Unit 12 (Surface Areas & Volumes)

Exercise 12.1 Multiple Choice Questions (MCQs)

Question 1:

A cylindrical pencil sharpened at one edge is the combination of

(a) a cone and a cylinder

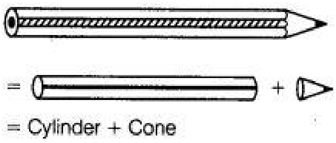
(b) frustum of a cone and a cylinder ‘

(c) a hemisphere and a cylinder

(d) two cylinders

Solution:

(a) Because the shape of sharpened pencil is



Question 2:

A surahi is the combination of

(a) a sphere and a cylinder

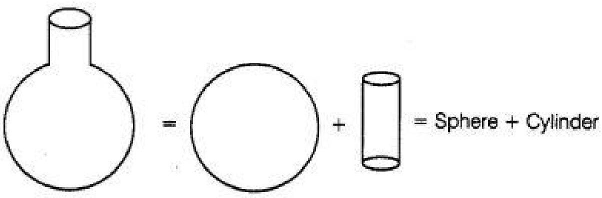
(b) a hemisphere and a cylinder

(c) two hemispheres

(d) a cylinder and a cone

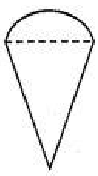
Solution:

(a) Because the shape of surahi is



Question 3:

A plumbline (sahul) is the combination of (see figure)



(a) a cone and a cylinder

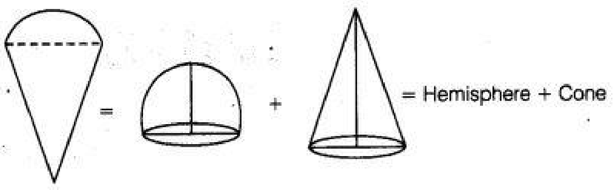
(b) a hemisphere and a cone

(c) frustum of a cone and a cylinder

(d) sphere and cylinder

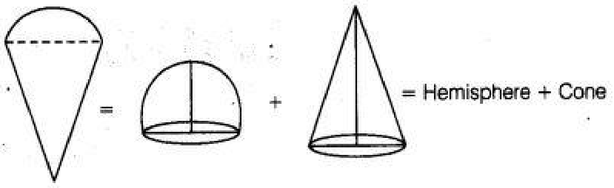
Solution:

(b)



Question 4:

The shape of a glass (tumbler) (see figure) is usually in the form of

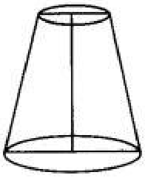


(a) a cone (b) frustum of a cone

(c) a cylinder (d) a sphere

Solution:

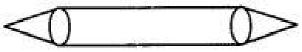
(b) We know that, the shape of frustum of a cone is



So, the given figure is usually in the form of frustum of a cone.

Question 5:

The shape of a gilli, in the gilli-danda game (see figure) is a combination of

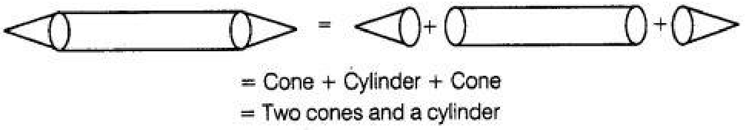


(a) two cylinders (b) a cone and a cylinder

(c) two cones and a cylinder (d) two cylinders and a cone

Solution:

(c)



Question 6:

A shuttle cock used for playing badminton has the shape of the combination of

(a) a cylinder and a sphere (b) a cylinder and a hemisphere

(c) a sphere and a cone (d) frustum of a cone and a hemisphere

Solution:

(d) Because the shape of the shuttle cock is equal to sum of frustum of a cone and hemisphere.

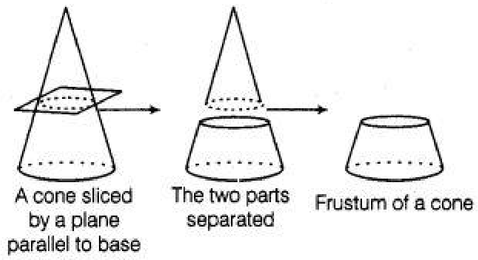
Question 7:

A cone is cut through a plane parallel to its base and then the cone that is formed on one side of that plane is removed. The new part that is left over on the other side of the plane is called

(a) a frustum of a cone (b) cone (c) cylinder (d) sphere

Solution:

