Unit 2 (Polynomials)

Exercise 2.1 Multiple Choice Questions (MCQs)

Question 1:

If one of the zeroes of the quadratic polynomial $(k-1)x^2 + kx + 1$ is -3, then the value of k is

(a) $\frac{4}{3}$ (b) $\frac{-4}{3}$ (c) $\frac{2}{3}$ (d) $\frac{-2}{3}$

Solution:

(a) Given that, one of the zeroes of the quadratic polynomial say p(x) = $(k-1)x^2 + kx + 1$ is -3, then

$$p(-3) = 0$$

$$\Rightarrow (k-1)(-3)^2 + k(-3) + 1 = 0$$

$$\Rightarrow 9(k-1) - 3k + 1 = 0$$

$$\Rightarrow 9k - 9 - 3k + 1 = 0$$

$$\Rightarrow 6k - 8 = 0$$

$$\therefore k = 4/3$$

Question 2:

A quadratic polynomial, whose zeroes are -3 and 4, is

(a) $x^2 - x + 12$ (b) $x^2 + x + 12$ (c) $\frac{x^2}{2} - \frac{x}{2} - 6$ (d) $2x^2 + 2x - 24$

Solution:

(c) Let $ax^2 + bx + c$ be a required polynomial whose zeroes are -3 and 4.

Then, sum of zeroes $= -3 + 4 = 1$ $\left[\because \text{sum of zeroes} = \frac{-b}{a}\right]$

$$\Rightarrow \frac{-b}{a} = \frac{1}{1} \Rightarrow \frac{-b}{a} = -\frac{(-1)}{1} \quad \ldots(i)$$

and product of zeroes $= -3 \times 4 = -12$ $\left[\because \text{product of zeroes} = \frac{c}{a}\right]$

$$\Rightarrow \frac{c}{a} = \frac{-12}{1} \quad \ldots(ii)$$

From Eqs. (i) and (ii),

$$a = 1, b = -1 \text{ and } c = -12$$

$$= ax^2 + bx + c$$

$$\therefore \text{ Required polynomial} = 1\cdot x^2 - 1\cdot x - 12$$
$$= x^2 - x - 12$$
$$= \frac{x^2}{2} - \frac{x}{2} - 6$$

We know that, if we multiply/divide any polynomial by any constant, then the zeroes of polynomial do not change.

Alternate Method

Let the zeroes of a quadratic polynomial are α = – 3 and β = 4.

Then, sum of zeroes =α + β = -3+4=1 and product of zeroes = αβ = (-3) (4) = -12

Question 3:

If the zeroes of the quadratic polynomial x̄ + (a +1)* + b are 2 and -3, then

(a) a = -7, b = -1 (b) a = 5,b = -1

(c) a=2, b = -6 (d)a=0,b = -6

Solution:

(d) Let p{x) =$x^2+(a+1)x$+ b

Given that, 2 and -3 are the zeroes of the quadratic polynomial p(x).

$$\therefore \quad p(2) = 0 \text{ and } p(-3)=0$$
$$\Rightarrow \quad 2^2 + (a+1)(2) + b = 0$$
$$\Rightarrow \quad 4 + 2a + 2 + b = 0$$
$$\Rightarrow \quad 2a + b = -6 \qquad \ldots(i)$$
$$\text{and} \quad (-3)^2 + (a+1)(-3) + b = 0$$
$$\Rightarrow \quad 9 - 3a - 3 + b = 0$$
$$\Rightarrow \quad 3a - b = 6 \qquad \ldots(ii)$$

On adding Eqs. (i) and (ii), we get

$$5a = 0 \Rightarrow a = 0$$

Put the value of a in Eq. (i), we get

$$2 \times 0 + b = -6 \Rightarrow b = -6$$

required values are a = 0 and b = – 6.

Question 4:

The number of polynomials having zeroes as -2 and 5 is

(a) 1 (b) 2 (c) 3 (d) more than 3

Solution:

(d) Let p (x) = ax^2 + bx + c be the required polynomial whose zeroes are -2 and 5.

$$\therefore \quad \text{Sum of zeroes} = \frac{-b}{a}$$
$$\Rightarrow \quad \frac{-b}{a} = -2 + 5 = \frac{3}{1} = \frac{-(-3)}{1} \qquad \ldots(i)$$
$$\text{and} \quad \text{product of zeroes} = \frac{c}{a}$$
$$\Rightarrow \quad \frac{c}{a} = -2 \times 5 = \frac{-10}{1} \qquad (ii)$$

From Eqs. (i) and (ii),

$$a = 1, b = -3 \text{ and } c = -10$$

$$\therefore \quad p(x) = ax^2 + bx + c = 1 \cdot x^2 - 3x - 10$$

$$= x^2 - 3x - 10$$

But we know that, if we multiply/divide any polynomial by any arbitrary constant. Then, the zeroes of polynomial never change.

$$\therefore \quad p(x) = kx^2 - 3kx - 10k \quad \text{[where, } k \text{ is a real number]}$$

$$\Rightarrow \quad p(x) = \frac{x^2}{k} - \frac{3}{k}x - \frac{10}{k}, \quad \text{[where, } k \text{ is a non-zero real number]}$$

Hence, the required number of polynomials are infinite i.e., more than 3.

Question 5:

If one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, the product of then other two zeroes is

(a) $\frac{-c}{a}$ (b) $\frac{c}{a}$ (c) 0 (d) $\frac{-b}{a}$

Solution:

(b) Let $p(x) = ax^3 + bx^2 + cx + d$

Given that, one of the zeroes of the cubic polynomial p(x) is zero.

Let α, β and γ are the zeroes of cubic polynomial p(x), where a = 0.

We know that,

$$\text{Sum of product of two zeroes at a time} = \frac{c}{a}$$

$$\Rightarrow \quad \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\Rightarrow \quad 0 \times \beta + \beta\gamma + \gamma \times 0 = \frac{c}{a} \quad [\because \alpha = 0, \text{ given}]$$

$$\Rightarrow \quad 0 + \beta\gamma + 0 = \frac{c}{a}$$

$$\Rightarrow \quad \beta\gamma = \frac{c}{a}$$

$$\text{Hence, product of other two zeroes} = \frac{c}{a}$$

Question 6:

If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is -1, then the product of the other two zeroes is

(a) b – a +1 (b) b – a -1 (c) a – b +1 (d) a – b -1

Solution:

(a) Let $p(x) = x^3 + ax^2 + bx + c$

Let a, p and y be the zeroes of the given cubic polynomial p(x).

