Unit 3(Pair of Liner Equation in Two Variable)

Exercise 3.1 Multiple Choice Questions (MCQs)

Question 1:

Graphically, the pair of equations

$6x - 3y + 10 = 0$

$2x - y + 9 = 0$

represents two lines which are

(a) intersecting at exactly one point

(b) intersecting exactly two points

(c) coincident

(d) parallel

Solution:

The given equations are

$6x-3y+10 = 0$

$\Rightarrow 2x-y+\frac{10}{3}= 0$ [dividing by 3]… (i)

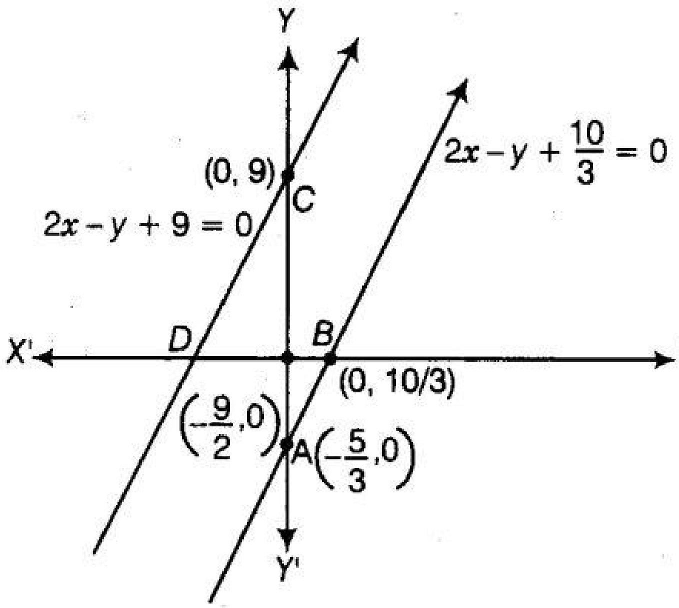
and $2x-y+9=0$ …(ii)

Now, table for $2x - y + \frac{10}{3}= 0$,

x	0	$-\frac{5}{3}$
$y = 2x + \frac{10}{3}$	$\frac{10}{3}$	0
Points	A	B

and table for $2x - y + 9 = 0$,

x	0	$-\frac{9}{2}$
$y = 2x + 9$	9	0
Points	C	D



Hence, the pair of equations represents two parallel lines.

Question 2:

The pair of equations x + 2y + 5 = 0 and – 3x – 6y +1 = 0 has

(a) a unique solution (b) exactly two solutions

(c) infinitely many solutions (d) no solution

Solution:

(d) Given, equations are $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$

Here, $a_1 = 1, b_1 = 2, c_1 = 5$ and $a_2 = -3, b_2 = -6, c_2 = 1$

$$\therefore \quad \frac{a_1}{a_2} = -\frac{1}{3}, \frac{b_1}{b_2} = -\frac{2}{6} = -\frac{1}{3},$$

$$\frac{c_1}{c_2} = \frac{5}{1}$$

$$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the pair of equations has no solution.

Question 3:

If a pair of linear equations is consistent, then the lines will be

(a) parallel (b) always coincident

(c) intersecting or coincident (d) always intersecting

Solution:

(c) Condition for a consistent pair of linear equations

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \quad \text{[intersecting lines having unique solution]}$$

and

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad \text{[coincident or dependent]}$$

Question 4:

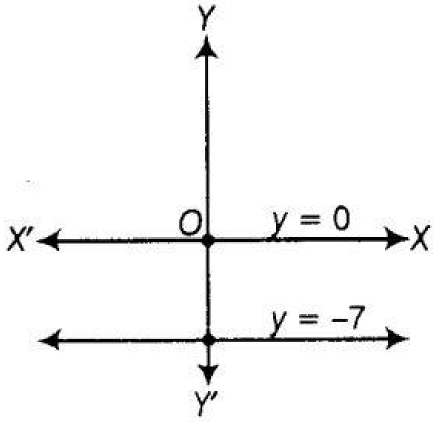
The pair of equations y = 0 and y = – 7 has

(a) one solution (b) two solutions

(c) infinitely many solutions (d) no solution

Solution:

(d) The given pair of equations are y = 0 and y = – 7.



By graphically, both lines are parallel and having no solution

Question 5:

The pair of equations x = a and y = b graphically represents lines which are

(a) parallel (b) intersecting at (b, a)

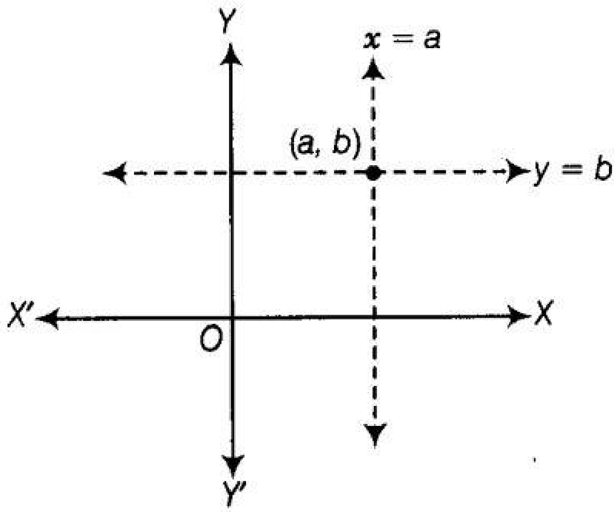
(c) coincident (d) intersecting at (a, b)

Solution:

(d) By graphically in every condition, if a, b>>0; a, b< 0, a>0, b< 0; a<0, b>0 but a = b≠ 0.

The pair of equations x = a and y = b graphically represents lines which are intersecting at (a, b).

If a, b > 0



Similarly, in all cases two lines intersect at (a, b).

Question 6:

For what value of k, do the equations 3x – y + 8 = 0 and 6x – ky = – 16

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 2 (d) -2

Solution:

(c) Condition for coincident lines is

(c) Condition for coincident lines is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Given lines, $3x - y + 8 = 0$

and $6x - ky + 16 = 0$

Here, $a_1 = 3, b_1 = -1, c_1 = 8$

and $a_2 = 6, b_2 = -k, c_2 = 16$

From Eq. (i), $\frac{3}{6} = \frac{-1}{-k} = \frac{8}{16}$

$\Rightarrow \quad \frac{1}{k} = \frac{1}{2}$

$\therefore \quad k = 2$

Question 7:

If the lines given by 3x+ 2ky = 2 and 2x + 5y = 1 are parallel, then the value of k is

(a) $-\frac{5}{4}$ (b) $\frac{2}{5}$

(c) $\frac{15}{4}$ (d) $\frac{3}{2}$

Solution:

(c) Condition for parallel lines is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Given lines, $3x + 2ky - 2 = 0$

and $2x + 5y - 1 = 0$

Here, $a_1 = 3, b_1 = 2k, c_1 = -2$

and $a_2 = 2, b_2 = 5, c_2 = -1$

From Eq. (i), $\frac{3}{2} = \frac{2k}{5}$

$\therefore \quad k = \frac{15}{4}$

Question 8:

The value of c for which the pair of equations cx- y = 2 and 6x – 2y = 3 will have infinitely many solutions is

(a) 3 (b) – 3 (c)-12 (d) no value

Solution:

(d) Condition for infinitely many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad \ldots(i)$$

The given lines are $cx - y = 2$ and $6x - 2y = 3$

Here, $a_1 = c, b_1 = -1, c_1 = -2$

and $a_2 = 6, b_2 = -2, c_2 = -3$

From Eq. (i), $\frac{c}{6} = \frac{-1}{-2} = \frac{-2}{-3}$

Here, $\frac{c}{6} = \frac{1}{2}$ and $\frac{c}{6} = \frac{2}{3}$

$\Rightarrow$ $c = 3$ and $c = 4$

Since, c has different values.

Hence, for no value of c the pair of equations will have infinitely many solutions.

Question 9:

One equation of a pair of dependent linear equations is – 5x+ 7y – 2 = 0. The second equation can be

(a) 10x + 14y + 4=0 (b)-10x-14y + 4 =0

(c) -10x + 14y + 4 = 0 (d) 10x-14y + 4=0

Solution:

(d) Condition for dependent linear equations

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{k} \quad \ldots(i)$$

Given equation of line is, $-5x + 7y - 2 = 0$

Here, $a_1 = -5, b_1 = 7, c_1 = -2$

From Eq. (i), $-\frac{5}{a_2} = \frac{7}{b_2} = -\frac{2}{c_2} = \frac{1}{k}$ [say]

$\Rightarrow$ $a_2 = -5k, b_2 = 7k, c_2 = -2k$

where, k is any arbitrary constant.

Putting $k = 2$, then $a_2 = -10, b_2 = 14$

and $c_2 = -4$

$\therefore$ The required equation of line becomes

$$a_2 x + b_2 y + c_2 = 0$$

$$\Rightarrow \quad -10x + 14y - 4 = 0$$

$$\Rightarrow \quad 10x - 14y + 4 = 0$$

Question 10:

A pair of linear equations which has a unique solution x – 2 and y = – 3 is

(a) x + y = 1 and 2x – 3y = – 5

(b) 2x+ 5y= -11 and 4x + 10y = -22

(c) 2x – y = 1 and 3x + 2y = 0

(d) x – 4y -14 = 0 and 5x – y -13 = 0

Solution:

(b) If x = 2, y = – 3 is a unique solution of any pair of equation, then these values must satisfy that pair of equations.

From option (b), LHS = 2x + 5y = 2(2) + 5(- 3) = 4 – 15 = – 11 = RHS

and LHS = 4x + 10y = 4(2) + 10(- 3)= 8 – 30 = – 22 = RHS

Question 11:

If x = a and y = b is the solution of the equations x- y = 2 and x + y = 4, then the values of a and b are, respectively

(a) 3 and 5 (b) 5 and 3 (c) 3 and 1 (d) – 1 and – 3

Solution:

(c) Since, x = a and y = b is the solution of the equations x – y = 2 and x+ y = 4, then these values will satisfy that equations

$a-b=2$,..(i)

and $a + b = 4$ … (ii)

On adding Eqs. (i) and (ii), we get

$2a = 6$

$a = 3$ and $b = 1$

Question 12:

Aruna has only ₹ 1 and ₹ 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is ₹ 75, then the number of ₹ 1 and ₹ 2 coins are, respectively

(a) 35 and 15 (b) 35 and 20 (c) 15 and 35 (d) 25 and 25

Solution:

(d) Let number of ₹ 1 coins = x

and number of ₹ 2 coins = y

Now, by given conditions $x+y=50$ …(i)

Also, $x\times1+y\times2=75$

$\Rightarrow$ $x + 2y = 75$ …(ii)

On subtracting Eq. (i) from Eq. (ii), we get

$(x + 2y) - (x + y) = 75 - 50$

$\Rightarrow$ $y = 25$

When $y = 25$, then $x = 25$

Question 13:

The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present ages (in year) of the son and the father are, respectively

(a) 4 and 24 (b) 5 and 30

(c) 6 and 36 (d) 3 and 24

Solution:

(c)

Let x yr be the present age of father and y yr be the present age of son.

Four years hence, it has relation by given condition,

$(x + 4) = 4(y + 4)$

$\Rightarrow$ $x-4y = 12$ …(i)

and $x = 6y$ …(ii)

On putting the value of x from Eq. (ii) in Eq. (i), we get

$6y-4y=12$

$\Rightarrow$ $2y = 12$

$\Rightarrow$ $y = 6$

When $y = 6$, then $x = 36$

Hence, present age of father is 36 yr and age of son is 6 yr.

