5 - 8$

$$= -3 < 0$$

Since, discriminant is negative, therefore quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has no real roots *i.e.*, imaginary roots.

Question 9:

Which of the following equations has two distinct real roots?

(a)$2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$

(b) $x^2 + x - 5 = 0$

(c) $x^2 + 3x + 2\sqrt{2} = 0$

(d)$5x^2 - 3x + 1 = 0$

Solution:

(b) The given equation is $x^2 + x - 5 = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = 1 \text{ and } c = -5$$

The discriminant of $x^2 + x - 5 = 0$ is

$$D = b^2 - 4ac = (1)^2 - 4(1)(-5)$$

$$= 1 + 20 = 21$$

$\Rightarrow$ $b^2 - 4ac > 0$

So, $x^2 + x - 5 = 0$ has two distinct real roots.

(a) Given equation is, $2x^2 - 3\sqrt{2}x + 9/4 = 0$.

On comparing with $ax^2 + bx + c = 0$

$$a = 2, b = -3\sqrt{2} \text{ and } c = 9/4$$

Now, $D = b^2 - 4ac = (-3\sqrt{2})^2 - 4(2)(9/4) = 18 - 18 = 0$

Thus, the equation has real and equal roots.

(c) Given equation is $x^2 + 3x + 2\sqrt{2} = 0$

On comparing with $ax^2 + bx + c = 0$

$$a = 1, b = 3 \text{ and } c = 2\sqrt{2}$$

Now, $D = b^2 - 4ac = (3)^2 - 4(1)(2\sqrt{2}) = 9 - 8\sqrt{2} < 0$

$\therefore$ Roots of the equation are not real.

(d) Given equation is, $5x^2 - 3x + 1 = 0$

On comparing with $ax^2 + bx + c = 0$

$$a = 5, b = -3, c = 1$$

Now, $D = b^2 - 4ac = (-3)^2 - 4(5)(1) = 9 - 20 < 0$

Hence, roots of the equation are not real.

Question 10:

Which of the following equations has no real roots?

(a) $x^2 - 4x + 3\sqrt{2} = 0$

(b) $x^2 + 4x - 3\sqrt{2} = 0$

(c) $x^2 - 4x - 3\sqrt{2} = 0$

(d) $3x^2 + 4\sqrt{3}x + 4 = 0$

Solution:

(a)

(a) The given equation is $x^2 - 4x + 3\sqrt{2} = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = -4 \text{ and } c = 3\sqrt{2}$$

The discriminant of $x^2 - 4x + 3\sqrt{2} = 0$ is

$$D = b^2 - 4ac$$

$$= (-4)^2 - 4(1)(3\sqrt{2}) = 16 - 12\sqrt{2} = 16 - 12 \times (1.41)$$

$$= 16 - 16.92 = -0.92$$

$$\Rightarrow \quad b^2 - 4ac < 0$$

(b) The given equation is $x^2 + 4x - 3\sqrt{2} = 0$

On comparing the equation with $ax^2 + bx + c = 0$, we get

$$a = 1, b = 4 \text{ and } c = -3\sqrt{2}$$

Then, $D = b^2 - 4ac = (-4)^2 - 4(1)(-3\sqrt{2})$

$$= 16 + 12\sqrt{2} > 0$$

Hence, the equation has real roots.

(c) Given equation is $x^2 - 4x - 3\sqrt{2} = 0$

On comparing the equation with $ax^2 + bx + c = 0$, we get

$$a = 1, b = -4 \text{ and } c = -3\sqrt{2}$$

Then, $D = b^2 - 4ac = (-4)^2 - 4(1)(-3\sqrt{2})$

$$= 16 + 12\sqrt{2} > 0$$

Hence, the equation has real roots.

(d) Given equation is $3x^2 + 4\sqrt{3}x + 4 = 0$.

On comparing the equation with $ax^2 + bx + c = 0$, we get

$$a = 3, b = 4\sqrt{3} \text{ and } c = 4$$

Then, $D = b^2 - 4ac = (4\sqrt{3})^2 - 4(3)(4) = 48 - 48 = 0$

Hence, the equation has real roots.

Hence, $x^2 - 4x + 3\sqrt{2} = 0$ has no real roots.

Question 11:

$(x^2 + 1)^2 - x^2 = 0$ has

(a) four real roots

(b) two real roots

(c) no real roots

(d) one real root

Solution:

(c) Given equation is $(x^2+1)^2 - x^2 = 0$

$\Rightarrow \quad x^4 + 1 + 2x^2 - x^2 = 0 \quad [\because (a+b)^2 = a^2 + b^2 + 2ab]$

$\Rightarrow \quad x^4 + x^2 + 1 = 0$

Let $\quad x^2 = y$

$\therefore \quad (x^2)^2 + x^2 + 1 = 0$

$y^2 + y + 1 = 0$

On comparing with $ay^2 + by + c = 0$, we get

$a = 1, b = 1$ and $c = 1$

Discriminant, $D = b^2 - 4ac$

$= (1)^2 - 4(1)(1)$

$= 1 - 4 = -3$

Since, $\quad D < 0$

$\therefore \; y^2 + y + 1 = 0$ *i.e.*, $x^4 + x^2 + 1 = 0$ or $(x^2+1)^2 - x^2 = 0$ has no real roots.