$\therefore$ α = -1 [given]

and p(−1) = 0

$\Rightarrow$ $(-1)^3 + a(-1)^2 + b(-1) + c = 0$

$\Rightarrow$ -1 + a- b + c = 0

$\Rightarrow$ c = 1 -a + b …(i)

We know that,

$$\text{Product of all zeroes} = (-1)^3 \cdot \frac{\text{Constant term}}{\text{Coefficient of } x^3} = -\frac{c}{1}$$

αβγ = -c

$\Rightarrow$ (-1)βγ = −c [∴α = -1]

$\Rightarrow$ βγ = c

$\Rightarrow$ βγ = 1 -a + b [from Eq. (i)]

Hence, product of the other two roots is 1 -a + b.

Alternate Method

Since, -1 is one of the zeroes of the cubic polynomial f(x) = $x^2 + ax^2 + bx + c$ i.e., (x + 1) is a factor of f{x).

Now, using division algorithm,

$$\begin{array}{r|l} & x^2 + (a-1)x + (b-a+1) \\ \hline x+1 & x^3 + ax^2 + bx + c \\ & \underline{x^3 + x^2} \\ & \quad (a-1)x^2 + bx \\ & \quad \underline{(a-1)x^2 + (a-1)x} \\ & \qquad (b-a+1)x + c \\ & \qquad \underline{(b-a+1)x (b-a+1)} \\ & \qquad\qquad (c-b+a-1) \end{array}$$

$\Rightarrow x^3 + ax^2 + bx + c = (x + 1) \times \{x^2 + (a - 1)x + (b - a + 1)> + (c - b + a - 1)$

$\Rightarrow x^3 + ax^2 + bx + (b - a + 1) = (x + 1) \{x^2 + (a - 1)x + (b - a + 1\}\}$

Let a and p be the other two zeroes of the given polynomial, then

$$\text{Product of zeroes} = (-1)\alpha\cdot\beta = \frac{-\text{Constant term}}{\text{Coefficient of } x^3}$$

$$\Rightarrow \qquad -\alpha\cdot\beta = \frac{-(b-a+1)}{1}$$

$$\Rightarrow \qquad \alpha\beta = -a + b + 1$$

Hence, the required product of other two roots is $(-a + b + 1)$.

Question 7:

The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are

(a) both positive (b) both negative

(c) one positive and one negative (d) both equal

Solution:

(b) Let given quadratic polynomial be p(x) $= x^2 + 99x + 127$.

On comparing p(x) with $ax^2 + bx + c$, we get

a = 1, b = 99 and c = 127

We know that,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{[by quadratic formula]}$$

$$= \frac{-99 \pm \sqrt{(99)^2 - 4 \times 1 \times 127}}{2 \times 1}$$

$$= \frac{-99 \pm \sqrt{9801 - 508}}{2}$$

$$= \frac{-99 \pm \sqrt{9293}}{2} = \frac{-99 \pm 96.4}{2}$$

$$= \frac{-99 + 96.4}{2}, \frac{-99 - 96.4}{2}$$

$$= \frac{-2.6}{2}, \frac{-195.4}{2}$$

$$= -1.3, -97.7$$

Hence, both zeroes of the given quadratic polynomial p(x) are negative.

Alternate Method

In quadratic polynomial, if $\left.\begin{array}{l} a > 0 \\ a < 0 \end{array} \text{ or } \begin{array}{l} b > 0, c > 0 \\ b < 0, c < 0 \end{array}\right\}$, then both zeroes are negative.

In given polynomial, we see that

$$a = 1 > 0,\ b = 99 > 0 \text{ and } c = 127 > 0$$

the above condition.

So, both zeroes of the given quadratic polynomial are negative.

Question 8:

The zeroes of the quadratic polynomial $x^2 + kx + k$ where $k \neq 0$,

(a) cannot both be positive (b) cannot both be negative

(c) are always unequal (d) are always equal

Solution:

(a)Let $p(x) = x^2 + kx + k, k \neq 0$

On comparing p(x) with $ax^2 + bx + c$, we get

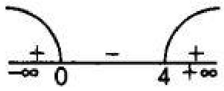
$$a = 1, b = k \text{ and } c = k$$

Now,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{[by quadratic formula]}$$

$$= \frac{-k \pm \sqrt{k^2 - 4k}}{2 \times 1}$$

$$= \frac{-k \pm \sqrt{k(k-4)}}{2}, k \neq 0$$



Here, we see that

$k(k - 4) > 0$

$\Rightarrow$ $k \in (-\infty, 0) \cup (4, \infty)$

Now, we know that

In quadratic polynomial $ax^2 + bx + c$

If $a > 0, b > 0, c > 0$ or $a < 0, b < 0, c < 0$,

then the polynomial has always all negative zeroes.

and if $a > 0, c < 0$ or $a < 0, c > 0$, then the polynomial has always zeroes of opposite sign

Case I If $k \in (-\infty, 0)$ i.e., $k<0$

$\Rightarrow$ $a = 1>0$, $b,c = k<0$

So, both zeroes are of opposite sign.

Case II If $k \in (4, \infty)$i.e., $k \geq 4$

=> $a = 1> 0$, $b,c>4$

So, both zeroes are negative.

Hence, in any case zeroes of the given quadratic polynomial cannot both be positive.

Question 9:

If the zeroes of the quadratic polynomial $ax^2 + bx + c$, where $c \neq 0$, are equal, then

(a) c and a have opposite signs (b) c and b have opposite signs

(c) c and a have same signs (d) c and b have the same signs

Solution:

(c) The zeroes of the given quadratic polynomial $ax^2 + bx + c$, $c \neq 0$ are equal. If coefficient of x^2 and constant term have the same sign i.e., c and a have the same sign. While b i.e., coefficient of x can be positive/negative but not zero.

e.g., (i) $x^2 + 4x + 4 = 0$ (ii) $x^2 - 4x + 4 = 0$

$\Rightarrow$ $(x + 2)^2 = 0$ $\Rightarrow$ $(x - 2)^2 = 0$

$\Rightarrow$ $x = -2, -2$ $\Rightarrow$ $x = 2, 2$

Alternate Method

Given that, the zeroes of the quadratic polynomial $ax^2 + bx + c$, where $c \neq 0$, are equal

i.e., discriminant $(D) = 0$

$$\Rightarrow \quad b^2 - 4ac = 0$$

$$\Rightarrow \quad b^2 = 4ac$$

$$\Rightarrow \quad ac = \frac{b^2}{4}$$

$$\Rightarrow \quad ac > 0$$

which is only possible when a and c have the same signs.