Exercise 3.2 Short Answer Type Questions

Question 1:

Do the following pair of linear equations have no solution? Justify your answer.

(i) $2x + 4y = 3$ and $12y + 6x = 6$

(ii) $x = 2y$ and $y = 2x$

(iii) $3x + y - 3 = 0$ and $2x + - y = 2$

Solution:

Condition for no solution $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(i) *Yes*, given pair of equations,

$$2x + 4y = 3 \text{ and } 12y + 6x = 6$$

Here, $a_1 = 2, b_1 = 4, c_1 = -3,$

$a_2 = 6, b_2 = 12, c_2 = -6$

$\therefore$ $\frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{4}{12} = \frac{1}{3}$

$\frac{c_1}{c_2} = \frac{-3}{-6} = \frac{1}{2}$

$\therefore$ $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the given pair of linear equations has no solution.

(ii) *No*, given pair of equations,

$$x = 2y \text{ and } y = 2x$$

or $x - 2y = 0 \text{ and } 2x - y = 0$

Here, $a_1 = 1, b_1 = -2, c_1 = 0;$

$a_2 = 2, b_2 = -1, c_2 = 0$

$\therefore$ $\frac{a_1}{a_2} = \frac{1}{2} \text{ and } \frac{b_1}{b_2} = \frac{2}{1}$

$\because$ $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the given pair of linear equations has unique solution.

(iii) *No*, given pair of equations,

$$3x + y - 3 = 0 \text{ and } 2x + \frac{2}{3}y - 2 = 0$$

Here, $a_1 = 3, b_1 = 1, c_1 = -3,$

$a_2 = 2, b_2 = \frac{2}{3}, c_2 = -2$

$\therefore$ $\frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{1}{2/3} = \frac{3}{2}$

$\frac{c_1}{c_2} = \frac{-3}{-2} = \frac{3}{2}$

$\because$ $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{3}{2}$

Hence, the given pair of linear equations is coincident and having infinitely many solutions.

Question 2:

Do the following equations represent a pair of coincident lines? Justify your answer.

(i) $3x + \frac{1}{7}y = 3$ and $7x + 3y = 7$

(ii) $-2x - 3y = 1$ and $6y + 4x = -2$

(iii) $\frac{x}{2} + y + \frac{2}{5} = 0$ and $4x + 8y + \frac{5}{16} = 0$

Solution:

Condition for coincident lines,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

(i) *No*, given pair of linear equations

$$3x + \frac{y}{7} - 3 = 0$$

and $$7x + 3y - 7 = 0,$$

where, $$a_1 = 3, b_1 = \frac{1}{7}, c_1 = -3;$$

$$a_2 = 7, b_2 = 3, c_2 = -7$$

Now, $$\frac{a_1}{a_2} = \frac{3}{7}, \frac{b_1}{b_2} = \frac{1}{21}, \frac{c_1}{c_2} = \frac{3}{7} \qquad \left[\because \frac{a_1}{a_2} \neq \frac{b_1}{b_2}\right]$$

Hence, the given pair of linear equations has unique solution.

(ii) *Yes*, given pair of linear equations

$$-2x - 3y - 1 = 0 \text{ and } 6y + 4x + 2 = 0$$

where, $$a_1 = -2, b_1 = -3, c_1 = -1;$$

$$a_2 = 4, b_2 = 6, c_2 = 2$$

Now, $$\frac{a_1}{a_2} = -\frac{2}{4} = -\frac{1}{2}$$

$$\frac{b_1}{b_2} = -\frac{3}{6} = -\frac{1}{2}, \frac{c_1}{c_2} = -\frac{1}{2}$$

$\because$ $$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = -\frac{1}{2}$$

Hence, the given pair of linear equations is coincident.

(iii) *No*, the given pair of linear equations are

$$\frac{x}{2} + y + \frac{2}{5} = 0 \text{ and } 4x + 8y + \frac{5}{16} = 0$$

Here, $$a_1 = \frac{1}{2}, b_1 = 1, c_1 = \frac{2}{5}$$

$$a_2 = 4, b_2 = 8, c_2 = \frac{5}{16}$$

Now, $$\frac{a_1}{a_2} = \frac{1}{8}, \frac{b_1}{b_2} = \frac{1}{8}, \frac{c_1}{c_2} = \frac{32}{25}$$

$\because$ $$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the given pair of linear equations has no solution.

Question 3:

Are the following pair of linear equations consistent? Justify your answer,

(i) $-3x - 4y = 12$ and $4y + 3x = 12$

(ii) $\frac{3}{5}x - y = \frac{1}{2}$ and $\frac{1}{5}x - 3y = \frac{1}{6}$

(iii) $2ax + by = a$ and $4ax + 2by - 2a = 0; a, b \neq 0$

(iv) $x + 3y = 11$ and $2(2x + 6y) = 22$

Solution:

Conditions for pair of linear equations are consistent

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \quad \text{[unique solution]}$$

and

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad \text{[infinitely many solutions]}$$

(i) *No*, the given pair of linear equations

$$-3x - 4y = 12 \text{ and } 3x + 4y = 12$$

Here, $a_1 = -3, b_1 = -4, c_1 = -12;$

$a_2 = 3, b_2 = 4, c_2 = -12$

Now, $\frac{a_1}{a_2} = -\frac{3}{3} = -1, \frac{b_1}{b_2} = -\frac{4}{4} = -1, \frac{c_1}{c_2} = \frac{-12}{-12} = 1$

$\because \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the pair of linear equations has no solution, *i.e.*, inconsistent.

(ii) *Yes*, the given pair of linear equations

$$\frac{3}{5}x - y = \frac{1}{2} \text{ and } \frac{1}{5}x - 3y = \frac{1}{6}$$

Here, $a_1 = \frac{3}{5}, b_1 = -1, c_1 = -\frac{1}{2}$

and $a_2 = \frac{1}{5}, b_2 = -3, c_2 = -\frac{1}{6}$

Now, $\frac{a_1}{a_2} = \frac{3}{1}, \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{3}{1}$ $\quad \left[\because \frac{a_1}{a_2} \neq \frac{b_1}{b_2}\right]$

Hence, the given pair of linear equations has unique solution, *i.e.*, consistent.

(iii) *Yes*, the given pair of linear equations

$2ax + by - a = 0$

and $4ax + 2by - 2a = 0; a, b \neq 0$

Here, $a_1 = 2a, b_1 = b, c_1 = -a;$

$a_2 = 4a, b_2 = 2b, c_2 = -2a$

Now, $\frac{a_1}{a_2} = \frac{2a}{4a} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{b}{2b} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-a}{-2a} = \frac{1}{2}$

$\because \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$

Hence, the given pair of linear equations has infinitely many solutions, *i.e.*, consistent or dependent.

(iv) *No*, the given pair of linear equations

$$x + 3y = 11 \text{ and } 2x + 6y = 11$$

Here, $a_1 = 1, b_1 = 3, c_1 = -11$ ...(i)

$a_2 = 2, b_2 = 6, c_2 = -11$

Now, $\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-11}{-11} = 1$

$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the pair of linear equation have no solution *i.e.*, inconsistent.

Question 4:

For the pair of equations $\lambda x + 3y + 7 = 0$ and $2x + 6y - 14 = 0$. To have infinitely many solutions, the value of λ should be 1. Is the statement true? Give reasons.

Solution:

No, the given pair of linear equations

$\lambda x + 3y + 7 = 0$ and $2x + 6y - 14 = 0$

Here, $a_1 = \lambda$, $b_1 = 3$ $c_1 = 7$; $a_2 = 2$, $b_2 = 6$, $c_2 = -14$

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then system has infinitely many solutions.

$\Rightarrow \qquad \frac{\lambda}{2} = \frac{3}{6} = -\frac{7}{14}$

$\because \qquad \frac{\lambda}{2} = \frac{3}{6} \Rightarrow \lambda = 1$

and $\qquad \frac{\lambda}{2} = -\frac{7}{14} \Rightarrow \lambda = -1$

Hence, λ = -1 does not have a unique value.

So, for no value of λ the given pair of linear equations has infinitely many solutions.

Question 5:

For all real values of c, the pair of equations x – 2y = 8 and 5x – lOy = c have a unique solution. Justify whether it is true or false.

Solution:

False, the given pair of linear equations

x-2y-8=0

5x-10y=c

Here, $\qquad a_1 = 1, b_1 = -2, c_1 = -8$

$a_2 = 5, b_2 = -10, c_2 = -c$

Now, $\qquad \frac{a_1}{a_2} = \frac{1}{5}, \frac{b_1}{b_2} = \frac{-2}{-10} = \frac{1}{5}$

$\frac{c_1}{c_2} = \frac{-8}{-c} = \frac{8}{c}$

But if c = 40 (real value), then the ratio $\frac{c_1}{c_2}$ becomes $\frac{1}{5}$ and then the system of linear equations has an infinitely many solutions.

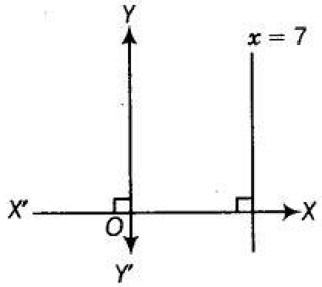
Hence, ate = 40, the system of linear equations does not have a unique solution.

Question 6:

The line represented by x = 7 is parallel to the X-axis, justify whether the statement is true or not.

Solution:

Not true, by graphically, we observe that x = 7 line is parallel to y-axis and perpendicular to X-axis.



Exercise 3.3 Short Answer Type Questions

Question 1:

For which value(s) of λ, do the pair of linear equations λx + y =λ^2 and x + λy = 1 have

(i) no solution? (ii) infinitely many solutions?

(iii) a unique solution?

Solution:

The given pair of linear equations is

$\lambda x + y = \lambda^2$ and $x + \lambda y = 1$

$a_1 = \lambda$, $b_1 = 1$, $c_1 = -\lambda^2$

$a_2 = 1$, $b_2 = \lambda$ $c_2 = -1$

(i) For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{\lambda}{1} = \frac{1}{\lambda} \neq \frac{-\lambda^2}{-1}$$

$$\Rightarrow \quad \lambda^2 - 1 = 0$$

$$\Rightarrow \quad (\lambda - 1)(\lambda + 1) = 0$$

$$\Rightarrow \quad \lambda = 1, -1$$

Here, we take only $\lambda = -1$ because at $\lambda = 1$ the system of linear equations has infinitely many solutions.

(ii) For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{\lambda}{1} = \frac{1}{\lambda} = \frac{\lambda^2}{1}$$

$$\Rightarrow \quad \frac{\lambda}{1} = \frac{\lambda^2}{1}$$

$$\Rightarrow \quad \lambda(\lambda - 1) = 0$$

When $\lambda \neq 0$, then $\lambda = 1$

(iii) For a unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \quad \Rightarrow \quad \frac{\lambda}{1} \neq \frac{1}{\lambda}$$

$$\Rightarrow \quad \lambda^2 \neq 1 \quad \Rightarrow \quad \lambda \neq \pm 1$$

So, all real values of λ except ± 1.

Question 2:

For which value (s) of k will the pair of equations

kx+3y = k – 3,

12 x + ky =k

has no solution?