(a)



[when we remove the upper portion of the cone cut off by plane, we get frustum of a cone]

Question 8:

If a hollow cube of internal edge 22 cm is filled with spherical marbles of diameter 0.5 cm and it is assumed that – space of the cube remains unfilled. Then, the number of marbles that the cube can accomodate is

(a) 142244 (b) 142344 (c) 142444 (d) 142544

Solution:

(a) Given, edge of the cube = 22 cm

$\therefore$ Volume of the cube $= (22)^3 = 10648 \text{ cm}^3$ [$\because$ volume of cube $= (\text{side})^3$]

Also, given diameter of marble = 0.5 cm

$\therefore$ Radius of a marble, $r = \frac{0.5}{2} = 0.25$ cm [$\because$ diameter $= 2 \times$ radius]

Volume of one marble $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (0.25)^3$

[$\because$ volume of sphere $= \frac{4}{3} \times \pi \times (\text{radius})^3$]

$= \frac{1.375}{21} = 0.0655 \text{ cm}^3$

Filled space of cube = Volume of the cube $\frac{1}{8} \times$ Volume of cube

$= 10648 - 10648 \times \frac{1}{8}$

$= 10648 \times \frac{7}{8} = 9317 \text{ cm}^3$

$\therefore$ Required number of marbles $= \frac{\text{Total space filled by marbles in a cube}}{\text{Volume of one marble}}$

$= \frac{9317}{0.0655} = 142244$ (approx)

Hence, the number of marbles that the cube can accomodate is 142244.

Question 9:

A metallic spherical shell of internal and external diameters 4 cm and 8 cm, respectively is melted and recast into the form a cone of base diameter 8 cm. The height of the cone is

(a) 12 cm (b) 14 cm (c) 15 cm (d) 18 cm

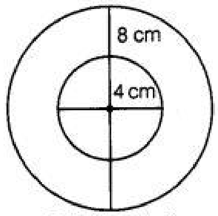
Solution:

(b) Given, internal diameter of spherical shell = 4 cm

and external diameter of shell = 8 cm

∴ Internal radius of spherical shell, $r_1 = \frac{4}{2}$ cm $= 2$ cm [∵ diameter $= 2 \times$ radius]

and external radius of shell, $r_2 = \frac{8}{2} = 4$ cm [∵ diameter $= 2 \times$ radius]



Spherical shell

Now, volume of the spherical shell $= \frac{4}{3}\pi[r_2^3 - r_1^3]$

[∵ volume of the spherical shell $= \frac{4}{3}\pi\{(\text{external radius})^3 - (\text{internal radius})^3\}$]

$$= \frac{4}{3}\pi(4^3 - 2^3)$$
$$= \frac{4}{3}\pi(64 - 8)$$
$$= \frac{224}{3}\pi \text{ cm}^3$$

Let height of the cone $= h$ cm

Diameter of the base of cone $= 8$ cm

∴ Radius of the base of cone $= \frac{8}{2} = 4$ cm [∵ diameter $= 2 \times$ radius]

According to the question,

Volume of cone = Volume of spherical shell

$$\Rightarrow \quad \frac{1}{3}\pi(4)^2 h = \frac{224}{3}\pi \Rightarrow h = \frac{224}{16} = 14 \text{ cm}$$

[∵ volume of cone $= \frac{1}{3} \times \pi \times (\text{radius})^2 \times (\text{height})$]

Hence, the height of the cone is 14 cm.

Question 10:

If a solid piece of iron in the form of a cuboid of dimensions 49 cm x 33 cm x 24 cm, is moulded to form a solid sphere. Then, radius of the sphere is

(a) 21 cm (b) 23 cm (c) 25 cm

(d)19cm

Solution:

(a) Given, dimensions of the cuboid = 49 cm x 33 cm x 24 cm

∴ Volume of the cuboid = 49 x 33 x 24 = 38808 cm^3

[∵ volume of cuboid = length × breadth × height]

Let the radius of the sphere is r, then

Volume of the sphere $= \frac{4}{3}\pi r^3$ [∵ voulme of the sphere $= \frac{4}{3}\pi \times (\text{radius})^3$]

According to the question,

Volume of the sphere = Volume of the cuboid

$$\Rightarrow \quad \frac{4}{3}\pi r^3 = 38808$$
$$\Rightarrow \quad 4 \times \frac{22}{7} r^3 = 38808 \times 3$$
$$\Rightarrow \quad r^3 = \frac{38808 \times 3 \times 7}{4 \times 22} = 441 \times 21$$
$$\Rightarrow \quad r^3 = 21 \times 21 \times 21$$
$$\therefore \quad r = 21 \text{ cm}$$

Hence, the radius of the sphere is 21 cm.