Question 10:

If one of the zeroes of a quadratic polynomial of the form $x^2 + ax + b$ is the negative of the other, then it

(a) has no linear term and the constant term is negative

(b) has no linear term and the constant term is positive

(c) can have a linear term but the constant term is negative

(d) can have a linear term but the constant term is positive

Solution:

(a) Let $p(x) = x^2 + ax + b$.

Put a = 0, then, $p(x) = x^2 + b = 0$

$\Rightarrow$ $x^2 = -b$

$\Rightarrow$ $x = \pm\pm\sqrt{-b}$

$[\therefore b < 0]$

Hence, if one of the zeroes of quadratic polynomial p(x) is the negative of the other, then it has no linear term i.e., a = O and the constant term is negative i.e., b< 0.

Alternate Method

Let $f(x) = x^2 + ax + b$

and by given condition the zeroes area and $-\alpha$.

Sum of the zeroes $= \alpha - \alpha = a$

$\Rightarrow a = 0$

$f(x) = x^2 + b$, which cannot be linear,

and product of zeroes $= \alpha . (-\alpha) = b$

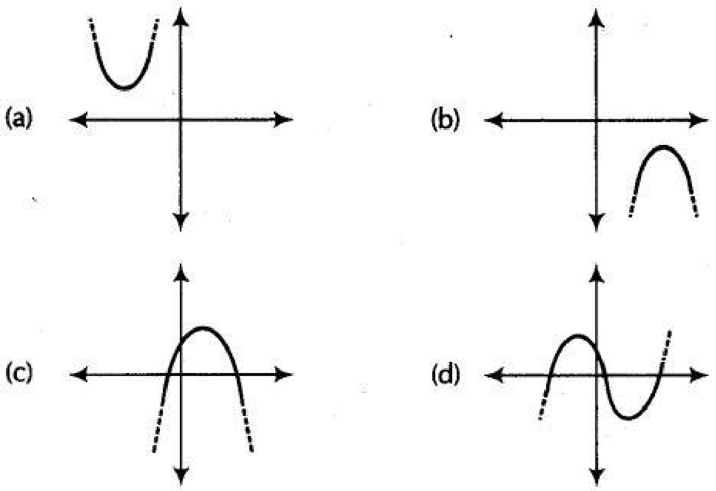
$\Rightarrow$ $-\alpha^2 = b$

which is possible when, $b < 0$.

Hence, it has no linear term and the constant term is negative.

Question 11:

Which of the following is not the graph of a quadratic polynomial?



Solution:

(d) For any quadratic polynomial $ax^2 + bx + c$, $a \neq 0$, the graph of the Corresponding equation $y = ax^2 + bx + c$ has one of the two shapes either open upwards like u or open downwards like ∩ depending on whether $a > 0$ or $a < 0$. These curves are called parabolas. So, option (d) cannot be possible.

Also, the curve of a quadratic polynomial crosses the X-axis on at most two points but in option (d) the curve crosses the X-axis on the three points, so it does not represent the quadratic polynomial.

Exercise 2.2 Very Short Answer Type Questions

Question 1:

Answer the following and justify.

(i) Can x^2 -1 be the quotient on division of x^6 +2x^3 +x-l by a polynomial in x of degree 5?

(ii) What will the quotient and remainder be on division of ox^2 + bx + c by px^3 +qx^2 + rx+ s, p≠ 0 ?

(iii) If on division of a polynomial p(x) by a polynomial g(x),the quotient is zero, what is the relation between the degree of p(x)and g(x)l

(vi) If on division of a non-zero polynomial p(x)by a polynomial g(x),the remainder is zero, what is the relation between the degrees of p(x) and g(x)?

(v) Can the quadratic polynomial x^2 + kx + k have equal zeroes for some odd integer k > 1?

Solution:

(i) No. because whenever we divide a polynomial x^6 + 2x^3 + x -1 by a polynomial in x of degree 5, then we get quotient always as in linear form i.e., polynomial in x of degree 1. Let divisor = a polynomial in x of degree 5

= ax^5 + bx^4 + cx^3 + dx^2 + ex + f

quotient = x^2 -1

and dividend = x^6 + 2x^3 + x -1

By division algorithm for polynomials,

Dividend = Divisor x Quotient + Remainder

= (ax^5 + bx^4 + cx^3 + dx^2 + ex + f)x(x^2 -1) + Remainder

= (a polynomial of degree 7) + Remainder

[in division algorithm, degree of divisor > degree of remainder]

= (a polynomial of degree 7)

But dividend = a polynomial of degree 6

So, division algorithm is not satisfied.

Hence, x^2 -1 is not a required quotient.

(ii) Given that, Divisor px^3 + gx^2 + rx + s, p≠0

and dividend = ax^2 + bx + c

We see that,

Degree of divisor > Degree of dividend

So, by division algorithm,

quotient = 0 and remainder = ax^2 + bx + c

If degree of dividend < degree of divisor, then quotient will be zero and remainder as same as dividend.

(iii) If division of a polynomial p(x) by a polynomial g(x), the quotient is zero, then relation between the degrees ofp(x) and g(x) is degree of p(x) < degree of g(x).

(iv) If division of a non-zero polynomial p(x) by a polynomial g(x), the remainder is zero, then g(x) is a factor of p(x) and has degree less than or equal to the degree of p(x). e., degree of g(x) < degree of p(x).

(v) No, let p(x) = x^2 + kx + k

If p(x) has equal zeroes, then its discriminant should be zero.

D = B^2 -4AC = 0 ,..(j)

On comparing p(x) with Ax^2 + Bx + C, we get

A =1 B = k and C = k

∴ $(k)^2$-4(1)(k) = 0 [from Eq. (i)]

⇒ k(k- 4)=0

⇒ k =0, 4

So, the quadratic polynomial p(x) have equal zeroes only at k =0, 4.

Question 2:

Are the following statements True' or 'False'? Justify your answer.

(i) If the zeroes of a quadratic polynomial ax^2 + bx + c are both positive, then a, b and c all have the same sign.