Exercise 4.2 Very Short Answer Type Questions

Question 1:

State whether the following quadratic equations have two distinct real roots. Justify your answer.

(i) $x^2 - 3x + 4 = 0$

(ii) $2x^2 + x - 1 = 0$

(iii) $2x^2 - 6x + \frac{9}{2} = 0$

(iv) $3x^2 - 4x + 1 = 0$

(v) $(x+4)^2 - 8x = 0$

(vi) $(x - \sqrt{2})^2 - \sqrt{2}(x+1) = 0$

(vii) $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + \frac{1}{\sqrt{2}} = 0$

(viii) $x(1-x) - 2 = 0$

(ix) $(x-1)(x+2) + 2 = 0$

(x) $(x+1)(x-2) + x = 0$

Solution:

(i) Given equation is $x^2 - 3x + 4 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = -3 \text{ and } c = 4$$

$\therefore$ Discriminant, $D = b^2 - 4ac = (-3)^2 - 4(1)(4)$

$$= 9 - 16 = -7 < 0 \quad i.e., D < 0$$

Hence, the equation $x^2 - 3x + 4 = 0$ has no real roots .

(ii) Given equation is, $2x^2 + x - 1 = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 2, b = 1 \text{ and } c = -1$$

$\therefore$ Discriminant, $D = b^2 - 4ac = (1)^2 - 4(2)(-1)$

$$= 1 + 8 = 9 > 0 \quad i.e., D < 0$$

Hence, the equation $2x^2 + x - 1 = 0$ has two distinct real roots .

(iii) Given equation is $2x^2 - 6x + \frac{9}{2} = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 2, b = -6 \text{ and } c = \frac{9}{2}$$

$\therefore$ Discriminant, $D = b^2 - 4ac$

$$= (-6)^2 - 4(2)\left(\frac{9}{2}\right) = 36 - 36 = 0 \quad i.e., D = 0$$

Hence, the equation $2x^2 - 6x + \frac{9}{2} = 0$ has equal and real roots.

(iv) Given equation is $3x^2 - 4x + 1 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 3, b = -4 \text{ and } c = 1$$

$\therefore$ Discriminant, $D = b^2 - 4ac = (-4)^2 - 4(3)(1)$

$$= 16 - 12 = 4 > 0 \quad i.e., D > 0$$

Hence, the equation $3x^2 - 4x + 1 = 0$ has two distinct real roots.

(v) Given equation is $(x + 4)^2 - 8x = 0$.

$\Rightarrow$ $x^2 + 16 + 8x - 8x = 0$ $\quad [\because (a + b)^2 = a^2 + 2ab + b^2]$

$\Rightarrow$ $x^2 + 16 = 0$

$\Rightarrow$ $x^2 + 0 \cdot x + 16 = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = 0 \text{ and } c = 16$$

$\therefore$ Discriminant, $D = b^2 - 4ac = (0)^2 - 4(1)(16) = -64 < 0 \quad i.e., D < 0$

Hence, the equation $(x + 4)^2 - 8x = 0$ has imaginary roots, *i.e.*, no real roots.

(vi) Given equation is $(x-\sqrt{2})^2-\sqrt{2}(x+1)=0.$

$\Rightarrow \quad x^2+(\sqrt{2})^2-2x\sqrt{2}-\sqrt{2}x-\sqrt{2}=0 \quad [\because (a-b)^2=a^2-2ab+b^2]$

$\Rightarrow \quad x^2+2-2\sqrt{2}\,x-\sqrt{2}x-\sqrt{2}=0$

$\Rightarrow \quad x^2-3\sqrt{2}x+(2-\sqrt{2})=0$

On comparing with $ax^2+bx+c=0$, we get

$$a=1, b=-3\sqrt{2} \text{ and } c=2-\sqrt{2}$$

$\therefore$ Discriminant, $D=b^2-4ac$

$$=(-3\sqrt{2})^2-4(1)(2-\sqrt{2})=9\times 2-8+4\sqrt{2}$$

$$=18-8+4\sqrt{2}=10+4\sqrt{2}>0 \text{ i.e., } D>0$$

Hence, the equation $(x-\sqrt{2})^2-\sqrt{2}(x+1)=0$ has two distinct real roots.

(vii) Given, equation is $\sqrt{2}x^2-\frac{3}{\sqrt{2}}x+\frac{1}{\sqrt{2}}=0.$

On comparing with $ax^2+bx+c=0$, we get

$$a=\sqrt{2}, b=-\frac{3}{\sqrt{2}} \text{ and } c=\frac{1}{\sqrt{2}}$$

$\therefore$ Discriminant, $D=b^2-4ac$

$$=\left(-\frac{3}{\sqrt{2}}\right)^2-4\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)$$

$$=\frac{9}{2}-4=\frac{9-8}{2}=\frac{1}{2}>0 \text{ i.e., } D>0$$

Hence, the equation $\sqrt{2}x^2-\frac{3}{\sqrt{2}}x+\frac{1}{\sqrt{2}}=0$ has two distinct real roots.