Hence, the radius of the sphere is 21 cm.

Question 11:

A mason constructs a wall of dimensions 270 cmx 300 cm x 350 cm with the bricks each of size 22.5 cm x 11.25 cmx 8.75 cm and it is assumed that $\frac{1}{8}$space is covered by the mortar. Then, the number of bricks used to construct the wall is

(a) 11100 (b) 11200 (c) 11000 (d) 11300

Solution:

(b) Volume of the wall = $270 \times 300 \times 350 = 28350000 \text{ cm}^3$

[∵ volume of cuboid = length × breadth × height]

Since, $\frac{1}{8}$ space of wall is covered by mortar.

So, remaining space of wall = Volume of wall − Volume of mortar

$= 28350000 - 28350000 \times \frac{1}{8}$

$= 28350000 - 3543750 = 24806250 \text{ cm}^3$

Now, volume of one brick = $22.5 \times 1125 \times 875 = 2214.844 \text{ cm}^3$

[∵ volume of cuboid = length × breadth × height]

∴ Required number of bricks = $\frac{24806250}{2214.844} = 11200$ (approx)

Hence, the number of bricks used to construct the wall is 11200.

Question 12:

Twelve solid spheres of the same size are made by melting a solid metallic cylinder of base diameter 2 cm and height 16 cm. The diameter of each sphere is

(a) 4 cm (b) 3 cm (c) 2 cm (d) 6 cm

Solution:

(c) Given, diameter of the cylinder = 2 cm

∴ Radius = 1 cm and height of the cylinder = 16 cm [∵ diameter = 2 x radius]

∴ Volume of the cylinder = $\pi \times (1)^2 \times 16 = 16\pi \text{ cm}^3$

[∵ volume of cylinder = $\pi \times (\text{radius})^2 \times$ height]

Now, let the radius of solid sphere = r cm

Then, its volume = $\frac{4}{3}\pi r^3 \text{ cm}^3$ [∵ volume of sphere = $\frac{4}{3} \times \pi \times (\text{radius})^3$]

According to the question,

Volume of the twelve solid sphere = Volume of cylinder

$\Rightarrow 12 \times \frac{4}{3}\pi r^3 = 16\pi$

$\Rightarrow r^3 = 1 \Rightarrow r = 1 \text{cm}$

∴ Diameter of each sphere, d=2r = 2×1=2 cm

Hence, the required diameter of each sphere is 2 cm.

Question 13:

The radii of the top and bottom of a bucket of slant height 45 cm are 28 cm and 7 cm, respectively. The curved surface area of the bucket is

(a) 4950 cm^2 (b) 4951 cm^2 (c) 4952 cm^2 (d) 4953 cm^2

Solution:

(a) Given, the radius of the top of the bucket, R = 28 cm

and the radius of the bottom of the bucket, r = 7 cm

Slant height of the bucket, l= 45 cm

Since, bucket is in the form of frustum of a cone.

∴ Curved surface area of the bucket = π l (R + r) = π x 45 (28 + 7)

[∵ curved surface area of frustum of a cone = $\pi(R + r)l$]

$= \pi \times 45 \times 35 = \frac{22}{7} \times 45 \times 35 = 4950 \text{ cm}^2$

Question 14:

A medicine-capsule is in the shape of a cylinder of diameter 0.5 cm with two hemispheres stuck to each of its ends. The length of entire capsule is 2 cm. The capacity of the capsule is

(a) 0.36 cm^3 (b) 0.35 cm^3 (c) 0.34 cm^3

(d) 0.33 cm^3

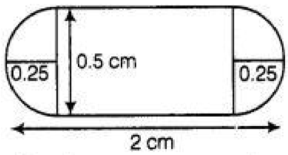
Solution:

(a) Given, diameter of cylinder = Diameter of hemisphere = 0.5 cm

[since, both hemispheres are attach with cylinder]

$\therefore$ Radius of cylinder (r) = radius of hemisphere (r) = $\frac{0.5}{2}$ = 0.25 cm