Solution:

Given pair of linear equations is

kx + 3y = k – 3

and 12x + ky = k

On comparing with ax + by + c = 0, we get

$a_1 = k$, $b_1 = 3$ and $c_1 = -(k-3)$

$a_2 = 12$, $b_2 = k$ and $c_2 = -k$

For no solution of the pair of linear equations,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{k}{12} = \frac{3}{k} \neq \frac{-(k-3)}{-k}$$

Taking first two parts, we get

$$\Rightarrow \quad \frac{k}{12} = \frac{3}{k}$$

$$\Rightarrow \quad k^2 = 36$$

$$\Rightarrow \quad k = \pm 6$$

Taking last two parts, we get

$$\frac{3}{k} \neq \frac{k-3}{k}$$

$$\Rightarrow \quad 3k \neq k(k-3)$$

$$\Rightarrow \quad 3k - k(k-3) \neq 0$$

$$\Rightarrow \quad k(3 - k + 3) \neq 0$$

$$\Rightarrow \quad k(6-k) \neq 0$$

$$\Rightarrow \quad k \neq 0 \text{ and } k \neq 6$$

Hence, required value of k for which the given pair of linear equations has no solution is -6.

Question 3:

For which values of a and b will the following pair of linear equations has infinitely many solutions?

x + 2y = 1

(a -b)x+(a + b)y = a + b – 2

Solution:

Given pair of linear equations are

x+2y=1

(a-b)x+(a+b)y=a+b-2

on comparing with ax+by+c=0,we get

$a_1 = 1, b_1 = 2$ and $c_1 = -1$ [from Eq. (i)]

$a_2 = (a-b), b_2 = (a+b)$ [from Eq. (ii)]

and $c_2 = -(a+b-2)$

For infinitely many solutions of the the pairs of linear equations,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{1}{a-b} = \frac{2}{a+b} = \frac{-1}{-(a+b-2)}$$

Taking first two parts,

$$\frac{1}{a-b} = \frac{2}{a+b}$$

$$\Rightarrow \quad a+b = 2a-2b$$

$$\Rightarrow \quad 2a-a = 2b+b$$

$$\Rightarrow \quad a = 3b \qquad \text{...(iii)}$$

Taking last two parts,

$$\frac{2}{a+b} = \frac{1}{(a+b-2)}$$

$$\Rightarrow \quad 2a+2b-4 = a+b$$

$$\Rightarrow \quad a+b = 4 \qquad \text{...(iv)}$$

Now, put the value of *a* from Eq. (iii) in Eq. (iv), we get

$$3b+b = 4$$

$$\Rightarrow \quad 4b = 4$$

$$\Rightarrow \quad b = 1$$

Put the value of *b* in Eq. (iii), we get

$$a = 3 \times 1$$

$$\Rightarrow \quad a = 3$$

So, the values (*a*, *b*) = (3, 1) satisfies all the parts. Hence, required values of *a* and *b* are 3 and 1 respectively for which the given pair of linear equations has infinitely many solutions.

Question 4:

Find the values of p in (i) to (iv) and p and q in (v) for the following pair of equations

(i) 3x – y – 5 = 0 and 6x – 2y – p = 0, if the lines represented by these equations are parallel.

(ii) – x + py = 1 and px – y = 1 if the pair of equations has no solution.

(iii) – 3x + 5y = 7 and 2px – 3y = 1,

if the lines represented by these equations are intersecting at a unique point.

(iv) 2x + 3y – 5 = 0 and px – 6y – 8 = 0,

if the pair of equations has a unique solution.

(v) 2x + 3y = 7 and 2px + py = 28 – qy,

if the pair of equations has infinitely many solutions.

Solution:

(i) Given pair of linear equations is

$$3x - y - 5 = 0 \quad \ldots\text{(i)}$$

and $$6x - 2y - p = 0 \quad \ldots\text{(ii)}$$

On comparing with $ax + by + c = 0$, we get

$$a_1 = 3, b_1 = -1$$

and $$c_1 = -5 \quad \text{[from Eq. (i)]}$$

$$a_2 = 6, b_2 = -2$$

and $$c_2 = -p \quad \text{[from Eq. (ii)]}$$

Since, the lines represented by these equations are parallel, then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{3}{6} = \frac{-1}{-2} \neq \frac{-5}{-p}$$

Taking last two parts, we get $\frac{-1}{-2} \neq \frac{-5}{-p}$

$\Rightarrow$ $$\frac{1}{2} \neq \frac{5}{p}$$

$\Rightarrow$ $$p \neq 10$$

Hence, the given pair of linear equations are parallel for all real values of p except 10 *i.e.*, $p \in R - \{10\}$.

(ii) Given pair of linear equations is

$$-x + py - 1 = 0 \quad \ldots\text{(i)}$$

and $$px - y - 1 = 0 \quad \ldots\text{(ii)}$$

On comparing with $ax + by + c = 0$, we get

$$a_1 = -1, b_1 = p \text{ and } c_1 = -1 \quad \text{[from Eq. (i)]}$$

$$a_2 = p, b_2 = -1 \text{ and } c_2 = -1 \quad \text{[from Eq. (ii)]}$$

Since, the pair of linear equations has no solution *i.e.*, both lines are parallel to each other.

$\therefore$ $$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{-1}{p} = \frac{p}{-1} \neq \frac{-1}{-1}$$

Taking last two parts, we get

$$\frac{p}{-1} \neq \frac{-1}{-1}$$

$\Rightarrow$ $$p \neq -1$$

Taking first two parts, we get

$$\frac{-1}{p} = \frac{p}{-1}$$

$\Rightarrow$ $$p^2 = 1$$

$\Rightarrow$ $$p = \pm 1$$

but $$p \neq -1$$

$\therefore$ $$p = 1$$

Hence, the given pair of linear equations has no solution for $p = 1$.

(iii) Given, pair of linear equations is

$$-3x + 5y - 7 = 0 \quad \text{...(i)}$$

and $$2px - 3y - 1 = 0 \quad \text{...(ii)}$$

On comparing with $ax + by + c = 0$, we get

$$a_1 = -3, b_1 = 5$$

and $$c_1 = -7 \quad \text{[from Eq. (i)]}$$

$$a_2 = 2p, b_2 = -3$$

and $$c_2 = -1 \quad \text{[from Eq. (ii)]}$$

Since, the lines are intersecting at a unique point *i.e.*, it has a unique solution.

$\therefore$ $$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\Rightarrow$ $$\frac{-3}{2p} \neq \frac{5}{-3}$$

$\Rightarrow$ $$9 \neq 10p$$

$\Rightarrow$ $$p \neq \frac{9}{10}.$$

Hence, the lines represented by these equations are intersecting at a unique point for all real values of p except $\frac{9}{10}$

(iv) Given pair of linear equations is

$$2x + 3y - 5 = 0 \quad \text{...(i)}$$

and $$px - 6y - 8 = 0 \quad \text{...(ii)}$$

On comparing with $ax + by + c = 0$, we get

$$a_1 = 2, b_1 = 3$$

and $$c_1 = -5 \quad \text{[from Eq. (i)]}$$

$$a_2 = p, \; b_2 = -6$$

and $$c_2 = -8 \quad \text{[from Eq. (ii)]}$$

Since, the pair of linear equations has a unique solution.

$\therefore$ $$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\Rightarrow$ $$\frac{2}{p} \neq \frac{3}{-6}$$

$\Rightarrow$ $$p \neq -4$$

Hence, the pair of linear equations has a unique solution for all values of p except -4

i.e., $$p \in R - \{-4,\}.$$

(v) Given pair of linear equations is

$$2x + 3y = 7 \quad \ldots(i)$$

and $$2px + py = 28 - qy$$

$\Rightarrow$ $$2px + (p+q)y = 28 \quad \ldots(ii)$$

On comparing with $ax + by + c = 0$, we get

$$a_1 = 2, b_1 = 3$$

and $$c_1 = -7 \quad \text{[from Eq. (i)]}$$

$$a_2 = 2p, b_2 = (p+q)$$

and $$c_2 = -28 \quad \text{[from Eq. (ii)]}$$

Since, the pair of equations has infinitely many solutions *i.e.,* both lines are coincident.

$\therefore$ $$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow$ $$\frac{2}{2p} = \frac{3}{(p+q)} = \frac{-7}{-28}$$

Taking first and third parts, we get

$$\frac{2}{2p} = \frac{-7}{-28}$$

$\Rightarrow$ $$\frac{1}{p} = \frac{1}{4}$$

$\Rightarrow$ $$p = 4$$

Again, taking last two parts, we get

$$\frac{3}{p+q} = \frac{-7}{-28} \Rightarrow \frac{3}{p+q} = \frac{1}{4}$$

$\Rightarrow$ $$p + q = 12$$

$\Rightarrow$ $$4 + q = 12 \quad [\because p = 4]$$

$\therefore$ $$q = 8$$

Here, we see that the values of $p = 4$ and $q = 8$ satisfies all three parts.

Hence, the pair of equations has infinitely many solutions for the values of $p = 4$ and $q = 8$

Question 5:

Two straight paths are represented by the equations x-3y = 2 and – 2x + 6y =5. Check whether the paths cross each other or not.

Solution:

Given linear equations are

$$x - 3y - 2 = 0 \quad \ldots(i)$$

and $$-2x + 6y - 5 = 0 \quad \ldots(ii)$$

On comparing both the equations with $ax + by + c = 0$, we get

$$a_1 = 1, b_1 = -3$$

and $$c_1 = -2 \quad \text{[from Eq. (i)]}$$

$$a_2 = -2, \; b_2 = 6$$

and $$c_2 = -5 \quad \text{[from Eq. (ii)]}$$

Here, $$\frac{a_1}{a_2} = \frac{1}{-2}$$

$$\frac{b_1}{b_2} = \frac{-3}{6} = -\frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$$

i.e., $$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad \text{[parallel lines]}$$

Hence, two straight paths represented by the given equations never cross each other, because they are parallel to each other.

Question 6:

Write a pair of linear equations which has the unique solution x = – 1 and y = 3. How many such pairs can you write?

Solution:

Condition for the pair of system to have unique solution

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Let the equations are,

$$a_1x + b_1y + c_1 = 0$$

and $$a_2x + b_2y + c_2 = 0$$

Since, $x = -1$ and $y = 3$ is the unique solution of these two equations, then

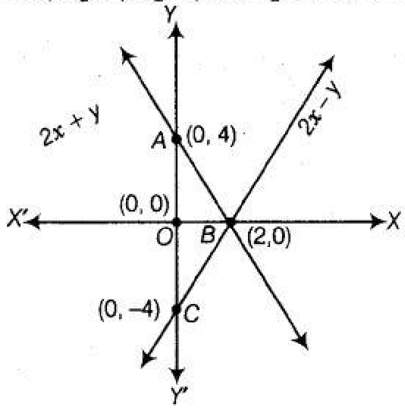
$$a_1(-1) + b_1(3) + c_1 = 0$$

$\Rightarrow$ $$-a_1 + 3b_1 + c_1 = 0 \quad \ldots\text{(i)}$$

and $$a_2(-1) + b_2(3) + c_2 = 0$$

$\Rightarrow$ $$-a_2 + 3b_2 + c_2 = 0 \quad \ldots\text{(ii)}$$

So, the different values of a_1, a_2, b_1, b_2, c_1 and c_2 satisfy the Eqs. (i) and (ii).



Hence, infinitely many pairs of linear equations are possible.

Question 7:

If 2x+ y = 23 and 4x- y = 19, then find the values of 5y – 2x and $\frac{y}{x} - 2$.