(viii) Given equation is $x(1-x)-2=0.$

$\Rightarrow \quad x-x^2-2=0$

$\Rightarrow \quad x^2-x+2=0$

On comparing with $ax^2+bx+c=0$, we get

$$a=1, b=-1 \text{ and } c=2$$

$\therefore$ Discriminant, $D=b^2-4ac$

$$=(-1)^2-4(1)(2)=1-8=-7<0 \text{ i.e., } D<0$$

Hence, the equation $x(1-x)-2=0$ has imaginary roots *i.e.*, no real roots.

(ix) Given equation is

$$(x-1)(x+2)+2=0$$

$\Rightarrow \quad x^2+x-2+2=0$

$\Rightarrow \quad x^2+x+0=0$

On comparing the equation with $ax^2+bx+c-0$, We have

$$a=1, b=1 \text{ and } c=0$$

$\therefore$ Discriminant, $D=b^2-4ac$

$$=1-4(1)(0)=1>0 \text{ i.e., } D>0$$

Hence, equation has two distinct real roots.

(x) Given equation is $(x+1)(x-2)+x=0$

$\Rightarrow \quad x^2+x-2x-2+x=0$

$\Rightarrow \quad x^2-2=0$

$\Rightarrow \quad x^2+0\cdot x-2=0$

On comparing with $ax^2+bx+c=0$, we get

$$a=1, b=0 \text{ and } c=-2$$

$\therefore$ Discriminant, $D=b^2-4ac=(0)^2-4(1)(-2)=0+8=8>0$

Hence, the equation $(x+1)(x-2)+x=0$ has two distinct real roots.

Question 2:

Write whether the following statements are true or false. Justify your answers.

(i) Every quadratic equation has exactly one root.

(ii) Every quadratic equation has atleast one real root.

(iii) Every quadratic equation has atleast two roots.

(iv) Every quadratic equation has atmost two roots.

(v) If the coefficient of x^2 and the constant term of a quadratic equation have opposite signs, then the quadratic equation has real roots.

(vi) If the coefficient of x^2 and the constant term have the same sign and if the coefficient of x term is zero, then the quadratic equation has no real roots.

Solution:

(i) False, since a quadratic equation has two and only two roots.

(ii) False, for example $x^2 + 4 = O$ has no real root.

(iii) False, since a quadratic equation has two and only two roots.

(vi) True, because every quadratic polynomial has atmost two roots.

(v) True, since in this case discriminant is always positive, so it has always real roots, e., ac < 0 and so, $b^2 - 4ac > 0$.

(vi) True, since in this case discriminant is always negative, so it has no real roots e., if b = 0, then $b^2 - 4ac \Rightarrow - 4ac < 0$and ac>0.

Question 3:

A quadratic equation with integral coefficient has integral roots. Justify your answer.

Solution:

No, consider the quadratic equation $2x^2 + x-6=0$ with integral coefficient. The roots of the given quadratic equation are -2 and $\frac{3}{2}$ which are not integers.

Question 4:

Does there exist a quadratic equation whose coefficients are rational but both of its roots are irrational? Justify your answer.

Solution:

Yes, consider the quadratic equation $2x^2 + x - 4 = O$with rational coefficient. The roots ofthe given quadratic equation are $\frac{-1+\sqrt{33}}{4}$ and $\frac{-1-\sqrt{33}}{4}$ are irrational.

Question 5:

Does there exist a quadratic equation whose coefficient are all distinct irrationals but both the roots are rationals? why?

Solution:

Yes, consider the quadratic equation with all distinct irrationals coefficients i.e., $\sqrt{3}x^2 - 7\sqrt{3}x + 12\sqrt{3} = 0$. The roots of this quadratic equation are 3 and 4, which are rationals.

Question 6:

Is 0.2 a root of the equation $x^2 - 0.4 = 0$? Justify your answer.

Solution:

No, since 0.2 does not satisfy the quadratic equation i.e., $(0.2)^2 - 0.4 = 0.04 - 0.4 \neq 0$.

Question 7:

If b = 0, c < 0, is it true that the roots of $x^2 + bx + c = 0$ are numerically equal and opposite in sign? Justify your answer.

Solution:

Given that, b = 0andc < 0and quadratic equation,

$$x^2 + bx + c = 0 \quad \text{...(i)}$$

Put $b = 0$ in Eq. (i), we get

$$x^2 + 0 + c = 0$$

$$\Rightarrow \quad x^2 = -c \quad \begin{bmatrix}\text{here } c > 0 \\ \therefore -c > 0\end{bmatrix}$$

$$\therefore \quad x = \pm\sqrt{-c}$$

So, the roots of x^2 + bx+c = O are numerically equal and opposite in sign.