[$\because$ diameter = 2 × radius]



and total length of capsule = 2 cm

$\therefore$ Length of cylindrical part of capsule,

h = Length of capsule − Radius of both hemispheres

= 2 − (0.25 + 0.25) = 1.5 cm

Now, capacity of capsule = Volume of cylindrical part + 2 × Volume of hemisphere

$$= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3$$

[$\because$ volume of cylinder = $\pi \times (\text{radius})^2 \times$ height and volume of hemispere = $\frac{2}{3}\pi(\text{radius})^3$]

$$= \frac{22}{7}\left[(0.25)^2 \times 1.5 + \frac{4}{3} \times (0.25)^3\right]$$

$$= \frac{22}{7}[0.09375 + 0.0208]$$

$$= \frac{22}{7} \times 0.11455 = 0.36 \text{ cm}^3$$

Hence, the capacity of capsule is 0.36 cm^3

Question 15:

If two solid hemispheres of same base radius r are joined together along their bases, then curved surface area of this new solid is

(a) $47\pi r^2$ (b) $6\pi r^2$ (c) $3\pi r^2$ (d) $8\pi r^2$

Solution:

(a) Because curved surface area of a hemisphere is 2 πr^2 and here, we join two solid hemispheres along their bases of radius r, from which we get a solid sphere.

Hence, the curved surface area of new solid = $2\pi r^2 + 2\pi r^2 = 4\pi r^2$

Question 16:

A right circular cylinder of radius r cm and height h cm (where, h>2r) just encloses a sphere of diameter

(a) r cm (b) 2r cm (c) h cm (d) 2h cm

Solution:

(b) Because the sphere encloses in the cylinder, therefore the diameter of sphere is equal to diameter of cylinder which is 2r cm.

Question 17:

During conversion of a solid from one shape to another, the volume of the new shape will

(a) increase (b) decrease

(c) remain unaltered (d) be doubled

Solution:

(c) During conversion of a solid from one shape to another, the volume of the new shape will remain unaltered.

Question 18:

The diameters of the two circular ends of the bucket are 44 cm and 24 cm. The height of the bucket is 35 cm. The capacity of the bucket is

(a) 32.7 L (b) 33.7 L (c) 34.7 L (d) 31.7 L

Solution:

(a) Given, diameter of one end of the bucket

$2R = 44 \Rightarrow R = 22$ cm [$\because$ diameter, $r = 2 \times$ radius]

and diameter of the other end,

$2r = 24 \Rightarrow r = 12$ cm [$\because$ diameter, $r = 2 \times$ radius]

Height of the bucket, $h = 35$ cm

Since, the shape of bucket is look like as frustum of a cone.

$\therefore$ Capacity of the bucket = Volume of the frustum of the cone

$$= \frac{1}{3}\pi h [R^2 + r^2 + Rr]$$

$$= \frac{1}{3} \times \pi \times 35 [(22)^2 + (12)^2 + 22 \times 12]$$

$$= \frac{35\pi}{3}[484 + 144 + 264]$$

$$= \frac{35\,\pi \times 892}{3} = \frac{35 \times 22 \times 892}{3 \times 7}$$

$$= 32706.6 \text{ cm}^3 = 32.7 \text{ L} \qquad [\because 1000 \text{ cm}^3 = 1 \text{ L}]$$

Hence, the capacity of bucket is 32.7 L.

Question 19:

In a right circular cone, the cross-section made by a plane parallel to the base is a

(a) circle (b) frustum of a cone (c) sphere (d) hemisphere

Solution:

(b) We know that, if a cone is cut by a plane parallel to the base of the cone, then the portion between the plane and base is called the frustum of the cone.

Question 20:

If volumes of two spheres are in the ratio 64 : 27, then the ratio of their surface areas is

(a) 3: 4 (b) 4 : 3 (c) 9 : 16 (d) 16 : 9

Solution:

(d) Let the radii of the two spheres are r_1 and r_2, respectively.