Solution:

Given equations are

$$2x + y = 23 \quad \ldots\text{(i)}$$

and $$4x - y = 19 \quad \ldots\text{(ii)}$$

On adding both equations, we get

$$6x = 42 \Rightarrow x = 7$$

Put the value of x in Eq. (i), we get

$$2(7) + y = 23$$

$\Rightarrow$ $$14 + y = 23$$

$\Rightarrow$ $$y = 23 - 14$$

$\Rightarrow$ $$y = 9$$

We have, $$5y - 2x = 5 \times 9 - 2 \times 7$$

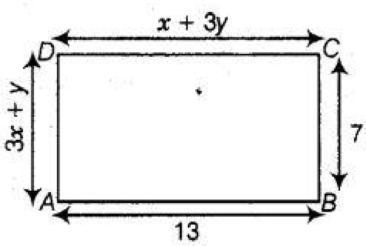
$$= 45 - 14 = 31$$

and $$\frac{y}{x} - 2 = \frac{4}{x} - 2 = \frac{9}{7} - 2 = \frac{9-14}{7} = -\frac{5}{7}$$

Hence, the values of $(5y - 2x)$ and $\left(\frac{y}{x} - 2\right)$ are 31 and $\frac{-5}{7}$, respectively.

Question 8:

Find the values of x and y in the following rectangle



Solution:

By property of rectangle,

Lengths are equal, *i.e.*, $CD = AB$

$\Rightarrow \quad x + 3y = 13 \quad \ldots(i)$

Breadth are equal, *i.e.*, $AD = BC$

$\Rightarrow \quad 3x + y = 7 \quad \ldots(ii)$

On multiplying Eq. (ii) by 3 and then subtracting Eq. (i), we get

$$\begin{array}{r} 9x + 3y = 21 \\ x + 3y = 13 \\ \underline{- \quad - \quad\quad -} \\ 8x \quad\quad = 8 \\ x = 1 \end{array}$$

On putting $x = 1$ in Eq. (i), we get

$$3y = 12 \Rightarrow y = 4$$

Hence, the required values of x and y are 1 and 4, respectively.

Question 9:

Solve the following pairs of equations

(i) $x + y = 3.3,\quad \frac{0.6}{3x - 2y} = -1, 3x - 2y \neq 0$

(ii) $\frac{x}{3} + \frac{y}{4} = 4,\quad \frac{5x}{6} - \frac{y}{8} = 4$

(iii) $4x + \frac{6}{y} = 15,\quad 6x - \frac{8}{y} = 14, y \neq 0$

(iv) $\frac{1}{2x} - \frac{1}{y} = -1,\quad \frac{1}{x} + \frac{1}{2y} = 8, x, y \neq 0$

(v) $43x + 67y = -24,\quad 67x + 43y = 24$

(vi) $\frac{x}{a} + \frac{y}{b} = a + b,\quad \frac{x}{a^2} + \frac{y}{b^2} = 2, a, b \neq 0$

(vii) $\frac{2xy}{x + y} = \frac{3}{2},\quad \frac{xy}{2x - y} = \frac{-3}{10}, x + y \neq 0, 2x - y \neq 0$

Solution:

(i) Given pair of linear equations are is

$$x + y = 3.3 \quad \ldots(i)$$

and $\frac{0.6}{3x - 2y} = -1$

$\Rightarrow \quad 0.6 = -3x + 2y$

$\Rightarrow \quad 3x - 2y = -0.6 \quad \ldots(ii)$

Now, multiplying Eq. (i) by 2 and then adding with Eq. (ii), we get

$$\Rightarrow \quad 2x + 2y = 6.6$$

$$\Rightarrow \quad 3x - 2y = -0.6$$

$$5x = 6 \Rightarrow x = \frac{6}{5} = 1.2$$

Now, put the value of x in Eq. (i), we get

$$1.2 + y = 3.3$$

$$\Rightarrow \quad y = 3.3 - 1.2$$

$$\Rightarrow \quad y = 2.1$$

Hence, the required values of x and y are 1.2 and 2.1, respectively.

(ii) Given, pair of linear equations is

$$\frac{x}{3} + \frac{y}{4} = 4$$

On multiplying both sides by LCM (3, 4) = 12, we get

$$4x + 3y = 48 \quad \text{...(i)}$$

and $$\frac{5x}{6} - \frac{y}{8} = 4$$

On multiplying both sides by LCM (6, 8) = 24, we get

$$20x - 3y = 96 \quad \text{...(ii)}$$

Now, adding Eqs. (i) and (ii), we get

$$24x = 144$$

$$\Rightarrow \quad x = 6$$

Now, put the value of x in Eq. (i), we get

$$4 \times 6 + 3y = 48$$

$$\Rightarrow \quad 3y = 48 - 24$$

$$\Rightarrow \quad 3y = 24 \Rightarrow y = 8$$

Hence, the required values of x and y are 6 and 8, respectively.

(iii) Given pair of linear equations are

$$4x + \frac{6}{y} = 15 \quad \text{...(i)}$$

and $$6x - \frac{8}{y} = 14, \; y \neq 0 \quad \text{...(ii)}$$

Let $u = \frac{1}{y}$, then above equation becomes

$$4x + 6u = 15 \quad \text{...(iii)}$$

and $$6x - 8u = 14 \quad \text{...(iv)}$$

On multiplying Eq. (iii) by 8 and Eq. (iv) by 6 and then adding both of them, we get

$$32x + 48u = 120$$

$$36x - 48u = 84 \Rightarrow 68x = 204$$

$$\Rightarrow \quad x = 3$$

Now, put the value of x in Eq. (iii), we get

$$4 \times 3 + 6u = 15$$

$$\Rightarrow \quad 6u = 15 - 12 \Rightarrow 6u = 3$$

$$\Rightarrow \quad u = \frac{1}{2} \Rightarrow \frac{1}{y} = \frac{1}{2} \quad \left[\because u = \frac{1}{y}\right]$$

$$\Rightarrow \quad y = 2$$

Hence, the required values of x and y are 3 and 2, respectively.

(iv) Given pair of linear equations is

$$\frac{1}{2x}-\frac{1}{y}=-1 \quad \text{...(i)}$$

and $$\frac{1}{x}+\frac{1}{2y}=8,\ x,y\neq 0 \quad \text{...(ii)}$$

Let $u=\frac{1}{x}$ and $v=\frac{1}{y}$, then the above equations becomes

$$\frac{u}{2}-v=-1$$

$\Rightarrow$ $$u-2v=-2 \quad \text{....(iii)}$$

and $$u+\frac{v}{2}=8$$

$\Rightarrow$ $$2u+v=16 \quad \text{...(iv)}$$

On, multiplying Eq. (iv) by 2 and then adding with Eq. (iii), we get

$$\begin{array}{r} 4u+2v=32 \\ \underline{u-2v=-2} \\ 5u=30 \end{array}$$

$\Rightarrow$ $$u=6$$

Now, put the value of u in Eq. (iv), we get

$$2\times 6+v=16$$

$\Rightarrow$ $$v=16-12=4$$

$\Rightarrow$ $$v=4$$

$\therefore$ $$x=\frac{1}{u}=\frac{1}{6} \text{ and } y=\frac{1}{v}=\frac{1}{4}$$

Hence, the required values of x and y are $\frac{1}{6}$ and $\frac{1}{4}$, respectively.

(v) Given pair of linear equations is

$$43x+67y=-24 \quad \text{...(i)}$$

and $$67x+43y=24 \quad \text{...(ii)}$$

On multiplying Eq. (i) by 43 and Eq. (ii) by 67 and then subtracting both of them, we get

$$\begin{array}{r} (67)^2x+43\times 67y=24\times 67 \\ \underline{\underset{-}{(43)^2x}+\underset{-}{43}\times 67y=\underset{+}{-24}\times 43} \\ \{(67)^2-(43)^2\}\,x=24\,(67+43) \end{array}$$

$\Rightarrow$ $$(67+43)(67-43)\,x=24\times 110 \quad [\because (a^2-b^2)=(a-b)(a+b)]$$

$\Rightarrow$ $$110\times 24\,x=24\times 110$$

$\Rightarrow$ $$x=1$$

Now, put the value of x in Eq. (i), we get

$$43\times 1+67y=-24$$

$\Rightarrow$ $$67y=-24-43$$

$\Rightarrow$ $$67y=-67$$

$\Rightarrow$ $$y=-1$$

Hence, the required values of x and y are 1 and -1, respectively.

(vi) Given pair of linear equations is

$$\frac{x}{a} + \frac{y}{b} = a + b \quad \text{...(i)}$$

and $$\frac{x}{a^2} + \frac{y}{b^2} = 2, a, b \neq 0 \quad \text{...(ii)}$$

On multiplying Eq. (i) by $\frac{1}{a}$ and then subtracting from Eq. (ii), we get

$$\frac{x}{a^2} + \frac{y}{b^2} = 2$$

$$\frac{x}{a^2} + \frac{y}{ab} = 1 + \frac{b}{a}$$

$$- \quad - \quad -$$

$$y\left(\frac{1}{b^2} - \frac{1}{ab}\right) = 2 - 1 - \frac{b}{a}$$

$\Rightarrow$ $$y\left(\frac{a-b}{ab^2}\right) = 1 - \frac{b}{a} = \left(\frac{a-b}{a}\right)$$

$\Rightarrow$ $$y = \frac{ab^2}{a} \Rightarrow y = b^2$$

Now, put the value of y in Eq. (ii), we get

$$\frac{x}{a^2} + \frac{b^2}{b^2} = 2$$

$\Rightarrow$ $$\frac{x}{a^2} = 2 - 1 = 1$$

$\Rightarrow$ $$x = a^2$$

Hence, the required values of x and y are a^2 and b^2, respectively.

(vii) Given pair of equations is

$$\frac{2xy}{x+y} = \frac{3}{2}, \text{ where } x + y \neq 0$$

$\Rightarrow$ $$\frac{x+y}{2xy} = \frac{2}{3}$$

$\Rightarrow$ $$\frac{x}{xy} + \frac{y}{xy} = \frac{4}{3}$$

$\Rightarrow$ $$\frac{1}{y} + \frac{1}{x} = \frac{4}{3} \quad \text{...(i)}$$

and $$\frac{xy}{2x-y} = \frac{-3}{10}, \text{ where } 2x - y \neq 0$$

$\Rightarrow$ $$\frac{2x-y}{xy} = \frac{-10}{3}$$

$\Rightarrow$ $$\frac{2x}{xy} - \frac{y}{xy} = \frac{-10}{3}$$

$\Rightarrow$ $$\frac{2}{y} - \frac{1}{x} = \frac{-10}{3} \quad \text{...(ii)}$$

Now, put $\frac{1}{x} = u$ and $\frac{1}{y} = v$, then the pair of equations becomes

$$v + u = \frac{4}{3} \quad \text{...(iii)}$$

and $$2v - u = \frac{-10}{3} \quad \text{...(iv)}$$

On adding both equations, we get

$$3v = \frac{4}{3} - \frac{10}{3} = \frac{-6}{3}$$

$$\Rightarrow \quad 3v = -2$$

$$\Rightarrow \quad v = \frac{-2}{3}$$

Now, put the value of v in Eq. (iii), we get

$$\frac{-2}{3} + u = \frac{4}{3}$$

$$\Rightarrow \quad u = \frac{4}{3} + \frac{2}{3} = \frac{6}{3} = 2$$

$$\therefore \quad x = \frac{1}{u} = \frac{1}{2}$$

and $$y = \frac{1}{v} = \frac{1}{(-2/3)} = \frac{-3}{2}$$

Hence, the required values of x and y are $\frac{1}{2}$ and $\frac{-3}{2}$, respectively.

Question 10:

Find the solution of the pair of equations $\frac{x}{10} + \frac{y}{5} - 1 = 0$
$\frac{x}{8} + \frac{y}{6}$= 15 and find A, if y = λx + 5.