Exercise 4.3 Short Answer Type Questions

Question 1:

Find the roots of the quadratic equations by using the quadratic formula irt each of the following

Solution:

(i) Given equation is $2x^2 - 3x - 5 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 2, b = -3 \text{ and } c = -5$$

By quadratic formula,
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-5)}}{2(2)} = \frac{3 \pm \sqrt{9 + 40}}{4}$$

$$= \frac{3 \pm \sqrt{49}}{4} = \frac{3 \pm 7}{4} = \frac{10}{4}, \frac{-4}{4} = \frac{5}{2}, -1$$

So, $\frac{5}{2}$ and -1 are the roots of the given equation.

(ii) Given equation is $5x^2 + 13x + 8 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 5, b = 13 \text{ and } c = 8$$

By quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-(13) \pm \sqrt{(13)^2 - 4(5)(8)}}{2(5)}$$

$$= \frac{-13 \pm \sqrt{169 - 160}}{10} = \frac{-13 \pm \sqrt{9}}{10}$$

$$= \frac{-13 \pm 3}{10} = -\frac{10}{10}, -\frac{16}{10} = -1, -\frac{8}{5}$$

(iii) Given equation is $-3x^2 + 5x + 12 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = -3, b = 5 \text{ and } c = 12$$

By quadratic formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-(5) \pm \sqrt{(5)^2 - 4(-3)(12)}}{2(-3)}$$

$$= \frac{-5 \pm \sqrt{25 + 144}}{-6} = \frac{-5 \pm \sqrt{169}}{-6}$$

$$= \frac{-5 \pm 13}{-6} = \frac{8}{-6}, \frac{-18}{-6} = -\frac{4}{3}, 3$$

So, $-\dfrac{4}{3}$ and 3 are two roots of the given equation.

(iv) Given equation is $-x^2 + 7x - 10 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = -1, b = 7 \text{ and } c = -10$$

By quadratic formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-(7) \pm \sqrt{(7)^2 - 4(-1)(-10)}}{2(-1)} = \frac{-7 \pm \sqrt{49 - 40}}{-2}$$

$$= \frac{-7 \pm \sqrt{9}}{-2} = \frac{-7 \pm 3}{-2} = \frac{-4}{-2}, \frac{-10}{-2} = 2, 5$$

So, 2 and 5 are two roots of the given equation.

(v) Given equation is $x^2 + 2\sqrt{2}\,x - 6 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = 2\sqrt{2} \text{ and } c = -6$$

By quadratic formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-(2\sqrt{2}) \pm \sqrt{(2\sqrt{2})^2 - 4(1)(-6)}}{2(1)} = \frac{-2\sqrt{2} \pm \sqrt{8 + 24}}{2}$$

$$= \frac{-2\sqrt{2} \pm \sqrt{32}}{2} = \frac{-2\sqrt{2} \pm 4\sqrt{2}}{2}$$

$$= \frac{-2\sqrt{2} + 4\sqrt{2}}{2}, \frac{-2\sqrt{2} - 4\sqrt{2}}{2} = \sqrt{2}, -3\sqrt{2}$$

So, $\sqrt{2}$ and $-3\sqrt{2}$ are the roots of the given equation.

(vi) Given equation is $x^2 - 3\sqrt{5}x + 10 = 0$.

On comparing with $ax^2 + bx + c = 0$, we have

$$a = 1, b = -3\sqrt{5} \text{ and } c = 10$$

By quadratic formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-(-3\sqrt{5}) \pm \sqrt{(-3\sqrt{5})^2 - 4(1)(10)}}{2(1)}$$

$$= \frac{3\sqrt{5} \pm \sqrt{45 - 40}}{2} = \frac{3\sqrt{5} \pm \sqrt{5}}{2}$$

$$= \frac{3\sqrt{5} + \sqrt{5}}{2}, \frac{3\sqrt{5} - \sqrt{5}}{2} = 2\sqrt{5}, \sqrt{5}$$

So, $2\sqrt{5}$ and $\sqrt{5}$ are the roots of the given equation.

(vii) Given equation is $\frac{1}{2}x^2 - \sqrt{11}x + 1 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = \frac{1}{2}, b = -\sqrt{11} \text{ and } c = 1$$

$\therefore$ By quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-(-\sqrt{11}) \pm \sqrt{(-\sqrt{11})^2 - 4 \times \frac{1}{2} \times 1}}{2\left(\frac{1}{2}\right)}$$

$$= \frac{\sqrt{11} \pm \sqrt{11-2}}{1} = \sqrt{11} \pm \sqrt{9}$$

$$= \sqrt{11} \pm 3 = 3 + \sqrt{11}, \sqrt{11} - 3$$

So, $3 + \sqrt{11}$ and $\sqrt{11} - 3$ are the roots of the given equation.

Question 2:

Find the roots of the following quadratic equations by the factorisation method.