$\therefore$ Volume of the sphere of radius, $r_1 = V_1 = \frac{4}{3}\pi r_1^3$ ...(i)

[$\because$ volume of sphere $= \frac{4}{3}\pi (\text{radius})^3$]

and volume of the sphere of radius, $r_2 = V_2 = \frac{4}{3}\pi r_2^3$ ...(ii)

Given, ratio of volumes $= V_1 : V_2 = 64 : 27 \Rightarrow \dfrac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \dfrac{64}{27}$ [using Eqs. (i) and (ii)]

$$\Rightarrow \frac{r_1^3}{r_2^3} = \frac{64}{27} \Rightarrow \frac{r_1}{r_2} = \frac{4}{3} \quad \text{...(iii)}$$

Now, ratio of surface area $= \dfrac{4\pi r_1^2}{4\pi r_2^2}$ [$\because$ surface area of a sphere $= 4\pi (\text{radius})^2$]

$$= \frac{r_1^2}{r_2^2}$$

$$= \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{4}{3}\right)^2 \qquad \text{[using Eq. (iii)]}$$

$$= 16 : 9$$

Hence, the required ratio of their surface area is 16 : 9.

Exercise 12.2 Very Short Answer Type Questions

Write whether True or False and justify your answer.

Question 1:

Two identical solid hemispheres of equal base radius r cm are stuck together along their bases. The total surface area of the combination is $6\pi r^2$.

Solution:

False

Curved surface area of a hemisphere = 2 πr^2

Here, two identical solid hemispheres of equal radius are stuck together. So, base of both hemispheres is

common.

∴ Total surface area of the combination

= 2 πr^2 + 2 πr^2 = 4π r^2

Question 2:

A solid cylinder of radius r and height h is placed over other cylinder of same height and radius. The total surface area of the shape so formed is 4πrh + 4πr^2.

Solution:

False

Since, the total surface area of cylinder of radius, rand height, h = 2πrh + 2πr^2When one cylinder is placed over the other cylinder of same height and radius,

then height of the new cylinder = 2 h

and radius of the new cylinder = r

∴ Total surface area of the new cylinder = 2πr(2h) + 2πr^2 = 4πrh + 2πr^2

Question 3:

A solid cone of radius r and height h is placed over a solid cylinder having same base radius and height as that of a cone The total surface area of thecombined solid is [$\sqrt{r^2+h^2}$+3r + 2h].

Solution:

False

We know that, total surface area of a cone of radius, r

and height, h = Curved surface Area + area of base = $\pi r l + \pi r^2$

where, $l = \sqrt{h^2 + r^2}$

and total surface area of a cylinder of base radius, r and height, h

= Curved surface area + Area of both base = $2\pi r h + 2\pi r^2$

Here, when we placed a cone over a cylinder, then one base is common for both.

So, total surface area of the combined solid

$$= \pi r l + 2\pi r h + \pi r^2 = \pi r\,[l + 2h + r]$$

$$= \pi r\left[\sqrt{r^2 + h^2} + 2h + r\right]$$

Question 4:

A solid ball is exactly fitted inside the cubical box of side a. The volume of the ball is $\frac{4}{3}\pi a^3$.

Solution:

False

Because solid ball is exactly fitted inside the cubical box of side a. So, a is the diameter for . the solid ball.

∴ Radius of the ball = $\frac{a}{2}$

So, volume of the ball = $\frac{4}{3}\pi\left(\frac{a}{2}\right)^3 = \frac{1}{6}\pi a^3$

Question 5:

The volume of the frustum of a cone is $\frac{1}{3}\pi h\,[r_1^2 + r_2^2 - r_1 r_2]$, where h is vertical height of the frustum and r_1, r_2 are the radii of the ends.

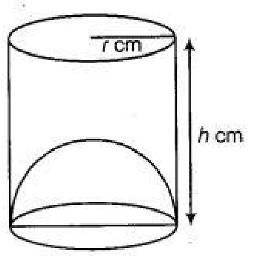
Solution:

False

Since, the volume of the frustum of a cone is $\frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1 r_2]$, where h is vertical height of the frustum and r_1, r_2 are the radii of the ends.

Question 6:

The capacity of a cylindrical vessel with a hemispherical portion raised upward at the bottom as shown in the figure is $\frac{\pi r^2}{3}[3h - 2r]$.



Solution:

True

We know that, capacity of cylindrical vessel = $\pi r^2 h$ cm^3

and capacity of hemisphere = $\frac{2}{3}\pi r^3$ cm

From the figure, capacity of the cylindrical vessel

$$= \pi r^2 h - \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2[3h - 2r]$$

Question 7:

The curved surface area of a frustum of a cone is $\pi l(r_1 + r_2)$, where $l = \sqrt{h^2 + (r_1 + r_2)^2}$, r_1 and r_2 are the radii of the two ends of the frustum and h is the vertical height.