(ii) If the graph of a polynomial intersects the X-axis at only one point, it cannot be a quadratic polynomial.

(iii) If the graph of a polynomial intersects the X-axis at exactly two points, it need not be a quadratic polynomial.

(iv) If two of the zeroes of a cubic polynomial are zero, then it does not have linear and constant terms.

(v) If all the zeroes of a cubic polynomial are negative, then all the coefficients and the constant term of the polynomial have the same sign.

(vi) If all three zeroes of a cubic polynomial $x^3 + ax^2 - bx + c$ are positive, then atleast one of a, b and c is non-negative.

(vii) The only value of k for which the quadratic polynomial $kx^z + x + k$ has equal zeroes is $\frac{1}{2}$

Solution:

(i) *False*, if the zeroes of a quadratic polynomial $ax^2 + bx + c$ are both positive, then

$$\alpha + \beta = -\frac{b}{a} \quad \text{and} \quad \alpha \cdot \beta = \frac{c}{a}$$

where α and β are the zeroes of quadratic polynomial.

$\therefore$ $c < 0, a < 0$ and $b > 0$

or $c > 0, a > 0$ and $b < 0$

(ii) True, if the graph of a polynomial intersects the X-axis at only one point, then it cannot be a quadratic polynomial because a quadratic polynomial may touch the X-axis at exactly one point or intersects X-axis at exactly two points or do not touch the X-axis.

(iii) True, if the graph of a polynomial intersects the X-axis at exactly two points, then it may or may not be a quadratic polynomial. As, a polynomial of degree more than z is possible which intersects the X-axis at exactly two points when it has two real roots and other imaginary roots.

(iv) True, let a, p and y be the zeroes of the cubic polynomial and given that two of the zeroes have value 0.

Let $\alpha = \beta = 0$

and $f(x) = (x-\alpha)(x-\beta)(x-\gamma)$

$= (x-a)(x-0)(x-0)$

$= x^3 - ax^2$

which does not have linear and constant terms.

(v) True, if $f(x) = ax^3 + bx^2 + cx + d$. Then, for all negative roots, a, b, c and d must have same sign.

(vi) *False*, let α, β and γ be the three zeroes of cubic polynomial $x^3 + ax^2 - bx + c$.

Then, product of zeroes $= (-1)^3 \dfrac{\text{Constant term}}{\text{Coefficient of } x^3}$

$$\Rightarrow \quad \alpha\beta\gamma = -\frac{(+c)}{1}$$

$$\Rightarrow \quad \alpha\beta\gamma = -c \quad \ldots(i)$$

Given that, all three zeroes are positive. So, the product of all three zeroes is also positive

i.e., $\alpha\beta\gamma > 0$

$\Rightarrow \quad -c > 0$ [from Eq. (i)]

$\Rightarrow \quad c < 0$

Now, sum of the zeroes $= \alpha + \beta + \gamma = (-1)\dfrac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

$$\Rightarrow \quad \alpha + \beta + \gamma = -\frac{a}{1} = -a$$

But α, β and γ are all positive.

Thus, its sum is also positive.

So, $\alpha + \beta + \gamma > 0$

$\Rightarrow \quad -a > 0$

$\Rightarrow \quad a < 0$

and sum of the product of two zeroes at a time $= (-1)^2 \cdot \dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \dfrac{-b}{1}$

$\Rightarrow \quad \alpha\beta + \beta\gamma + \gamma\alpha = -b$

$\because \quad \alpha\beta + \beta\gamma + \alpha\gamma > 0 \quad \Rightarrow \quad -b > 0$

$\Rightarrow \quad b < 0$

So, the cubic polynomial $x^3 + ax^2 - bx + c$ has all three zeroes which are positive only when all constants a, b and c are negative.

(vii) False, let $f(x) = kx^2 + x + k$

For equal roots. Its discriminant should be zero i.e., $D = b^2 - 4ac = 0$

$\Rightarrow \quad 1 - 4k.k = 0$

$\Rightarrow \quad k = \pm \frac{1}{2}$

So, for two values of k, given quadratic polynomial has equal zeroes

Exercise 2.3 Short Answer TypeQuestions

Question 1:

Find the zeroes of the following polynomials by factorisation method and verify the relations between the zeroes and the coefficients of the polynomials

(i) $4x^2 - 3x - 1$.

Solution:

Let
$$\begin{aligned} f(x) &= 4x^2 - 3x - 1 \\ &= 4x^2 - 4x + x - 1 \quad \text{[by splitting the middle term]} \\ &= 4x(x-1) + 1(x-1) \\ &= (x-1)(4x+1) \end{aligned}$$

So, the value of $4x^2 - 3x - 1$ is zero when $x - 1 = 0$ or $4x + 1 = 0$ *i.e.,* when $x = 1$ or $x = -\frac{1}{4}$.

So, the zeroes of $4x^2 - 3x - 1$ are 1 and $-\frac{1}{4}$.

$$\therefore \quad \text{Sum of zeroes} = 1 - \frac{1}{4} = \frac{3}{4} = \frac{-(-3)}{4} = (-1)\left(\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}\right)$$

and
$$\text{product of zeroes} = (1)\left(-\frac{1}{4}\right) = -\frac{1}{4} = (-1)^2 \cdot \left(\frac{\text{Constant term}}{\text{Coefficient of } x^2}\right)$$

Hence, verified the relations between the zeroes and the coefficients of the polynomial.

(ii) $3x^2 + 4x - 4$.

Solution:

Let $f(x) = 3x^2 + 4x - 4$

$= 3x^2 + 6x - 2x - 4$ [by splitting the middle term]

$= 3x(x+2) - 2(x+2)$

$= (x+2)(3x-2)$

So, the value of $3x^2 + 4x - 4$ is zero when $x + 2 = 0$ or $3x - 2 = 0$, *i.e.*, when $x = -2$ or $x = \frac{2}{3}$. So, the zeroes of $3x^2 + 4x - 4$ are -2 and $\frac{2}{3}$.

$\therefore$ Sum of zeroes $= -2 + \frac{2}{3} = -\frac{4}{3}$

$= (-1) \cdot \left(\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}\right)$

and product of zeroes $= (-2)\left(\frac{2}{3}\right) = \frac{-4}{3}$

$= (-1)^2 \cdot \left(\frac{\text{Constant term}}{\text{Coefficient of } x^2}\right)$

Hence, verified the relations between the zeroes and the coefficients of the polynomial.