Solution:

Given pair of equations is

$$\frac{x}{10} + \frac{y}{5} - 1 = 0 \quad \text{...(i)}$$

and $$\frac{x}{8} + \frac{y}{6} = 15 \quad \text{...(ii)}$$

Now, multiplying both sides of Eq. (i) by LCM (10, 5) = 10 , we get

$$x + 2y - 10 = 0$$

$$\Rightarrow \quad x + 2y = 10 \quad \text{...(iii)}$$

Again, multiplying both sides of Eq. (iv) by LCM (8,6) = 24, we get

$$3x + 4y = 360 \quad \text{...(iv)}$$

On, multiplying Eq. (iii) by 2 and then subtracting from Eq. (iv), we get

$$\begin{array}{r} 3x + 4y = 360 \\ \underline{\overset{-}{2x} + \overset{-}{4y} = \overset{-}{20}} \\ x = 340 \end{array}$$

Put the value of x in Eq. (iii), we get

$$340 + 2y = 10$$

$$\Rightarrow \quad 2y = 10 - 340 = -330$$

$$\Rightarrow \quad y = -165$$

Given that, the linear relation between x, y and λ is

$$y = \lambda x + 5$$

Now, put the values of x and y in above relation, we get

$$-165 = \lambda(340) + 5$$

$$\Rightarrow \quad 340\lambda = -170$$

$$\Rightarrow \quad \lambda = -\frac{1}{2}$$

Hence, the solution of the pair of equations is $x = 340, y = -165$ and the required value of λ is $-\frac{1}{2}$.

Question 11:

By the graphical method, find whether the following pair of equations are consistent or not. If consistent, solve them.

(i) 3x + y + 4 = 0,6x- 2y + 4 = 0

(ii) x — 2y — 6, 3x — 6y = 0

(iii) x + y = 3, 3x + 3y =9

Solution:

(i) Given pair of equations is

$$3x + y + 4 = 0 \quad \text{...(i)}$$

and $$6x - 2y + 4 = 0 \quad \text{...(ii)}$$

On comparing with $ax + by + c = 0$, we get

$$a_1 = 3, b_1 = 1$$

and $$c_1 = 4 \quad \text{[from Eq. (i)]}$$

$$a_2 = 6, b_2 = -2$$

and $$c_2 = 4 \quad \text{[from Eq. (ii)]}$$

Here, $$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}; \frac{b_1}{b_2} = \frac{1}{-2}$$

and $$\frac{c_1}{c_2} = \frac{4}{4} = \frac{1}{1}$$

$\because$ $$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

So, the given pair of linear equations are intersecting at one point, therefore these lines have unique solution.

Hence, given pair of linear equations is consistent.

We have, $$3x + y + 4 = 0$$

$\Rightarrow$ $$y = -4 - 3x$$

When $x = 0$, then $y = -4$

When $x = -1$, then $y = -1$

When $x = -2$, then $y = 2$

x	0	-1	-2
y	-4	-1	2
Points	B	C	A

and $$6x - 2y + 4 = 0$$

$\Rightarrow$ $$2y = 6x + 4$$

$\Rightarrow$ $$y = 3x + 2$$

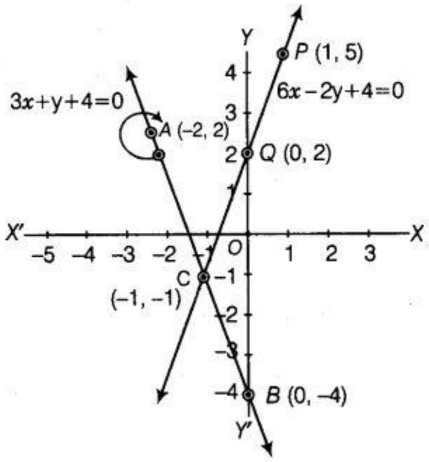
When $x = 0$, then $y = 2$

When $x = -1$, then $y = -1$

When $x = 1$, then $y = 5$

x	-1	0	1
y	-1	2	5
Points	C	Q	P

Plotting the points $B(0, -4)$ and $A(-2, 2)$, we get the straight tine AB. Plotting the points Q $(0, 2)$ and P $(1, 5)$, we get the straight line PQ. The lines AB and PQ intersect at C $(-1, -1)$.



(ii) Given pair of equations is $x - 2y = 6$ ...(i)

and $3x - 6y = 0$ ...(ii)

On comparing with $ax + by + c = 0$, we get

$a_1 = 1, b_1 = -2$ and $c_1 = -6$ [from Eq. (i)]

$a_2 = 3, b_2 = -6$ and $c_2 = 0$ [from Eq. (ii)]

Here, $\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} \frac{-2}{-6} = \frac{1}{3}$ and $\frac{c_1}{c_2} = \frac{-6}{0}$

$\therefore$ $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the lines represented by the given equations are parallel. Therefore, it has no solution. So, the given pair of lines is inconsistent.

(iii) Given pair of equations is $x + y = 3$ ...(i)

and $3x + 3y = 9$ ...(ii)

On comparing with $ax + by + c = 0$, we get

$a_1 = 1, b_1 = 1$ and $c_1 = -3$ [from Eq. (i)]

$a_2 = 3, b_2 = 3$ and $c_2 = -9$ [from Eq. (ii)]

Here, $\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{1}{3}$ and $\frac{c_1}{c_2} = \frac{-3}{-9} = \frac{1}{3}$

$\Rightarrow$ $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, the given pair of lines is coincident. Therefore, these lines have infinitely many solutions. Hence, the given pair of linear equations is consistent.

Now, $x + y = 3 \Rightarrow y = 3 - x$

If $x = 0$, then $y = 3$, If $x = 3$, then $y = 0$

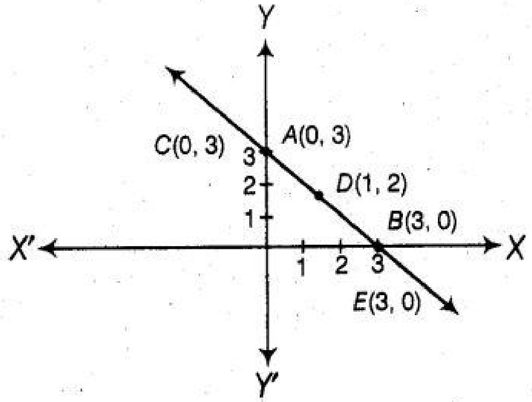
x	0	3
y	3	0
Points	A	B

and $3x + 3y = 9 \Rightarrow 3y = 9 - 3x$

$\Rightarrow$ $y = \frac{9 - 3x}{3}$

If $x = 0$, then $y = 3$; if $x = 1$, then $y = 2$ and if $x = 3$, then $y = 0$

x	0	1	3
y	3	2	0
Points	C	D	E



Plotting the points A (0,3) and B (3,0), we get the line AB. Again, plotting the points C (0,3) D (1, 2) and E (3,0), we get the line CDE.
We observe that the lines represented by Eqs. (i) and (ii) are coincident.

Question 12:

Draw the graph of the pair of equations 2x + y= 4 and 2x – y = 4. Write the vertices of the triangle formed by these lines and the y-axis, find the area of this triangle? '

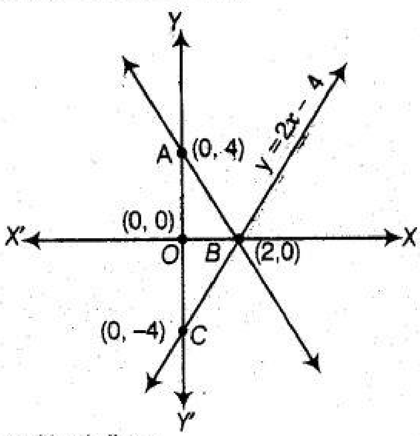
Solution:

The given pair of linear equations
Table for line $2x + y = 4$,

x	0	2
$y = 4 - 2x$	4	0
Points	A	B

and table for line $2x - y = 4$,

x	0	2
$y = 2x - 4$	-4	0
Points	C	B



Graphical representation of both lines.
Here, both lines and Y-axis form a ΔABC.

Hence, the vertices of a ΔABC are $A(0, 4)$ B (2, 0) and C (0, – 4).

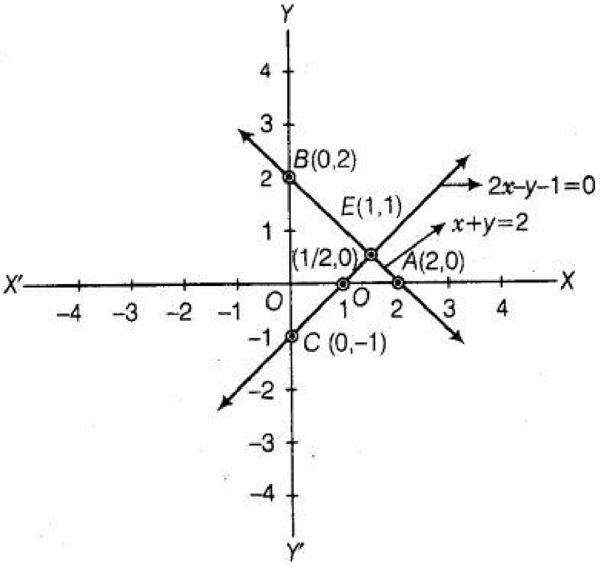
$\therefore$ Required area of $\Delta ABC = 2 \times$ Area of ΔAOB

$$= 2 \times \frac{1}{2} \times 4 \times 2 = 8 \text{ sq units}$$

Hence, the required area of the triangle is 8 sq units.

If $x = 0$, then $y = -1$; if $x = \frac{1}{2}$, then $y = 0$ and if $x = 1$, then $y = 1$

x	0	1/2	1
y	-1	0	1
Points	C	D	E



Question 13:

Write an equation of a Line passing through the point representing solution of the pair of Linear equations x + y = 2 and 2x – y = 1, How many such lines can we find?

Solution:

Plotting the points A (2,0) and B (0, 2), we get the straight line AB. Plotting the points C (0, –1)andD (1/2, 0), we get the straight line CD. The lines AB and CD intersect at E(1,1), Hence, infinite lines can pass through the intersection point of linear equations x + y = 2 and 2x – y = 1 i.e., E(1,1) like as y = x, 2x + y = 3, x + 2y = 3. so on.

Question 14:

If (x + 1) is a factor of $2x^3 + ax^2 + 2bx+l$, then find the value of a and b given that 2a – 3b = 4.

Solution:

Given that, (x + 1) is a factor of $f(x) = 2x^s + ax^2 + 2bx + 1$, then f(-1) = 0.

[if (x + a) is a factor of $f(x) = ax^2 + bx + c$, then f(-) = 0]

$$\Rightarrow \quad 2(-1)^3 + a(-1)^2 + 2b(-1) + 1 = 0$$
$$\Rightarrow \quad -2 + a - 2b + 1 = 0$$
$$\Rightarrow \quad a - 2b - 1 = 0 \qquad \ldots(i)$$

Also,
$$2a - 3b = 4$$
$$\Rightarrow \quad 3b = 2a - 4$$
$$\Rightarrow \quad b = \left(\frac{2a-4}{3}\right)$$

Now, put the value of b in Eq. (i), we get

$$a - 2\left(\frac{2a-4}{3}\right) - 1 = 0$$
$$\Rightarrow \quad 3a - 2(2a - 4) - 3 = 0$$
$$\Rightarrow \quad 3a - 4a + 8 - 3 = 0$$
$$\Rightarrow \quad -a + 5 = 0$$
$$\Rightarrow \quad a = 5$$

Now, put the value of a in Eq. (i), we get

$$5 - 2b - 1 = 0$$
$$\Rightarrow \quad 2b = 4$$
$$\Rightarrow \quad b = 2$$

Hence, the required values of a and b are 5 and 2, respectively.