(i) $2x^2 + \frac{5}{3}x - 2 = 0$ (ii) $\frac{2}{5}x^2 - x - \frac{3}{5} = 0$

(iii) $3\sqrt{2}x^2 - 5x - \sqrt{2} = 0$ (iv) $3x^2 + 5\sqrt{5}x - 10 = 0$

(v) $21x^2 - 2x + \frac{1}{21} = 0$

Solution:

(i) Given equation is $2x^2 + \frac{5}{3}x - 2 = 0$

On multiplying by 3 on both sides, we get

$$6x^2 + 5x - 6 = 0$$

$\Rightarrow$ $6x^2 + (9x - 4x) - 6 = 0$ [by splitting the middle term]

$\Rightarrow$ $6x^2 + 9x - 4x - 6 = 0$

$\Rightarrow$ $3x(2x + 3) - 2(2x + 3) = 0$

$\Rightarrow$ $(2x + 3)(3x - 2) = 0$

Now, $2x + 3 = 0$

$\Rightarrow$ $x = -\frac{3}{2}$

and $3x - 2 = 0$

$\Rightarrow$ $x = \frac{2}{3}$

Hence, the roots of the equation $6x^2 + 5x - 6 = 0$ are $\frac{-3}{2}$ and $\frac{2}{3}$.

(ii) Given equation is $\frac{2}{5}x^2 - x - \frac{3}{5} = 0$.

On multiplying by 5 on both sides, we get

$\Rightarrow \quad 2x^2 - 5x - 3 = 0$

$2x^2 - (6x - x) - 3 = 0$ [by splitting the middle term]

$\Rightarrow \quad 2x^2 - 6x + x - 3 = 0$

$\Rightarrow \quad 2x(x - 3) + 1(x - 3) = 0$

$\Rightarrow \quad (x - 3)(2x + 1) = 0$

Now, $x - 3 = 0 \Rightarrow x = 3$

and $2x + 1 = 0$

$\Rightarrow \quad x = -\frac{1}{2}$

Hence, the roots of the equation $2x^2 - 5x - 3 = 0$ are $-\frac{1}{2}$ and 3.

(iii) Given equation is $3\sqrt{2}\,x^2 - 5x - \sqrt{2} = 0$.

$3\sqrt{2}\,x^2 - (6x - x) - \sqrt{2} = 0$ [by splitting the middle term]

$3\sqrt{2}x^2 - 6x + x - \sqrt{2} = 0$

$3\sqrt{2}x^2 - 3\sqrt{2} \cdot \sqrt{2}x + x - \sqrt{2} = 0$

$\Rightarrow \quad 3\sqrt{2}\,x(x - \sqrt{2}) + 1(x - \sqrt{2}) = 0$

$\Rightarrow \quad (x - \sqrt{2})(3\sqrt{2}x + 1) = 0$

Now, $x - \sqrt{2} = 0 \Rightarrow x = \sqrt{2}$

and $3\sqrt{2}\,x + 1 = 0$

$\Rightarrow \quad x = -\frac{1}{3\sqrt{2}} = \frac{-\sqrt{2}}{6}$

Hence, the roots of the equation $3\sqrt{2}x^2 - 5x - \sqrt{2} = 0$ are $-\frac{\sqrt{2}}{6}$ and $\sqrt{2}$.

(iv) Given equation is $3x^2 + 5\sqrt{5}\,x - 10 = 0$.

$3x^2 + 6\sqrt{5}x - \sqrt{5}x - 2\sqrt{5}.\sqrt{5} = 0$ [by splitting the middle term]

$\Rightarrow \quad 3x^2 + 6\sqrt{5}x - \sqrt{5}x - 10 = 0$

$3x^2 + 6\sqrt{5}x - \sqrt{5}x - 2\sqrt{5} \cdot \sqrt{5} = 0$

$\Rightarrow \quad 3x(x + 2\sqrt{5}) - \sqrt{5}(x + 2\sqrt{5}) = 0$

$\Rightarrow \quad (x + 2\sqrt{5})(3x - \sqrt{5}) = 0$

Now, $x + 2\sqrt{5} = 0$

$\Rightarrow \quad x = -2\sqrt{5}$ and $3x - \sqrt{5} = 0$

$\Rightarrow \quad x = \frac{\sqrt{5}}{3}$

Hence, the roots of the equation $3x^2 + 5\sqrt{5}\,x - 10 = 0$ are $-2\sqrt{5}$ and $\frac{\sqrt{5}}{3}$.

(v) Given equation is $21x^2 - 2x + \frac{1}{21} = 0$.

On multiplying by 21 on both sides, we get

$441x^2 - 42x + 1 = 0$

$441x^2 - (21x + 21x) + 1 = 0$ [by splitting the middle term]

$\Rightarrow \quad 441x^2 - 21x - 21x + 1 = 0$

$\Rightarrow \quad 21x(21x - 1) - 1(21x - 1) = 0$

$\Rightarrow \quad (21x - 1)(21x - 1) = 0$

Now, $21x - 1 = 0 \Rightarrow x = \frac{1}{21}$ and $21x - 1 = 0$

$\therefore \quad x = \frac{1}{21}$

Hence, the roots of the equation $441x^2 - 42x + 1 = 0$ are $\frac{1}{21}$ and $\frac{1}{21}$.