(iii) $51^2 + 12t + 7$.

Solution:

Let $f(t) = 5t^2 + 12t + 7$

$= 5t^2 + 7t + 5t + 7$ [by splitting the middle term]

$= t(5t+7) + 1(5t+7)$

$= (5t+7)(t+1)$

So, the value of $5t^2 + 12t + 7$ is zero when $5t + 7 = 0$ or $t + 1 = 0$,

i.e., when $t = \frac{-7}{5}$ or $t = -1$.

So, the zeroes of $5t^2 + 12t + 7$ are $-7/5$ and -1.

$\therefore$ Sum of zeroes $= -\frac{7}{5} - 1 = \frac{-12}{5}$

$= (-1) \cdot \left(\frac{\text{Coefficient of } t}{\text{Coefficient of } t^2}\right)$

and product of zeroes $= \left(-\frac{7}{5}\right)(-1) = \frac{7}{5}$

$= (-1)^2 \cdot \left(\frac{\text{Constant term}}{\text{Coefficient of } t^2}\right)$

Hence, verified the relations between the zeroes and the coefficients of the polynomial.

(iv) $t^3 - 2t^2 - 15t$.

Solution:

Let $f(t) = t^3 - 2t^2 - 15t$

$= t(t^2 - 2t - 15)$

$= t(t^2 - 5t + 3t - 15)$ [by splitting the middle term]

$= t[t(t-5) + 3(t-5)]$

$= t(t-5)(t+3)$

So, the value of $t^3 - 2t^2 - 15t$ is zero when $t = 0$ or $t - 5 = 0$ or $t + 3 = 0$

i.e., when $t = 0$ or $t = 5$ or $t = -3$.

So, the zeroes of $t^3 - 2t^2 - 15t$ are -3, 0 and 5.

$\therefore$ Sum of zeroes $= -3 + 0 + 5 = 2 = \frac{-(-2)}{1}$

$= (-1) \cdot \left(\frac{\text{Coefficient of } t^2}{\text{Coefficient of } t^3}\right)$

Sum of product of two zeroes at a time

$= (-3)(0) + (0)(5) + (5)(-3)$

$= 0 + 0 - 15 = -15$

$= (-1)^2 \cdot \left(\frac{\text{Coefficient of } t}{\text{Coefficient of } t^3}\right)$

and product of zeroes $= (-3)(0)(5) = 0$

$= (-1)^3 \left(\frac{\text{Constant term}}{\text{Coefficient of } t^3}\right)$

Hence, verified the relations between the zeroes and the coefficients of the polynomial.

(v) $2x^2 + \frac{7}{2}x + \frac{3}{4}$

Solution:

Let $f(x) = 2x^2 + \frac{7}{2}x + \frac{3}{4} = 8x^2 + 14x + 3$

$= 8x^2 + 12x + 2x + 3$ [by splitting the middle term]

$= 4x(2x + 3) + 1(2x + 3)$

$= (2x + 3)(4x + 1)$

So, the value of $8x^2 + 14x + 3$ is zero when $2x + 3 = 0$ or $4x + 1 = 0$,

i.e., when $x = -\frac{3}{2}$ or $x = -\frac{1}{4}$.

So, the zeroes of $8x^2 + 14x + 3$ are $-\frac{3}{2}$ and $-\frac{1}{4}$.

$\therefore$ Sum of zeroes $= -\frac{3}{2} - \frac{1}{4} = -\frac{7}{4} = \frac{-7}{2 \times 2}$

$= -\frac{(\text{Coefficient of } x)}{(\text{Coefficient of } x^2)}$

And roduct of zeroes $= \left(-\frac{3}{2}\right)\left(-\frac{1}{4}\right) = \frac{3}{8} = \frac{3}{2 \times 4}$

$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Hence, verified the relations between the zeroes and the coefficients of the polynomial.

(vi) 4×2 +5√2x – 3.

Solution:

Let $\quad f(x) = 4x^2 + 5\sqrt{2}x - 3$

$$= 4x^2 + 6\sqrt{2}x - \sqrt{2}x - 3 \quad \text{[by splitting the middle term]}$$

$$= 2\sqrt{2}x(\sqrt{2}x + 3) - 1(\sqrt{2}x + 3)$$

$$= (\sqrt{2}x + 3)(2\sqrt{2} \cdot x - 1)$$

So, the value of $4x^2 + 5\sqrt{2}x - 3$ is zero when $\sqrt{2}x + 3 = 0$ or $2\sqrt{2} \cdot x - 1 = 0$,

i.e., $\quad$ when $x = -\frac{3}{\sqrt{2}}$ or $x = \frac{1}{2\sqrt{2}}$.

So, the zeroes of $4x^2 + 5\sqrt{2}\,x - 3$ are $-\frac{3}{\sqrt{2}}$ and $\frac{1}{2\sqrt{2}}$.

$\therefore \quad$ Sum of zeroes $= -\frac{3}{\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$$= -\frac{5}{2\sqrt{2}} = \frac{-5\sqrt{2}}{4}$$

$$= -\frac{(\text{Coefficient of } x)}{(\text{Coefficient of } x^2)}$$

and $\quad$ product of zeroes $= -\frac{3}{\sqrt{2}} \cdot \frac{1}{2\sqrt{2}} = -\frac{3}{4}$

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Hence, verified the relations between the zeroes and the coefficients of the polynomial.

(vii) $2s^2 - (1+2\sqrt{2})s + \sqrt{2}$

Solution:

Let $\quad f(s) = 2s^2 - (1 + 2\sqrt{2})\,s + \sqrt{2}$

$$= 2s^2 - s - 2\sqrt{2}\,s + \sqrt{2}$$

$$= s\,(2s - 1) - \sqrt{2}\,(2s - 1)$$

$$= (2s - 1)(s - \sqrt{2})$$

So, the value of $2s^2 - (1 + 2\sqrt{2})\,s + \sqrt{2}$ is zero when $2s - 1 = 0$ or $s - \sqrt{2} = 0$,

i.e., $\quad$ when $s = \frac{1}{2}$ or $s = \sqrt{2}$.

So, the zeroes of $2s^2 - (1 + 2\sqrt{2})\,s + \sqrt{2}$ are $\frac{1}{2}$ and $\sqrt{2}$.