Solution:

Fasle

We know that, if r_1 and r_2 are the radii of the two ends of the frustum and h is the vertical height, then curved surface area of a frustum is $\pi l(r_1 + r_2)$, where $l = \sqrt{h^2 + (r_1 - r_2)^2}$.

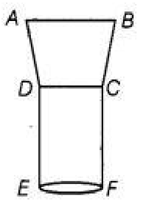
Question 8:

An open metallic bucket is in the shape of a frustum of a cone, mounted on a hollow cylindrical base made of the same metallic sheet. The surface area of the metallic sheet used is equal to curved surface area of frustum of a cone + area of circular base + curved surface area of cylinder.

Solution:

True

Because the resulting figure is



Here, ABCD is a frustum of a cone and CDEF is a hollow cylinder.

Exercise 12.3 Short Answer Type Questions

Question 1:

Three metallic solid cubes whose edges are 3 cm, 4 cm and 5 cm are melted and formed into a single cube. Find the edge of the cube so formed.

Solution:

Given, edges of three solid cubes are 3 cm, 4 cm and 5 cm, respectively.

$\therefore$ Volume of first cube = $(3)^3 = 27\ cm^3$

Volume of second cube = $(4)^3 = 64\ cm^3$

and volume of third cube = $(5)^3 = 125\ cm^3$

$\therefore$ Sum of volume of three cubes = $(27 + 64 + 125) = 216\ cm^3$

Let the edge of the resulting cube = R cm

Then, volume of the resulting cube, $R^3 = 216 \Rightarrow R = 6cm$

Question 2:

How many shots each having diameter 3 cm can be made from a cuboidal lead solid of dimensions 9 cm x 11 cm x 12 cm?

Solution:

Given, dimensions of cuboidal = 9 cm x 11 cm x 12 cm

$\therefore$ Volume of cuboidal = 9 x 11 x 12 = 1188 cm^3

and diameter of shot = 3 cm

$$\therefore \quad \text{Radius of shot, } r = \frac{3}{2} = 1.5 \text{ cm}$$

$$\text{Volume of shot} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (1.5)^3$$

$$= \frac{297}{21} = 14.143 \text{ cm}^3$$

$$\therefore \quad \text{Required number of shots} = \frac{1188}{14.143} = 84 \text{ (approx)}$$

Question 3:

A backet is in the form of a frustum of a cone and holds 28.490 L of water. The radii of the top and bottom are 28 cm and 21 cm, respectively. Find the height of the bucket.

Solution:

Given, volume of the frustum = 28.49 L = 28.49 x 1000 cm^3 [$\therefore$ 1 L = 1000 cm^3]

= 28490 cm^3

and radius of the top (r_1) = 28 cm

radius of the bottom (r_2) = 21 cm

Let height of the bucket = h cm

$$\text{Now, volume of the bucket} = \frac{1}{3}\pi h (r_1^2 + r_2^2 + r_1 r_2) = 28490 \quad \text{[given]}$$

$$\Rightarrow \quad \frac{1}{3} \times \frac{22}{7} \times h (28^2 + 21^2 + 28 \times 21) = 28490$$

$$\Rightarrow \quad h(784 + 441 + 588) = \frac{28490 \times 3 \times 7}{22}$$

$$\Rightarrow \quad 1813\, h = 1295 \times 21$$

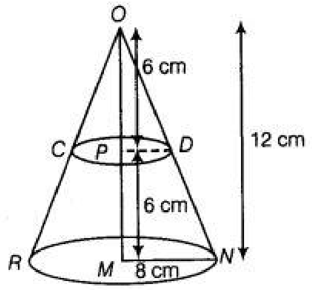
$$\therefore \quad h = \frac{1295 \times 21}{1813} = \frac{27195}{1813} = 15 \text{ cm}$$

Question 4:

A cone of radius 8 cm and height 12 cm is divided into two parts by a plane through the mid-point of its axis parallel to its base. Find the ratio of the volumes of two parts.