Exercise 4.4 Long Answer Type Questions

Question 1:

Find whether the following equations have real roots. If real roots exist, find them

(i) $8x^2 + 2x - 3 = 0$

(ii) $-2x^2 + 3x + 2 = 0$

(iii) $5x^2 - 2x - 10 = 0$

(iv) $\frac{1}{2x-3} + \frac{1}{x-5} = 1, x \neq \frac{3}{2}, 5$

(v) $x^2 + 5\sqrt{5}x - 70 = 0$

Solution:

(i) Given equation is $8x^2 + 2x - 3 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 8, b = 2 \text{ and } c = -3$$

$\therefore$ Discriminant, $D = b^2 - 4ac$

$$= (2)^2 - 4(8)(-3)$$

$$= 4 + 96 = 100 > 0$$

Therefore, the equation $8x^2 + 2x - 3 = 0$ has two distinct real roots because we know that, if the equation $ax^2 + bx + c = 0$ has discriminant greater than zero, then if has two distinct real roots .

Roots, $$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-2 \pm \sqrt{100}}{16} = \frac{-2 \pm 10}{16}$$

$$= \frac{-2+10}{16}, \frac{-1-10}{16}$$

$$= \frac{8}{16}, -\frac{12}{16} = \frac{1}{2}, -\frac{3}{4}$$

(ii) Given equation is $-2x^2 + 3x + 2 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = -2, b = 3 \text{ and } c = 2$$

$\therefore$ Discriminant, $D = b^2 - 4ac$

$$= (3)^2 - 4(-2)(2)$$

$$= 9 + 16 = 25 > 0$$

Therefore, the equation $-2x^2 + 3x + 2 = 0$ has two distinct real roots because we know that if the equation $ax^2 + bx + c = 0$ has its discriminant greater than zero, then it has two distinct real roots.

$$\text{Roots, } x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-3 \pm \sqrt{25}}{2(-2)}$$

$$= \frac{-3 \pm 5}{-4} = \frac{-3+5}{-4}, \frac{-3-5}{-4}$$

$$= \frac{2}{-4}, \frac{-8}{-4} = -\frac{1}{2}, 2$$

(iii) Given equation is $5x^2 - 2x - 10 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 5, b = -2 \text{ and } c = -10$$

$\therefore$ Discriminant, $D = b^2 - 4ac$

$$= (-2)^2 - 4(5)(-10)$$

$$= 4 + 200 = 204 > 0$$

Therefore, the equation $5x^2 - 2x - 10 = 0$ has two distinct real roots.

$$\text{Roots, } x = \frac{-b \pm \sqrt{D}}{2a}$$
$$= \frac{-(-2) \pm \sqrt{204}}{2 \times 5} = \frac{2 \pm 2\sqrt{51}}{10}$$
$$= \frac{1 \pm \sqrt{51}}{5} = \frac{1+\sqrt{51}}{5}, \frac{1-\sqrt{51}}{5}$$

(iv) Given equation is $\frac{1}{2x-3} + \frac{1}{x-5} = 1, x \neq \frac{3}{2}, 5$

$$\Rightarrow \quad \frac{x-5+2x-3}{(2x-5)(x-5)} = 1$$
$$\Rightarrow \quad \frac{3x-8}{2x^2-5x-10x+25} = 1$$
$$\Rightarrow \quad \frac{3x-8}{2x^2-15x+25} = 1$$
$$\Rightarrow \quad 3x-8 = 2x^2-15x+25$$
$$\Rightarrow \quad 2x^2-15x-3x+25+8 = 0$$
$$\Rightarrow \quad 2x^2-18x+33 = 0$$

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 2, b = -18 \text{ and } c = 33$$

$\therefore$ Discriminant, $D = b^2 - 4ac$

$$= (-18)^2 - 4 \times 2\,(33)$$
$$= 324 - 264 = 60 > 0$$

Therefore, the equation $2x^2 - 18x + 33 = 0$ has two distinct real roots.

$$\text{Roots, } x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-18) \pm \sqrt{60}}{2\,(2)}$$
$$= \frac{18 \pm 2\sqrt{15}}{4} = \frac{9 \pm \sqrt{15}}{2}$$
$$= \frac{9+\sqrt{15}}{2}, \frac{9-\sqrt{15}}{2}$$

(v) Given equation is $x^2 + 5\sqrt{5}x - 70 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = 5\sqrt{5} \text{ and } c = -70$$

$\therefore$ Discriminant, $D = b^2 - 4ac = (5\sqrt{5})^2 - 4\,(1)\,(-70)$

$$= 125 + 280 = 405 > 0$$

Therefore, the equation $x^2 + 5\sqrt{5}x - 70 = 0$ has two distinct real roots.

$$\text{Roots, } x = \frac{-b \pm \sqrt{D}}{2a}$$
$$= \frac{-5\sqrt{5} \pm \sqrt{405}}{2\,(1)} = \frac{-5\sqrt{5} \pm 9\sqrt{5}}{2}$$
$$= \frac{-5\sqrt{5}+9\sqrt{5}}{2}, \frac{-5\sqrt{5}-9\sqrt{5}}{2}$$
$$= \frac{4\sqrt{5}}{2}, -\frac{14\sqrt{5}}{2} = 2\sqrt{5}, -7\sqrt{5}$$

Question 2:

Find a natural number whose square diminished by 84 is equal to thrice of 8 more than the given number.