$\therefore \quad$ Sum of zeroes $= \frac{1}{2} + \sqrt{2} = \frac{1 + 2\sqrt{2}}{2} = \frac{-[-(1 + 2\sqrt{2})]}{2} = \frac{(\text{Coefficient of } s)}{(\text{Coefficient of } s^2)}$

and $\quad$ product of zeroes $= \frac{1}{2} \cdot \sqrt{2} = \frac{1}{\sqrt{2}} = \frac{\text{Constant term}}{\text{Coefficient of } s^2}$

Hence, verified the relations between the zeroes and the coefficients of the polynomial.

(viii) $v^2 + 4\sqrt{3}v - 15$.

Solution:

Let $f(v) = v^2 + 4\sqrt{3}v - 15$

$= v^2 + (5\sqrt{3} - \sqrt{3})v - 15$ [by splitting the middle term]

$= v^2 + 5\sqrt{3}v - \sqrt{3}v - 15$

$= v(v + 5\sqrt{3}) - \sqrt{3}(v + 5\sqrt{3})$

$= (v + 5\sqrt{3})(v - \sqrt{3})$

So, the value of $v^2 + 4\sqrt{3}v - 15$ is zero when $v + 5\sqrt{3} = 0$ or $v - \sqrt{3} = 0$,

i.e., when $v = -5\sqrt{3}$ or $v = \sqrt{3}$.

So, the zeroes of $v^2 + 4\sqrt{3}v - 15$ are $-5\sqrt{3}$ and $\sqrt{3}$.

$\therefore$ Sum of zeroes $= -5\sqrt{3} + \sqrt{3} = -4\sqrt{3}$

$= (-1) \cdot \left(\dfrac{\text{Coefficient of } v}{\text{Coefficient of } v^2}\right)$

and product of zeroes $= (-5\sqrt{3})(\sqrt{3})$

$= -5 \times 3 = -15$

$= (-1)^2 \cdot \left(\dfrac{\text{Constant term}}{\text{Coefficient of } v^2}\right)$

Hence, verified the relations between the zeroes and the coefficients of the polynomial.

(ix) $y^2 + \frac{3}{2}\sqrt{5}y - 5$.

Solution:

Let $f(y) = y^2 + \dfrac{3}{2}\sqrt{5}y - 5 = 2y^2 + 3\sqrt{5}\,y - 10$

$= 2y^2 + 4\sqrt{5}\,y - \sqrt{5}\,y - 10$ [by splitting the middle term]

$= 2y\,(y + 2\sqrt{5}) - \sqrt{5}\,(y + 2\sqrt{5})$

$= (y + 2\sqrt{5})(2y - \sqrt{5})$

So, the value of $y^2 + \dfrac{3}{2}\sqrt{5}y - 5$ is zero when $(y + 2\sqrt{5}) = 0$ or $(2y - \sqrt{5}) = 0$,

i.e., when $y = -2\sqrt{5}$ or $y = \dfrac{\sqrt{5}}{2}$.

So, the zeroes of $2y^2 + 3\sqrt{5}y - 10$ are $-2\sqrt{5}$ and $\dfrac{\sqrt{5}}{2}$.

$\therefore$ Sum of zeroes $= -2\sqrt{5} + \dfrac{\sqrt{5}}{2} = \dfrac{-3\sqrt{5}}{2} = -\dfrac{(\text{Coefficient of } y)}{(\text{Coefficient of } y^2)}$

And product of zeroes $= -2\sqrt{5} \times \dfrac{\sqrt{5}}{2} = -5 = \dfrac{\text{Constant term}}{\text{Coefficient of } y^2}$

Hence, verified the relations between the zeroes and the coefficients of the polynomial.

(x) $7y^2 - \frac{11}{3}y - \frac{2}{3}$.

Solution:

Let $f(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3}$

$= 21y^2 - 11y - 2$

$= 21y^2 - 14y + 3y - 2$ [by splitting the middle term]

$= 7y(3y - 2) + 1(3y - 2)$

$= (3y - 2)(7y + 1)$

So, the value of $7y^2 - \frac{11}{3}y - \frac{2}{3}$ is zero when $3y - 2 = 0$ or $7y + 1 = 0$,

i.e., when $y = \frac{2}{3}$ or $y = -\frac{1}{7}$.

So, the zeroes of $7y^2 - \frac{11}{3}y - \frac{2}{3}$ are $\frac{2}{3}$ and $-\frac{1}{7}$.

$\therefore$ Sum of zeroes $= \frac{2}{3} - \frac{1}{7} = \frac{14 - 3}{21} = \frac{11}{21} = -\left(\frac{-11}{3 \times 7}\right)$

$= (-1) \cdot \left(\frac{\text{Coefficient of } y}{\text{Coefficient of } y^2}\right)$

and product of zeroes $= \left(\frac{2}{3}\right)\left(-\frac{1}{7}\right) = \frac{-2}{21} = \frac{-2}{3 \times 7}$

$= (-1)^2 \cdot \left(\frac{\text{Constant term}}{\text{Coefficient of } y^2}\right)$

Hence, verified the relations between the zeroes and the coefficients of the polynomial.

Exercise 2.4 Long Answer Type Questions

Question 1:

For each of the following, find a quadratic polynomial whose sum and product respectively of the zeroes are as given. Also, find the zeroes of these polynomials by factorisation.

(i) $\frac{-8}{3}, \frac{4}{3}$ (ii) $\frac{21}{8}, \frac{5}{16}$ (iii) $-2\sqrt{3}, -9$ (iv) $\frac{-3}{2\sqrt{5}}, -\frac{1}{2}$

Solution:

(i) Given that, sum of zeroes $(S) = -\frac{8}{3}$

and product of zeroes $(P) = \frac{4}{3}$

$\therefore$ Required quadratic expression, $f(x) = x^2 - Sx + P$

$= x^2 + \frac{8}{3}x + \frac{4}{3} = 3x^2 + 8x + 4$

Using factorisation method, $= 3x^2 + 6x + 2x + 4$

$= 3x(x + 2) + 2(x + 2) = (x + 2)(3x + 2)$

Hence, the zeroes of $f(x)$ are -2 and $-\frac{2}{3}$.