Question 15:

If the angles of a triangle are x, y and 40° and the difference between the two angles x and y is 30°. Then, find the value of x and y,

Solution:

Given that, x, y and 40° are the angles of a triangle.

$x + y + 40° = 180°$

[since, the sum of all the angles of a triangle is 180°]

$\Rightarrow \quad x + y = 140° \quad \ldots(i)$

Also, $\quad x - y = 30° \quad \ldots(ii)$

On adding Eqs. (i) and (ii), we get

$$2x = 170°$$

$\Rightarrow \quad x = 85°$

On putting $x = 85°$ in Eq. (i), we get

$$85° + y = 140°$$

$\Rightarrow \quad y = 55°$

Hence, the required values of x and y are 85° and 55°, respectively.

Question 16:

Two years ago, Salim was thrice as old as his daughter and six years later, he will be four year older than twice her age. How old are they now?

Solution:

Let Salim and his daughter's age be x and y yr respectively.

Now, by first condition

Two years ago, Salim was thrice as old as his daughter.

i.e., $\quad x - 2 = 3(y - 2) \Rightarrow x - 2 = 3y - 6$

$\Rightarrow \quad x - 3y = -4 \quad \ldots(i)$

and by second condition, six years later, Salim will be four years older than twice her age.

$$x + 6 = 2(y + 6) + 4$$

$\Rightarrow \quad x + 6 = 2y + 12 + 4$

$\Rightarrow \quad x - 2y = 16 - 6$

$\Rightarrow \quad x - 2y = 10 \quad \ldots(ii)$

On subtracting Eq. (i) from Eq. (ii), we get

$$\begin{array}{r} x - 2y = 10 \\ x - 3y = -4 \\ - \quad + \quad\quad + \\ \hline y = 14 \end{array}$$

Put the value of y in Eq. (ii), we get

$$x - 2 \times 14 = 10$$

$\Rightarrow \quad x = 10 + 28 \Rightarrow x = 38$

Hence, Salim and his daughter's age are 38 yr and 14 yr, respectively.

Question 17:

The age of the father is twice the sum of the ages of his two children. After 20 yr, his age will be equal to the sum of the ages of his children. Find the age of the father.

Solution:

Let the present age (in year) of father and his two children be x, y and z yr, respectively.

Now by given condition, $\quad x=2(y+z) \quad \ldots(i)$

and after 20 yr, $\quad (x + 20) = (y + 20) + (z + 20)$

$\Rightarrow \quad y+z +40 = x +20$

$\Rightarrow \quad y + z = x - 20$

On putting the value of (y + z) in Eq. (i) and get the present age of father

$x =2(x -20)$ x

$= 2x - 40 = 40$

Hence, the father's age is 40 yr.

Question 18:

Two numbers are in the ratio 5 : 6. If 8 is subtracted from each of the numbers, the ratio becomes 4 : 5, then find the numbers. .

Solution:

Let the two numbers be x and y.

Then, by first Condition, ratio of these two numbers = 5:6

$$x : y = 5 : 6$$

$$\Rightarrow \quad \frac{x}{y} = \frac{5}{6} \Rightarrow y = \frac{6x}{5} \quad \text{...(i)}$$

and by second condition, then, 8 is subtracted from each of the numbers, then ratio becomes 4 : 5.

$$\frac{x-8}{y-8} = \frac{4}{5}$$

$$\Rightarrow \quad 5x - 40 = 4y - 32$$

$$\Rightarrow \quad 5x - 4y = 8 \quad \text{...(ii)}$$

Now, put the value of y in Eq. (ii), we get

$$5x - 4\left(\frac{6x}{5}\right) = 8$$

$$\Rightarrow \quad 25x - 24x = 40$$

$$\Rightarrow \quad x = 40$$

Put the value of x in Eq. (i), we get

$$y = \frac{6}{5} \times 40$$

$$= 6 \times 8 = 48$$

Hence, the required numbers are 40 and 48.

Question 19:

There are some students in the two examination halls A and B. To make the number of students equal in each hall, 10 students are sent from A to B but, if 20 students are sent from B to A, the number of students in A becomes double the number of students in B, then find the number of students in the both halls.

Solution:

Let the number of students in halls A and 8 are x and y, respectively.

Now, by given condition, $x-10=y+10$

$\Rightarrow$ $x - y = 20$ … (i)

and $(x + 20) = 2 (y-20)$

$\Rightarrow$ $x-2y=-60$ …(ii)

On subtracting Eq. (ii) from Eq. (i), we get

(x-y)-(x- 2 y) = 20+60 „ x-y-x + 2y~ 80 => y= 80

On putting y = 80 in Eq. (i), we get

x – 80 = 20 => x = 100 and y = 80

Hence, 100 students are in hall A and 80 students are in hall 8.

Question 20:

A shopkeeper gives books on rent for reading. She takes a fixed charge for the first two days and an additional charge for each day thereafter. Latika paid ₹ 22 for a book kept for six days, while Anand paid ₹ 16 for the book kept for four days. Find the fixed charges and the charge for each extra day.

Solution:

Let Latika takes a fixed charge for the first two day is ₹ x and additional charge for each day thereafter is ₹ y.

Now by first condition.

Latika paid ₹ 22 for a book kept for six days i.e.,

$x + 4y = 22$ …(i)

and by second condition,

Anand paid ₹ 16 for a book kept for four days i.e.,

$x+2y=16$ …(ii)

Now, subtracting Eq. (ii) from Eq. (i), we get

$2y=6 \Rightarrow y = 3$

On putting the value of y in Eq. (ii), we get

$x + 2 \times 3 = 16$

$x = 16-6 = 10$

Hence, the fixed charge = ₹ 10

and the charge for each extra day = ₹ 3

Question 21:

In a competitive examination, 1 mark is awarded for each correct answer while 1/2 mark is deducted for every wrong answer. Jayanti answered 120 questions and got 90 marks. How many questions did she answer correctly?

Solution:

Let x be the number of correct answers of the questions in a competitive examination, then (120 – x) be the number of wrong answers of the questions.

Then, by given condition,

$$x \times 1 - (120 - x) \times \frac{1}{2} = 90$$

$$\Rightarrow \quad x - 60 + \frac{x}{2} = 90$$

$$\Rightarrow \quad \frac{3x}{2} = 150$$

$$\therefore \quad x = \frac{150 \times 2}{3} = 50 \times 2 = 100$$

Hence, Jayanti answered correctly 100 questions.

Question 22:

The angles of a cyclic quadrilateral ABCD are ∠A – (6x +10)°, ∠B = (5x)°, ∠C = (x+ y)° and ∠D = (3y – 10)°.Find x and y and hence the values of the four angles.

Solution:

We know that, by property of cyclic quadrilateral,

Sum of opposite angles = 180°

∠A + ∠C = (6x + 10)° + (x + y)° = 180°

$$[\because \angle A = (6x + 10)°, \angle C = (x + y)°, \text{ given}]$$

$$\Rightarrow \quad 7x + y = 170 \quad \ldots(i)$$

and $$\angle B + \angle D = (5x)° + (3y - 10)° = 180°$$

$$[\because \angle B = (5x)°, \angle D = (3y - 10)°, \text{ given}]$$

$$\Rightarrow \quad 5x + 3y = 190° \quad \ldots(ii)$$

On multiplying Eq. (i) by 3 and then subtracting, we get

$$3 \times (7x + y) - (5x + 3y) = 510° - 190°$$

$$\Rightarrow \quad 21x + 3y - 5x - 3y = 320°$$

$$\Rightarrow \quad 16x = 320°$$

$$\therefore \quad x = 20°$$

On putting $x = 20°$ in Eq. (i), we get

$$7 \times 20 + y = 170°$$

$$\Rightarrow \quad y = 170° - 140° \Rightarrow y = 30°$$

$$\therefore \quad \angle A = (6x + 10)° = 6 \times 20° + 10°$$

$$= 120° + 10° = 130°$$

$$\angle B = (5x)° = 5 \times 20° = 100°$$

$$\angle C = (x + y)° = 20° + 30° = 50°$$

$$\angle D = (3y - 10)° = 3 \times 30° - 10°$$

$$= 90° - 10° = 80°$$

Hence, the required values of x and y are 20° and 30° respectively and the values of the four angles ;.e., ZA, ZB, ZC and ZD are 130°, 100°, 50° and 80°, respectively.

Exercise 3.4 Long Answer Type Questions

Question 1:

Graphically, solve the following pair of equations 2x + y = 6 and 2x – y + 2 = 0
Find the ratio of the areas of the two triangles formed by the lines representing these equations with the X-axis and the lines with the Y-axis.

Solution:

Given equations are $2x + y = 6$ and $2x - y + 2 = 0$

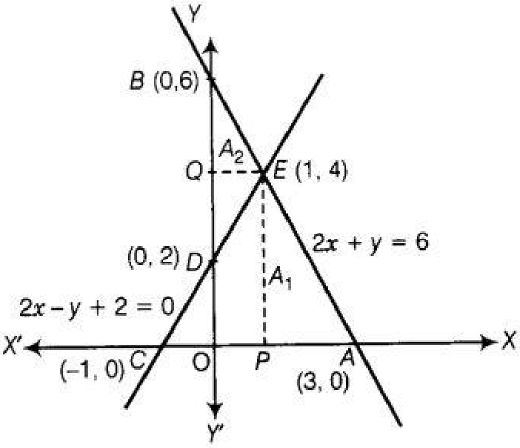
Table for equation $2x + y = 6$,

x	0	3
y	6	0
Points	B	A

Table for equation $2x - y + 2 = 0$,

x	0	−1
y	2	0
Points	D	C

Let A_1 and A_2 represent the areas of ΔACE and ΔBDE, respectively.



Now, A_1 = Area of $\Delta ACE = \frac{1}{2} \times AC \times PE$

$= \frac{1}{2} \times 4 \times 4 = 8$

and A_2 = Area of $\Delta BDE = \frac{1}{2} \times BD \times QE$

$= \frac{1}{2} \times 4 \times 1 = 2$

$\therefore A_1 : A_2 = 8 : 2 = 4 : 1$

Hence, the pair of equations intersect graphically at point $E(1, 4)$, *i.e.*, $x = 1$ and $y = 4$.

Question 2:

Determine graphically, the vertices of the triangle formed by the lines
y = x, 3y = x and x + y = 8

Solution:

Given linear equations are

$$y = x \quad \text{...(i)}$$
$$3y = x \quad \text{...(ii)}$$
and
$$x + y = 8 \quad \text{...(iii)}$$

For equation $y = x$,
If $x = 1$, then $y = 1$
If $x = 0$, then $y = 0$
If $x = 2$, then $y = 2$
Table for line $y = x$,

x	0	1	2
y	0	1	2
Points	O	A	B

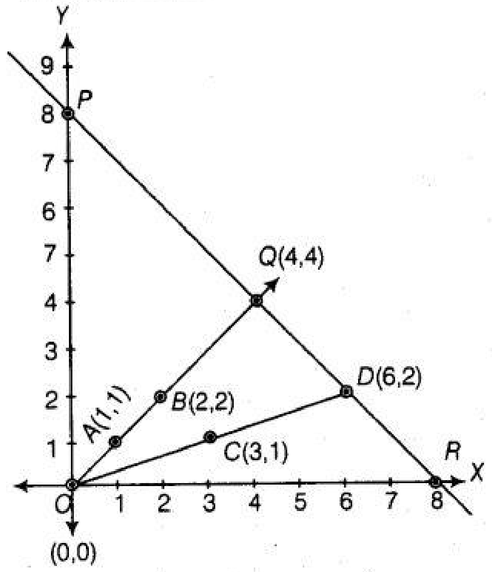
For equation $x = 3y$,
If $x = 0$, then $y = 0$; if $x = 3$, then $y = 1$ and if $x = 6$, then $y = 2$
Table for line $x = 3y$,

x	0	3	6
y	0	1	2
Points	O	C	D

For equation $x + y = 8 \Rightarrow y = 8 - x$
If $x = 0$, then $y = 8$; if $x = 8$, then $y = 0$ and if $x = 4$, then $y = 4$
Table for line $x + y = 8$,.