Solution:

Let ORN be the cone then given, radius of the base of the cone r = 8cm



and height of the cone, (h) $OM = 12$ cm

Let P be the mid-point of OM, then

$$OP = PM = \frac{12}{2} = 6\,\text{cm}$$

Now, $\Delta OPD \sim \Delta OMN$

$\therefore$ $$\frac{OP}{OM} = \frac{PD}{MN}$$

$\Rightarrow$ $$\frac{6}{12} = \frac{PD}{8} \Rightarrow \frac{1}{2} = \frac{PD}{8}$$

$\Rightarrow$ $$PD = 4\,\text{cm}$$

The plane along CD divides the cone into two parts, namely

(i) a smaller cone of radius 4 cm and height 6cm and (ii) frustum of a cone for which

Radius of the top of the frustum, $r_1 = 4$ cm

Radius of the bottom, $r_2 = 8$ cm

and height of the frustum, h = 6 cm

$\therefore$ Volume of smaller cone $= \left(\frac{1}{3}\pi \times 4 \times 4 \times 6\right) = 32\pi\,\text{cm}^3$

and volume of the frustum of cone $= \frac{1}{3} \times \pi \times 6\,[(8)^2 + (4)^2 + 8 \times 4]$

$$= 2\pi\,(64 + 16 + 32) = 224\,\pi\,\text{cm}^3$$

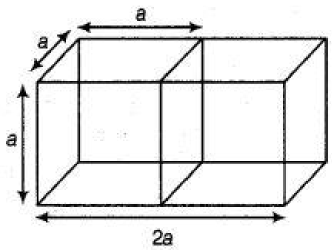
$\therefore$ Required ratio = Volume of frustum : Volume of cone = 24 π: 32 π = 1:7

Question 5:

Two identical cubes each of volume 64 cm^3 are joined together end to end. What is the surface area of the resulting cuboid?

Solution:

Let the length of side of a cube = a cm



Given, volume of the cube, $a^3 = 64\,\text{cm}^3 \Rightarrow a = 4\,\text{cm}$

On joining two cubes, we get a cuboid whose

length, $l = 2a$ cm

breadth, $b = a$ cm

and height, $h = a$ cm

Now, surface area of the resulting cuboid $= 2\,(lb + bh + hl)$

$$= 2\,(2a \cdot a + a \cdot a + a \cdot 2a)$$

$$= 2\,(2a^2 + a^2 + 2a^2) = 2\,(5a^2)$$

$$= 10\,a^2 = 10\,(4)^2 = 160\,\text{cm}^2$$

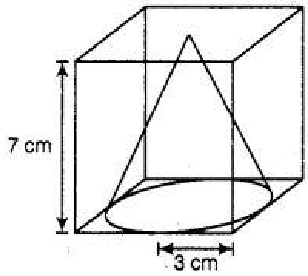
Question 6:

From a solid cube of side 7 cm, a conical cavity of height 7 cm and radius 3 cm is hollowed out. Find the volume of the remaining solid.

Solution:

Given that, side of a solid cube (a) = 7 cm

Height of conical cavity i.e., cone, h = 7 cm



Since, the height of conical cavity and the side of cube is equal that means the conical cavity fit vertically in the cube.

Radius of conical cavity i.e., cone, r = 3 cm

⇒ Diameter = 2 x r = 2 x 3= 6 cm

Since, the diameter is less than the side of a cube that means the base of a conical cavity is not fit inhorizontal face of cube.

Now, volume of cube = $(\text{side})^3 = a^3 = (7)^3 = 343 \text{ cm}^3$

and volume of conical cavity *i.e.*, cone $= \frac{1}{3}\pi \times r^2 \times h$

$$= \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 7$$

$$= 66 \text{ cm}^3$$

∴ Volume of remaining solid = Volume of cube – Volume of conical cavity

$$= 343 - 66 = 277 \text{ cm}^3$$

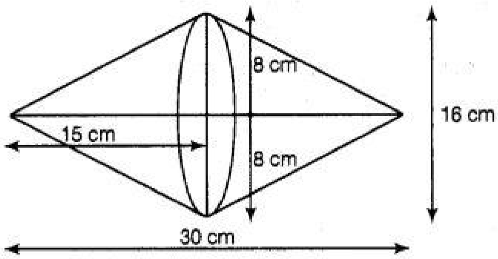
Hence, the required volume of solid is 277 cm^3

Question 7:

Two cones with same base radius 8 cm and height 15 cm are joined together along their bases. Find the surface area of the shape so formed.

Solution:

If two cones with same base and height are joined together along their bases, then the shape so formed is look like as figure shown.