(ii) Given that, $S = \frac{21}{8}$ and $P = \frac{5}{16}$

$\therefore$ Required quadratic expression, $f(x) = x^2 - Sx + P$

$= x^2 - \frac{21}{8}x + \frac{5}{16} = 16x^2 - 42x + 5$

Using factorisation method $= 16x^2 - 40x - 2x + 5$

$= 8x(2x - 5) - 1(2x - 5) = (2x - 5)(8x - 1)$

Hence, the zeroes of $f(x)$ are $\frac{5}{2}$ and $\frac{1}{8}$

(iii) Given that, $S = -2\sqrt{3}$ and $P = -9$

∴ Required quadratic expression,

$$f(x) = x^2 - Sx + P = x^2 + 2\sqrt{3}x - 9$$
$$= x^2 + 3\sqrt{3}x - \sqrt{3}x - 9 \quad \text{[using factorisation method]}$$
$$= x(x + 3\sqrt{3}) - \sqrt{3}(x + 3\sqrt{3})$$
$$= (x + 3\sqrt{3})(x - \sqrt{3})$$

Hence, the zeroes of $f(x)$ are $-3\sqrt{3}$ and $\sqrt{3}$.

(iv) Given that, $S = -\frac{3}{2\sqrt{5}}$ and $P = -\frac{1}{2}$

∴ Required quadratic expression,

$$f(x) = x^2 - Sx + P = x^2 + \frac{3}{2\sqrt{5}}x - \frac{1}{2}$$
$$= 2\sqrt{5}x^2 + 3x - \sqrt{5}$$

Using factorisation method, = $2\sqrt{5}\, x^2 + 5x - 2x - \sqrt{5}$

= $\sqrt{5}x(2x + \sqrt{5}) - 1(2x + \sqrt{5})$

= $(2x + \sqrt{5})(\sqrt{5}x - 1)$

Hence,the zeroes of f(x) are $-\frac{\sqrt{5}}{2}$ and $\frac{1}{\sqrt{5}}$

Question 2:

If the zeroes of the cubic polynomial $x^3 - 6x^2 + 3x + 10$ are of the form a,a + b and a + 2b for some real numbers a and b, find the values of a and b as well as the zeroes of the given polynomial.

Solution:

Let $f(x) = x^3 - 6x^2 + 3x + 10$

Given that, a, $(a + b)$ and $(a + 2b)$ are the zeroes of $f(x)$. Then,

$$\text{Sum of the zeroes} = -\frac{(\text{Coefficient of } x^2)}{(\text{Coefficient of } x^3)}$$

$$\Rightarrow \quad a + (a + b) + (a + 2b) = -\frac{(-6)}{1}$$

$$\Rightarrow \quad 3a + 3b = 6$$

$$\Rightarrow \quad a + b = 2 \quad \ldots(i)$$

$$\text{Sum of product of two zeroes at a time} = \left(\frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}\right).$$

$$\Rightarrow \quad a(a + b) + (a + b)(a + 2b) + a(a + 2b) = \frac{3}{1}$$

$$\Rightarrow \quad a(a + b) + (a + b)\{(a + b) + b\} + a\{(a + b) + b\} = 3$$

$$\Rightarrow \quad 2a + 2(2 + b) + a(2 + b) = 3 \quad \text{[using Eq. (i)]}$$

$$\Rightarrow \quad 2a + 2(2 + 2 - a) + a(2 + 2 - a) = 3 \quad \text{[using Eq. (i)]}$$

$$\Rightarrow \quad 2a + 8 - 2a + 4a - a^2 = 3$$

$$\Rightarrow \quad -a^2 + 8 = 3 - 4a$$

$$\Rightarrow \quad a^2 - 4a - 5 = 0$$

Using factorisation method,

a^2-5a+a-5 = 0 =3

⇒ a (a – 5) + 1 (a – 5) = 0

⇒ (a – 5) (a + 1) = 0

⇒ a = -1, 5

when a = -1, then b = 3

When a = 5, then b = – 3 [using Eq. (i)]

∴Required zeroes of f(x) are

When a = -1 and b = 3

then, a,(a+b),(a + 2) = -1, (-1+3), (-1+6) or -1,2, 5

When a = 5and b = -3 then

a, (a + b), (a + 2b) = 5, (5 -3), (5 -6) or 5,2,-1.

Hence, the required values of a and b are a = – 1 and d = 3ora = 5, b = -3 and the zeroes

are -1,2 and 5.

Question 3:

If √2 is a zero of the cubic polynomial $6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$, the find its other two zeroes.

Solution:

Let $f(x) = 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$ and given that. √2 is one of the zeroes of f(x) i.e.,$(x - \sqrt{2})$ is one of the factor of given cubic polynomial.

Now, using division algorithm,

$$
\begin{array}{r|l}
 & 6x^2 + 7\sqrt{2}x + 4 \\
\hline
(x-\sqrt{2}) & 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2} \\
 & 6x^3 - 6\sqrt{2}x^2 \\
 & \underline{- \quad +} \\
 & 7\sqrt{2}x^2 - 10x - 4\sqrt{2} \\
 & 7\sqrt{2}x^2 - 14x \\
 & \underline{- \quad +} \\
 & 4x - 4\sqrt{2} \\
 & 4x - 4\sqrt{2} \\
 & \underline{- \quad -} \\
 & \times
\end{array}
$$

$$\therefore \quad 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2} = (6x^2 + 7\sqrt{2}x + 4) \times (x - \sqrt{2}) + 0$$

[∵ dividend = divisor × quotient + remainder]

$$= (x - \sqrt{2})(6x^2 + 4\sqrt{2}x + 3\sqrt{2}x + 4)$$
$$= (x - \sqrt{2})\{\sqrt{2}x(3\sqrt{2}x + 4) + 1(3\sqrt{2}x + 4)\}$$
$$= (x - \sqrt{2})\{(3\sqrt{2}x + 4)(\sqrt{2}x + 1)\}$$
$$= (x - \sqrt{2})(\sqrt{2}x + 1)(3\sqrt{2}x + 4)$$

So, its other zeroes are $-\frac{1}{\sqrt{2}}$ and $-\frac{4}{3\sqrt{2}}$.

Question 4:

Find k, so that $x^2 + 2x + k$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$. Also, find all the zeroes of the two polynomials.