For equation $x + y = 8 \Rightarrow y = 8 - x$
If $x = 0$, then $y = 8$; if $x = 8$, then $y = 0$ and if $x = 4$, then $y = 4$
Table for line $x + y = 8$,.

x	0	4	8
y	8	4	0
Points	P	Q	R



Plotting the points A(1,1) and 6(2,2), we get the straight line AB. Plotting the points C(3,1) and 0(6,2), we get the straight line CD. Plotting the points P(0, 8), Q(4, 4) and R{8, 0), we get the straight line PQR. We see that lines AB and CD intersecting the line PR on Q and D, respectively.

So, AOQD is formed by these lines. Hence, the vertices of the A00D formed by the given lines are0(0, 0),Q(4, 4)and 0(6,2).

Question 3:

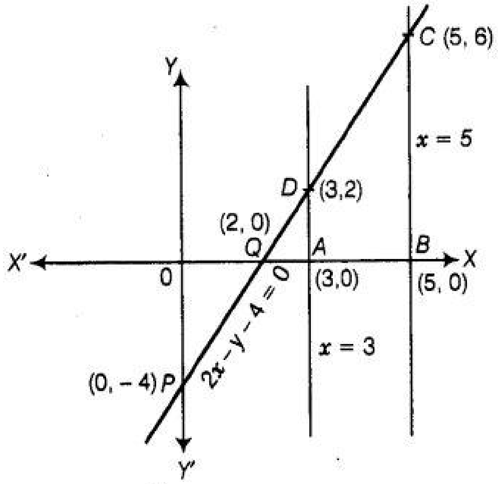
Draw the graphs of the equations x = 3, x = 5 and 2x – y – 4 = 0. Also find the area of the quadrilateral formed by the lines and the X-axis.

Solution:

Given equation of lines 2x – y – 4 = 0, x = 3 and x = 5 Table for line 2x – y – 4 = 0 Also find the area of the quadrilateral formed by the lines and the X-axis.

x	0	2
$y = 2x - 4$	-4	0
Points	P	Q

Draw the points $P(0, -4)$ and $Q(2, 0)$ and join these points and form a line PQ also draw the lines $x = 3$ and $x = 5$.



$\therefore$ Area of quadrilateral $ABCD = \frac{1}{2} \times$ distance between parallel lines $(AB) \times (AD + BC)$

[since, quadrilateral $ABCD$ is a trapezium]

$= \frac{1}{2} \times 2 \times (6 + 2)$

$[\because AB = OB - OA = 5 - 3 = 2, AD = 2$ and $BC = 6]$

$= 8$ sq units

Hence, the required area of the quadrilateral formed by the lines and the X-axis is 8 sq units.

Question 4:

The cost of 4 pens and 4 pencils boxes is 1100. Three times the cost of a pen is ₹ 15 more than the cost of a pencil box. Form the pair of linear equations for the above situation. Find the cost of a pen and a pencil box.

Solution:

Let the cost of a pen be ₹ x and the cost of a pencil box be ₹ y.

Then, by given condition,

$$4x + 4y = 100 \Rightarrow x + y = 25 \quad \ldots(i)$$

and $$3x = y + 15$$

$\Rightarrow$ $$3x - y = 15 \quad \ldots(ii)$$

On adding Eqs. (i) and (ii), we get

$$4x = 40$$

$\Rightarrow$ $$x = 10$$

By substituting $x = 10$, in Eq. (i) we get

$$y = 25 - 10 = 15$$

Hence, the cost of a pen and a pencil box are ₹ 10 and ₹ 15, respectively.

Question 5:

Determine, algebraically, the vertices of the triangle formed by the lines

3 x – y = 3

2x – 3y = 2

and x + 2y = 8

Solution:

Given equation of lines are

3 x – y = 3 …………..(i)

2x – 3y = 2 ………….(ii)

and x + 2y = 8 ………….(iii)

Let lines (i), (ii) and (iii) represent the sides of a ΔABC i.e., AB, BC and CA, respectively. On solving lines (i) and (ii), we will get the intersecting point B.

On multiplying Eq. (i) by 3 in Eq. (i) and then subtracting, we get

$$\begin{array}{r} 9x - 3y = 9 \\ \underline{\underset{-}{2x} \underset{+}{-\,3y} = \underset{-}{2}} \\ 7x = 7 \Rightarrow x = 1 \end{array}$$

On putting the value of x in Eq. (i), we get

$$3 \times 1 - y = 3$$

$\Rightarrow$ $y = 0$

So, the coordinate of point or vertex B is (1, 0).

On solving lines (ii) and (iii), we will get the intersecting point C.

On multiplying Eq. (iii) by 2 and then subtracting, we get

$$\begin{array}{r} 2x + 4y = 16 \\ \underline{\underset{-}{2x} \underset{+}{-\,3y} = \underset{-}{2}} \\ 7y = 14 \end{array}$$

$\Rightarrow$ $y = 2$

On putting the value of y in Eq. (iii), we gat

$$x + 2 \times 2 = 8$$

$\Rightarrow$ $x = 8 - 4$

$\Rightarrow$ $x = 4$

Hence, the coordinate of point or vertex C is (4, 2).

On solving lines (iii) and (i), we will get the intersecting point A.

On multiplying in Eq. (i) by 2 and then adding Eq. (iii), we get

$$\begin{array}{r} 6x - 2y = 6 \\ \underline{x + 2y = 8} \\ 7x = 14 \end{array}$$

$\Rightarrow$ $x = 2$

On putting the value of x in Eq. (i), we get

$$3 \times 2 - y = 3$$

$\Rightarrow$ $y = 6 - 3$

$\Rightarrow$ $y = 3$

So, the coordinate of point or vertex A is (2, 3).

Hence, the vertices of the ΔABC formed by the given lines are A (2, 3), B (1, 0) and C(4,2).

Question 6:

Ankita travels 14 km to her home partly by rickshaw and partly by bus. She takes half an hour, if she travels 2 km by rickshaw and the remaining distance by bus. On the other hand, if she travels 4 km by rickshaw and the remaining distance by bus, she takes 9 min longer. Find the speed of the rickshaw and of the bus.

Solution:

Let the speed of the rickshaw and the bus are x and y km/h, respectively.

Now, she has taken time to travel 2 km by rickshaw, $t_1 = \frac{2}{x}$ h. $\left[\because \text{speed} = \frac{\text{distance}}{\text{time}}\right]$

and she has taken time to travel remaining distance *i.e.*, $(14 - 2) = 12$ km by bus $= t_2 = \frac{12}{y}$ h.

By first condition, $t_1 + t_2 = \frac{1}{2} \Rightarrow \frac{2}{x} + \frac{12}{y} = \frac{1}{2}$ ...(i)

Now, she has taken time to travel 4 km by rickshaw, $t_3 = \frac{4}{x}$ h

and she has taken time to travel remaining distance *i.e.*, $(14 - 4) = 10$ km by bus $= t_4 = \frac{10}{y}$ h.

By second condition, $t_3 + t_4 = \frac{1}{2} + \frac{9}{60} = \frac{1}{2} + \frac{3}{20}$

$\Rightarrow \frac{4}{x} + \frac{10}{y} = \frac{13}{20}$ ...(ii)

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$, then Eqs. (i) and (ii) becomes

$$2u + 12v = \frac{1}{2} \quad \text{...(iii)}$$

and $$4u + 10v = \frac{13}{20} \quad \text{...(iv)}$$

On multiplying in Eq. (iii) by 2 and then subtracting, we get

$$4u + 24v = 1$$
$$4u + 10v = \frac{13}{20}$$
$$- \quad - \quad -$$
$$14v = 1 - \frac{13}{20} = \frac{7}{20}$$

$$\Rightarrow \quad 2v = \frac{1}{20} \Rightarrow v = \frac{1}{40}$$

Now, put the value of v in Eq. (iii), we get

$$2u + 12\left(\frac{1}{40}\right) = \frac{1}{2}$$

$$\Rightarrow \quad 2u = \frac{1}{2} - \frac{3}{10} = \frac{5-3}{10}$$

$$\Rightarrow \quad 2u = \frac{2}{10} \Rightarrow u = \frac{1}{10}$$

$$\therefore \quad \frac{1}{x} = u$$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{10} \Rightarrow x = 10 \text{ km/h}$$

and $$\frac{1}{y} = v \Rightarrow \frac{1}{y} = \frac{1}{40}$$

$$\Rightarrow \quad y = 40 \text{ km/h}$$

Hence, the speed of rickshaw and the bus are 10 km/h and 40 km/h, respectively.

Question 7:

A person,rowing at the rate of 5 km/h in still water,takes thrice as much time in going 40 km upstream as in going 40 km downstream. Find the speed of the stream.

Solution:

Let the speed of the stream be v km/h.

Given that, a person rowing in still water = 5 km/h

The speed of a person rowing in downstream = (5+ v) km/h

and the speed of a person has rowing in upstream = (5 – v) km/h

Now, the person taken time to cover 40 km downstream, Now, the person taken time to cover 40 km downstream,

$$t_1 = \frac{40}{5+v} \text{ h} \qquad \left[\because \text{speed} = \frac{\text{distance}}{\text{time}}\right]$$

and the person has taken time to cover 40 km upstream,

$$t_2 = \frac{40}{5-v} \text{ h.}$$

By condition, $t_2 = t_1 \times 3$

$$\Rightarrow \quad \frac{40}{5-v} = \frac{40}{5+v} \times 3$$

$$\Rightarrow \quad \frac{1}{5-v} = \frac{3}{5+v}$$

$$\Rightarrow \quad 5 + v = 15 - 3v \Rightarrow 4v = 10$$

$$\therefore \quad v = \frac{10}{4} = 2.5 \text{ km/h}$$

Hence, the speed of the stream is 2.5 km/h,

Question 8:

A motorboat can travel 30 km upstream and 28 km downstream in 7 h. It can travel 21 km upstream and return in 5 h. Find the speed of the boat in still water and the speed of the stream.

Solution:

Let the speed of the motorboat in still water and the speed of the stream are u km/h and v km/h, respectively.

Then, a motorboat speed in downstream = (u + v) km/h and a motorboat speed in upstream = (u -v) km/h.