Solution:

Let n be a required natural number.

Square of a natural number diminished by 84 = $n^2 - 84$ and thrice of 8 more than the natural number = 3 (n + 8)

Now, by given condition,

$$n^2 - 84 = 3(n+8)$$

$\Rightarrow$ $$n^2 - 84 = 3n + 24$$

$\Rightarrow$ $$n^2 - 3n - 108 = 0$$

$\Rightarrow$ $$n^2 - 12n + 9n - 108 = 0$$ [by splitting the middle term]

$\Rightarrow$ $$n(n-12) + 9(n-12) = 0$$

$\Rightarrow$ $$(n-12)(n+9) = 0$$

$\Rightarrow$ $$n = 12$$ [$\because n \neq -9$ because n is a natural number]

Hence, the required natural number is 12.

Question 3:

A natural number, when increased by 12, equals 160 times its reciprocal.Find the number.

Solution:

Let the natural number be x.

According to the question,

$$x + 12 = \frac{160}{x}$$

On multiplying by x on both sides, we get

$\Rightarrow$ $$x^2 + 12x - 160 = 0$$

$$x^2 + (20x - 8x) - 160 = 0$$

$\Rightarrow$ $$x^2 + 20x - 8x - 160 = 0$$ [by factorisation method]

$\Rightarrow$ $$x(x+20) - 8(x+20) = 0$$

$\Rightarrow$ $$(x+20)(x-8) = 0$$

Now, x + 20 = 0 ⇒ x = – 20 which is not possible because natural number is always greater than zero and x-8 = 0⇒x = 8.

Hence, the required natural number is 8.

Question 4:

A train, travelling at a uniform speed for 360 km, would have taken 48 min less to travel the same distance, if its speed were 5 km/h more. Find the original speed of the train.

Solution:

Let the original speed of the train = x km/h

Then, the increased speed of the train = (x + 5) km/h [by given condition]

and distance = 360 km

According to the question,

$$\frac{360}{x} - \frac{360}{x+5} = \frac{4}{5}$$ $\left[\because \text{time} = \frac{\text{Distance}}{\text{Speed}} \text{ and } 48 \text{ min} = \frac{48}{60}\text{h} = \frac{4}{5}\text{h}\right]$

$\Rightarrow$ $$\frac{360(x+5) - 360x}{x(x+5)} = \frac{4}{5}$$ $\left[\because 48 \text{ min} = \frac{48}{60}\text{h} = \frac{4}{5}\text{h}\right]$

$\Rightarrow$ $$\frac{360x + 1800 - 360x}{x^2 + 5x} = \frac{4}{5}$$

$\Rightarrow$ $$\frac{1800}{x^2 + 5x} = \frac{4}{5}$$

$\Rightarrow$ $$x^2 + 5x = \frac{1800 \times 5}{4} = 2250$$

$\Rightarrow$ $$x^2 + 5x - 2250 = 0$$

$$x^2 + (50x - 45x) - 2250 = 0$$

$\Rightarrow$ $$x^2 + 50x - 45x - 2250 = 0$$ [by factorisation method]

$\Rightarrow$ $$x(x+50) - 45(x+50) = 0$$

$\Rightarrow$ $$(x+50)(x-45) = 0$$

Now, x+50=0⇒x=-50

which is not possible because speed cannot be negative and x – 45 = 0⇒x = 45. Hence, the original speed of the train = 45 km/h

Question 5:

If Zeba were younger by 5 yr than what she really is, then the square of, her age (in years) would have been 11, more than five times her actual age, what is her age now?

Solution:

Let the actual age of Zeba = x yr,

Herage when she was 5 yr younger = (x – 5)yr.

Now, by given condition,

Square of her age = 11 more than five times her actual age

$$(x-5)^2 = 5 \times \text{actual age} + 11$$

$$\Rightarrow \quad (x-5)^2 = 5x + 11$$

$$\Rightarrow \quad x^2 + 25 - 10x = 5x + 11$$

$$\Rightarrow \quad x^2 - 15x + 14 = 0$$

$$\Rightarrow \quad x^2 - 14x - x + 14 = 0 \quad \text{[by splitting the middle term]}$$

$$\Rightarrow \quad x(x-14) - 1(x-14) = 0$$

$$\Rightarrow \quad (x-1)(x-14) = 0$$

$$\Rightarrow \quad x = 14$$

[here, **x** ≠ 1 because her age is **x – 5**. So, **x** – 5 = 1 – 5 = – 4 **i.e.,** age cannot be negative]

Hence, required Zeba's age now is 14 yr.

Question 6:

At present Asha's age (in years) is 2 more than the square of her daughter Nisha's age. When Nisha grows to her mother's present age. Asha's age would be one year less than 10 times the present age of Nisha. Find the present ages of both Asha and Nisha.

Solution:

Let Nisha's present age be x yr.

Then, Asha's present age = $x^2 + 2$ [by given condition]

Now, when Nisha grows to her mother's present age i.e., after [(x^2 +2)- x] yr. Then, Asha's age also increased by [(x^2 + 2) – x]yr.