Solution:

Given that, $x^2 + 2x + k$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$, then we apply division algorithm,

$$
\begin{array}{r|l}
 & 2x^2 - 3x + (-8 - 2k) \\
\hline
x^2 + 2x + k & 2x^4 + x^3 - 14x^2 + 5x + 6 \\
 & 2x^4 + 4x^3 + 2kx^2 \\
 & \underline{- \quad - \quad -} \\
 & -3x^3 - (2k + 14)x^2 + 5x + 6 \\
 & -3x^3 - 6x^2 - 3kx \\
 & \underline{+ \quad + \quad +} \\
 & (6 - 2k - 14)x^2 + (3k + 5)x + 6 \\
 & (-8 - 2k)x^2 + 2(-8 - 2k)x + k(-8 - 2k) \\
 & \underline{- \quad - \quad -} \\
 & (3k + 5 + 16 + 4k)x + (6 + 8k + 2k^2)
\end{array}
$$

Since, $(x^2 + 2x + k)$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$.

So, when we apply division algorithm remainder should be zero.

$\therefore \quad (7k+21)x + (2k^2+8k+6) = 0\cdot x + 0$

$\Rightarrow \quad 7k+21 = 0$ and $2k^2+8k+6=0$

$\Rightarrow \quad k = -3$ or $k^2+4k+3=0$

$\Rightarrow \quad k^2+3k+k+3=0$ [by splitting middle term]

$\Rightarrow \quad k(k+3)+1(k+3)=0$

$\Rightarrow \quad (k+1)(k+3)=0$

$\Rightarrow \quad k=-1$ or -3

Here, if we take $k=-3$, then remainder will be zero.

Thus, the required value of k is -3.

Now, Dividend = Divisor × Quotient + Remainder

$\Rightarrow \quad 2x^4+x^3-14x^2+5x+16 = (x^2+2x-3)(2x^2-3x-2)$

Using factorisation method,

$= (x^2+3x-x-3)(2x^2-4x+x-2)$ [by splitting middle term]

$= \{x(x+3)-1(x+3)\}\{2x(x-2)+1(x-2)\}$

$= (x-1)(x+3)(x-2)(2x+1)$

Hence, the zeroes of x^2+2x-3 are 1, –3 and the zeroes of $2x^4+x^3-14x^2+5x+6$ are 1, –3, 2, $\frac{-1}{2}$.

Question 5:

If x – √5 is a factor of the cubic polynomial $x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$, then find all the zeroes of the polynomial.

Solution:

Let $f(x) = x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$ and given that, $(x - \sqrt{5})$ is a one of the factor of f(x).

Now, using division algorithm,

$$\begin{array}{r|l} & x^2 - 2\sqrt{5}x + 3 \\ x-\sqrt{5} & x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5} \\ & \underline{\overset{-}{x^3} \overset{+}{-} \sqrt{5}x^2} \\ & -2\sqrt{5}x^2 + 13x - 3\sqrt{5} \\ & \underline{\overset{+}{-}2\sqrt{5}x^2 \overset{-}{+} 10x} \\ & 3x - 3\sqrt{5} \\ & \underline{\overset{-}{3x} \overset{+}{-} 3\sqrt{5}} \\ & \times \end{array}$$

$\therefore \quad x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5} = (x^2 - 2\sqrt{5}x + 3)\times(x-\sqrt{5})$

[$\because$ dividend = divisor × quotient + remainder]

$= (x-\sqrt{5})[x^2 - \{(\sqrt{5}+\sqrt{2})+(\sqrt{5}-\sqrt{2})\}x + 3]$ [by splitting the middle term]

$= (x-\sqrt{5})[x^2 - (\sqrt{5}+\sqrt{2})x - (\sqrt{5}-\sqrt{2})x + (\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})]$

[$\because 3 = (\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})$]

$= (x-\sqrt{5})[x\{x-(\sqrt{5}+\sqrt{2})\} - (\sqrt{5}-\sqrt{2})\{x-(\sqrt{5}+\sqrt{2})\}]$

$= (x-\sqrt{5})\{x-(\sqrt{5}+\sqrt{2})\}\{x-(\sqrt{5}-\sqrt{2})\}$

Hence, all the zeroes of polynomial are $\sqrt{5}$, $(\sqrt{5}+\sqrt{2})$ and $(\sqrt{5}-\sqrt{2})$.

Question 6:

For which values of a and b, the zeroes of $q(x) = x^3 + 2x^2 + a$ are also the zeroes of the polynomial $p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$? Which zeroes of p(x) are not the zeroes of p(x)?

Solution:

Given that the zeroes of q(x) = $x^3 + 2x^2 + a$ are also the zeroes of the polynomial p(x) = $x^5 - x^4 - 4x^3 + 3x^z + 3x + b$ i.e., q(x) is a factor of p(x). Then, we use a division algorithm.

$$x^3 + 2x^2 + a \,\overline{)\, x^5 - x^4 - 4x^3 + 3x^2 + 3x + b \,}\quad x^2 - 3x + 2$$

$$\underline{x^5 + 2x^4 + ax^2}$$

$$-3x^4 - 4x^3 + (3 - a)x^2 + 3x + b$$

$$\underline{-3x^4 - 6x^3 - 3ax}$$

$$2x^3 + (3 - a)x^2 + (3 + 3a)x + b$$

$$\underline{2x^3 + 4x^2 + 2a}$$

$$-(1 + a)x^2 + (3 + 3a)x + (b - 2a)$$

If $(x^3 + 2x^2 + a)$ is a factor of $(x^5 - x^4 - 4x^3 + 3x^2 + 3x + b)$, then remainder should be zero.

i.e., $-(1 + a)x^2 + (3 + 3a)x + (b - 2a) = 0$

$= 0.x^2 + 0.x + 0$

On comparing the coefficient of x, we get

$a + 1 = 0$

$\Rightarrow a = -1$

and $b - 2a = 0$

$\Rightarrow b = 2a$

$b = 2(-1) = -2$ [va = -1]

For a = -1 and b = – 2, the zeroes ofq(x) are also the zeroes of the polynomial p(x).

∴ $q(x) = x^3 + 2x^2 - 1$

and $p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x - 2$

Now, Divident = divisor xquotient + remainder

$p(x) = (x^3 + 2x^2 - 1)(x^2 - 3x + 2) + 0$

$= (x^3 + 2x^2 - 1)\{x^2 - 2x - x + 2\}$

$= (x^3 + 2x^2 - 1)(x - 2)(x - 1)$

Hence, the zeroes of p(x) are land 2 which are not the zeroes of q(x).