Motorboat has taken time to travel 30 km upstream,

$$t_1 = \frac{30}{u-v} \text{ h}$$

and motorboat has taken time to travel 28 km downstream,

$$t_2 = \frac{28}{u+v} \text{ h}$$

By first condition, a motorboat can travel 30 km upstream and 28 km downstream in 7 h

i.e., $t_1 + t_2 = 7\text{h}$

$$\Rightarrow \quad \frac{30}{u-v} + \frac{28}{u+v} = 7 \qquad \text{...(i)}$$

Now, motorboat has taken time to travel 21 km upstream and return *i.e.*, $t_3 = \frac{21}{u-v}$. [for upstream]

and $t_4 = \frac{21}{u+v}$ [for downstream]

By second condition, $t_4 + t_3 = 5\text{ h}$

$$\Rightarrow \quad \frac{21}{u+v} + \frac{21}{u-v} = 5 \qquad \text{...(ii)}$$

Let $$x = \frac{1}{u+v} \text{ and } y = \frac{1}{u-v}$$

Eqs. (i) and (ii) becomes $30x + 28y = 7$ ...(iii)

and $21x + 21y = 5$

$\Rightarrow$ $x + y = \frac{5}{21}$ ...(iv)

Now, multiplying in Eq. (iv) by 28 and then subtracting from Eq. (iii), we get

$$30x + 28y = 7$$
$$28x + 28y = \frac{140}{21}$$
$$\underline{- \quad - \quad -}$$
$$2x = 7 - \frac{20}{3} = \frac{21-20}{3}$$

$\Rightarrow$ $2x = \frac{1}{3} \Rightarrow x = \frac{1}{6}$

On putting the value of x in Eq. (iv), we get

$$\frac{1}{6} + y = \frac{5}{21}$$

$\Rightarrow$ $y = \frac{5}{21} - \frac{1}{6} = \frac{10-7}{42} = \frac{3}{42} \Rightarrow y = \frac{1}{14}$

$\therefore$ $x = \frac{1}{u+v} = \frac{1}{6} \Rightarrow u + v = 6$ ...(v)

and $y = \frac{1}{u-v} = \frac{1}{14}$

$\Rightarrow$ $u - v = 14$ ... (vi)

Now, adding Eqs. (v) and (vi), we get

$$2u = 20 \Rightarrow u = 10$$

On putting the value of u in Eq. (v), we get

$$10 + v = 6$$

$\Rightarrow$ $v = -4$

Hence, the speed of the motorboat in still water is 10 km/h and the speed of the stream 4 km/h.

Question 9:

A two-digit number is obtained by either multiplying the sum of the digits by 8 and then subtracting 5 or by multiplying the difference of the digits by 16 and then adding 3. Find the number.

Solution:

Let the two-digit number $= 10x + y$

***Case* I** Multiplying the sum of the digits by 8 and then subtracting 5 = two-digit number

$\Rightarrow$ $8 \times (x + y) - 5 = 10x + y$

$\Rightarrow$ $8x + 8y - 5 = 10x + y$

$\Rightarrow$ $2x - 7y = -5$ ...(i)

***Case* II** Multiplying the difference of the digits by 16 and then adding 3 = two-digit number

$\Rightarrow$ $16 \times (x - y) + 3 = 10x + y$

$\Rightarrow$ $16x - 16y + 3 = 10x + y$

$\Rightarrow$ $6x - 17y = -3$ ...(ii)

Now, multiplying in Eq. (i) by 3 and then subtracting from Eq. (ii), we get

$$6x - 17y = -3$$
$$6x - 21y = -15$$
$$\underline{- \quad + \quad +}$$
$$4y = 12 \Rightarrow y = 3$$

Now, put the value of y in Eq. (i), we get

$$2x - 7 \times 3 = -5$$

$\Rightarrow$ $2x = 21 - 5 = 16 \Rightarrow x = 8$

Hence, the required two-digit number

$$= 10x + y$$
$$= 10 \times 8 + 3 = 80 + 3 = 83$$

Question 10:

A railway half ticket cost half the full fare but the reservation charges are the same on a half ticket as on a full ticket. One reserved first class ticket from the stations A to B costs ₹ 2530. Also, one reserved first class ticket and one reserved first class half ticket from stations A to B costs ₹ 3810. Find the full first class fare from stations A to B and also the reservation charges for a ticket.

Solution:

Let the cost of full and half first class fare be ₹ $\frac{X}{2}$ and ₹ respectively and reservation charges be ₹ y per ticket.

Case I The cost of one reserved first class ticket from the stations A to B

$$= ₹\,2530$$

$$\Rightarrow \quad x + y = 2530 \quad \ldots(i)$$

Case II The cost of one reserved first class ticket and one reserved first class half ticket from stations A to B = ₹ 3810

$$\Rightarrow \quad x + y + \frac{x}{2} + y = 3810$$

$$\Rightarrow \quad \frac{3x}{2} + 2y = 3810$$

$$\Rightarrow \quad 3x + 4y = 7620 \quad \ldots(ii)$$

Now, multiplying Eq. (i) by 4 and then subtracting from Eq. (ii), we get

$$\begin{array}{r} 3x + 4y = 7620 \\ 4x + 4y = 10120 \\ - \quad - \quad\quad - \\ \hline -x = -2500 \end{array}$$

$$\Rightarrow \quad x = 2500$$

On putting the value of x in Eq. (i), we get

$$2500 + y = 2530$$

$$\Rightarrow \quad y = 2530 - 2500$$

$$\Rightarrow \quad y = 30$$

Hence, full first class fare from stations A to 6 is ₹ 2500 and the reservation for a ticket is ₹30.

Question 11:

A shopkeeper sells a saree at 8% profit and a sweater at 10% discount, thereby, getting a sum ₹ 1008. If she had sold the saree at 10% profit and the sweater at 8% discount, she would have got ₹ 1028 then find the cost of the saree and the list price (price before discount) Of the sweater.

Solution:

Let the cost price of the saree and the list price of the sweater be ₹ x and ₹ y, respectively.

Case I Sells a saree at 8% profit + Sells a sweater at 10% discount = ₹ 1008

$$\Rightarrow \quad (100 + 8)\% \text{ of } x + (100 - 10)\% \text{ of } y = 1008$$

$$\Rightarrow \quad 108\% \text{ of } x + 90\% \text{ of } y = 1008$$

$$\Rightarrow \quad 1.08x + 0.9y = 1008 \quad \ldots(i)$$

Case II Sold the saree at 10% profit + Sold the sweater at 8% discount = ₹ 1028

$$\Rightarrow \quad (100+10)\% \text{ of } x + (100-8)\% \text{ of } y = 1028$$

$$\Rightarrow \quad 110\% \text{ of } x + 92\% \text{ of } y = 1028$$

$$\Rightarrow \quad 1.1x + 0.92y = 1028 \quad \ldots\text{(ii)}$$

On putting the value of y from Eq. (i) into Eq. (ii), we get

$$1.1x + 0.92\left(\frac{1008 - 1.08x}{0.9}\right) = 1028$$

$$\Rightarrow \quad 1.1 \times 0.9x + 927.36 - 0.9936x = 1028 \times 0.9$$

$$\Rightarrow \quad 0.99x - 0.9936x = 9252 - 927.36$$

$$\Rightarrow \quad -0.0036x = -2.16$$

$$\therefore \quad x = \frac{2.16}{0.0036} = 600$$

On putting the value of x in Eq. (i), we get

$$1.08 \times 600 + 0.9y = 1008$$

$$\Rightarrow \quad 108 \times 6 + 0.9y = 1008$$

$$\Rightarrow \quad 0.9y = 1008 - 648$$

$$\Rightarrow \quad 0.9y = 360$$

$$\Rightarrow \quad y = \frac{360}{0.9} = 400$$

Hence, the cost price of the saree and the list price (price before discount) of the sweater are ₹ 600 and ₹ 400, respectively.

Question 12:

Susan invested certain amount of money in two schemes A and B, which offer interest at the rate of 8% per annum and 9% per annum, respectively. She received ₹ 1860 as annual interest. However, had she interchanged the amount of investments in the two schemes, she would have received ₹ 20 more as annual interest. How much money did she invest in each scheme?

Solution:

Let the amount of investments in schemes A and 6 be ₹ x and ₹ y, respectively.

Case I Interest at the rate of 8% per annum on scheme A+ Interest at the rate of 9% per annum on scheme 6 = Total amount received

$$\Rightarrow \quad \frac{x \times 8 \times 1}{100} + \frac{y \times 9 \times 1}{100} = ₹\,1860 \quad \left[\because \text{simple interest} = \frac{\text{principal} \times \text{rate} \times \text{time}}{100}\right]$$

$$\Rightarrow \quad 8x + 9y = 186000 \quad \ldots\text{(i)}$$

Case II Interest at the rate of 9% per annum on scheme A + Interest at the rate of 8% per annum on scheme B = ₹ 20 more as annual interest

$$\Rightarrow \quad \frac{x \times 9 \times 1}{100} + \frac{y \times 8 \times 1}{100} = ₹\,20 + ₹\,1860$$

$$\Rightarrow \quad \frac{9x}{100} + \frac{8y}{100} = 1880$$

$$\Rightarrow \quad 9x + 8y = 188000 \quad \ldots\text{(ii)}$$

On multiplying Eq. (i) by 9 and Eq. (ii) by 8 and then subtracting them, we get

$$72x + 81y = 9 \times 186000$$

$$\underline{\underset{-}{72x} \underset{-}{+} 64y = \underset{-}{8} \times 188000}$$

$$\Rightarrow \quad 17y = 1000\,[(9 \times 186) - (8 \times 188)]$$

$$= 1000\,(1674 - 1504) = 1000 \times 170$$

$$17y = 170000 \Rightarrow y = 10000$$

On putting the value of y in Eq. (i), we get

$$8x + 9 \times 10000 = 186000$$

$$\Rightarrow \quad 8x = 186000 - 90000$$

$$\Rightarrow \quad 8x = 96000$$

$$\Rightarrow \quad x = 12000$$

On putting the value of y in Eq. (i), we get

$$8x + 9 \times 10000 = 186000$$

$$\Rightarrow \quad 8x = 186000 - 90000$$

$$\Rightarrow \quad 8x = 96000$$

$$\Rightarrow \quad x = 12000$$

Question 13:

Vijay had some bananas and he divided them into two lots A and B. He sold the first lot at the rate of ₹ 2 for 3 bananas and the second lot at the rate of ₹ 1 per banana and got a total of ₹ 400 If he had sold the first lot at the rate of ₹ 1 per banana and the second lot at the rate of ₹ 4 for 5 bananas, his total collection would have been ₹ 460. Find the total nmber of bananas he had.

Solution:

Let the number of bananas in lots A and B be x and y, respectively

Case I Cost of the first lot at the rate of ₹ 2 for 3 bananas + Cost of the second lot at the rate of ₹ 1 per banana = Amount received

$$\Rightarrow \quad \frac{2}{3}x + y = 400$$

$$\Rightarrow \quad 2x + 3y = 1200 \quad \text{...(i)}$$

Case II Cost of the first lot at the rate of ₹ 1 per banana + Cost of the second lot at the rate of ₹ 4 for 5 bananas = Amount received

$$\Rightarrow \quad x + \frac{4}{5}y = 460$$

$$\Rightarrow \quad 5x + 4y = 2300 \quad \text{...(ii)}$$

On multiplying in Eq. (i) by 4 and Eq. (ii) by 3 and then subtracting them, we get

$$8x + 12y = 4800$$

$$15x + 12y = 6900$$

$$-7x = -2100$$

$$\Rightarrow \quad x = 300$$

Now, put the value of x in Eq. (i), we get

$$2 \times 300 + 3y = 1200$$

$$\Rightarrow \quad 600 + 3y = 1200$$

$$\Rightarrow \quad 3y = 1200 - 600$$

$$\Rightarrow \quad 3y = 600$$

$$\Rightarrow \quad y = 200$$

$\therefore$ Total number of bananas = Number of bana nas in lot A + Number of bananas in lot B

$= x + y$

$= 300 + 200 = 500$

Hence, he had 500 bananas.