Again by given condition,

Age of Asha = One years less than 10 times the present age of Nisha

$$(x^2 + 2) + \{(x^2 + 2) - x\} = 10x - 1$$

$$\Rightarrow \quad 2x^2 - x + 4 = 10x - 1$$

$$\Rightarrow \quad 2x^2 - 11x + 5 = 0$$

$$\Rightarrow \quad 2x^2 - 10x - x + 5 = 0$$

$$\Rightarrow \quad 2x(x-5) - 1(x-5) = 0$$

$$\Rightarrow \quad (x-5)(2x-1) = 0$$

$$\therefore \quad x = 5$$

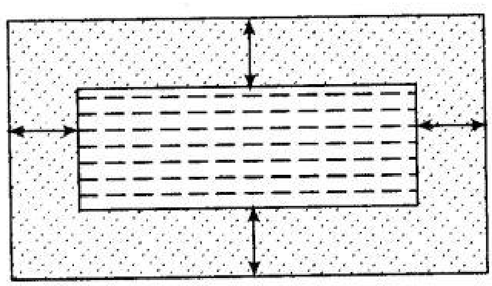
[here, $x = \frac{1}{2}$ cannot be possible, because at $x = \frac{1}{2}$, Asha's age is $2\frac{1}{4}$ yr which is not possible]

Hence, required age of Nisha = 5yr

and required age of Asha = $x^2 + 2 = (5)^2 + 2 = 25 + 2 = 27$ yr

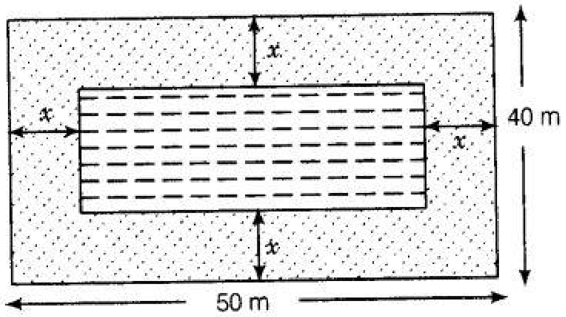
Question 7:

In the centre of a rectangular lawn of dimensions 50m x 40m, a rectangular pond has to be constructed, so that the area of the grass surrounding the pond would be 1184 m^2[see figure]. Find the length and breadth of the pond.



Solution:

Given that a rectangular pond has to be constructed in the centre of a rectangular lawn of dimensions 50 mx 40 m So, the distance between pond and lawn would be same around the pond. Say x m.



Now, length of rectangular lawn (l_1) = 50 m and breadth of rectangular lawn (b_1) = 40 m

Length of rectangular pond (l_2)= 50 – (x + x) = 50 – 2x

and breadth of rectangular pond (b_2) = 40 – (x+ x)= 40-2x

Also, area of the grass surrounding the pond =1184 m^2

Area of rectangular lawn – Area of rectangular pond

= Area of grass surrounding the pond

$l_1 \times b_1 - l_2 \times b_2 = 1184$ [$\because$ area of rectangle = length × breadth]

$\Rightarrow 50 \times 40 - (50 - 2x)(40 - 2x) = 1184$

$\Rightarrow 2000 - (2000 - 80x - 100x + 4x^2) = 1184$

$\Rightarrow 80x + 100x - 4x^2 = 1184$

$\Rightarrow 4x^2 - 180x + 1184 = 0$

$\Rightarrow x^2 - 45x + 296 = 0$

$\Rightarrow x^2 - 37x - 8x + 296 = 0$ [by splitting the middle term]

$\Rightarrow x(x - 37) - 8(x - 37) = 0$

$\Rightarrow (x - 37)(x - 8) = 0$

$\therefore$ x=8

[At x = 37, length and breadth of pond are -24 and -34, respectively but length and breadth cannot be negative. So, x = 37 cannot be possible]

$\therefore$ Length of pond = 50- 2x= 50- 2 (8)= 50- 16= 34 m

and breadth of pond = 40 – 2x = 40 – 2(8) = 40 – 16 = 24 m

Hence, required length and breadth of pond are 34 m and 24 m, respectively.

Question 8:

At t min past 2 pm, the time needed by the minute hand of a clock to show 3 pm was found to be 3 min less than $\frac{t^2}{4}$min. Find t.

Solution:

We know that, the time between 2 pm to 3 pm = 1 h = 60 min

Given that, at f min past 2pm, the time needed by the min hand of a clock to show 3 pm was found to be 3 min less than $\frac{t^2}{4}$ min i,e

4

$$t + \left(\frac{t^2}{4} - 3\right) = 60$$

$\Rightarrow$ $4t + t^2 - 12 = 240$

$\Rightarrow$ $t^2 + 4t - 252 = 0$

$\Rightarrow$ $t^2 + 18t - 14t - 252 = 0$ [by splitting the middle term]

$\Rightarrow$ $t(t + 18) - 14(t + 18) = 0$ [since, time cannot be negative, so $t \neq -18$]

$\Rightarrow$ $(t + 18)(t - 14) = 0$

$\therefore$ $t = 14$ min

Hence, the required value off is 14 min.