Given that, radius of cone, $r = 8$ cm and height of cone, $h = 15$ cm

So, surface area of the shape so formed

= Curved area of first cone + Curved surface area of second cone

= 2 · Surface area of cone [since, both cones are identical]

$$= 2 \times \pi r l = 2 \times \pi \times r \times \sqrt{r^2 + h^2}$$

$$= 2 \times \frac{22}{7} \times 8 \times \sqrt{(8)^2 + (15)^2}$$

$$= \frac{2 \times 22 \times 8 \times \sqrt{64 + 225}}{7}$$

$$= \frac{44 \times 8 \times \sqrt{289}}{7}$$

$$= \frac{44 \times 8 \times 17}{7}$$

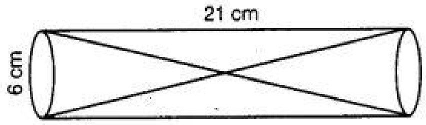
$$= \frac{5984}{7} = 854.85 \text{ cm}^2$$

$$= 855 \text{ cm}^2 \text{ (approx)}$$

Hence, the surface area of shape so formed is 855 cm^2

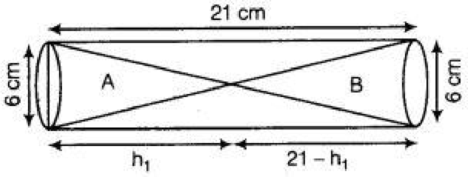
Question 8:

Two solid cones A and B are placed in a cylindrical tube as shown in the figure. The ratio of their capacities is 2 : 1. Find the heights and capacities of cones. Also, find the volume of the remaining portion of the cylinder.



Solution:

Let volume of cone A be 2 V and volume of cone B be V. Again, let height of the cone A = h_1 cm, then height of cone B = $(21 – h_1)$ cm



Given, diameter of the cone = 6 cm

$\therefore$ Radius of the cone $= \frac{6}{2} = 3$ cm

Now, volume of the cone, $A = 2V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (3)^2 h_1$

$\Rightarrow \quad V = \frac{1}{6}\pi 9 h_1 = \frac{3}{2} h_1 \pi$ ...(i)

and volume of the cone, $B = V = \frac{1}{3}\pi (3)^2 (21 - h_1) = 3\pi (21 - h_1)$ ...(ii)

From Eqs. (i) and (ii),

$$\frac{3}{2} h_1 \pi = 3\pi (21 - h_1)$$

$\Rightarrow \quad h_1 = 2(21 - h_1)$

$\Rightarrow \quad 3h_1 = 42$

$\Rightarrow \quad h_1 = \frac{42}{3} = 14$ cm

$\therefore$ Height of cone, $B = 21 - h_1 = 21 - 14 = 7$ cm

Now, volume of the cone, $A = 3 \times 14 \times \frac{22}{7} = 132$ cm^3 [using Eq. (i)]

and volume of the cone, $B = \frac{1}{3} \times \frac{22}{7} \times 9 \times 7 = 66$ cm^3 [using Eq. (ii)]

Now, volume of the cylinder $= \pi r^2 h = \frac{22}{7}(3)^2 \times 21 = 594$ cm^3

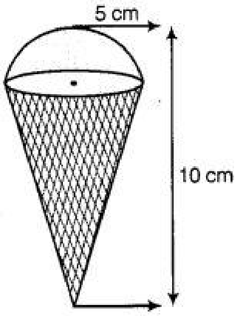
$\therefore$ Required volume of the remaining portion = Volume of the cylinder – (Volume of cone A + Volume of cone B)

= 594 – (132 + 66)

= 396 cm^3

Question 9:

An ice-cream cone full of ice-cream having radius 5 cm and height 10 cm as shown in figure



Calculate the volume of ice-cream, provided that its $\frac{1}{6}$ part is left unfilled with ice-cream.

Solution:

Given, ice-cream cone is the combination of a hemisphere and a cone.

Also , radius of hemisphere = 5 cm

$\therefore$ Volume of hemisphere $= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (5)^3$

$= \frac{5500}{21} = 261.90 \text{ cm}^3$

Now, radius of the cone = 5 cm

and height of the cone = 10 − 5 = 5 cm

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 5$

$= \frac{2750}{21} = 130.95 \text{ cm}^3$

Now, total volume of ice-cream cone = 261.90 + 130.95 = 392.85 cm^3

Since, $\frac{1}{6}$ part is left unfilled with ice-cream.

$\therefore$ Required volume of ice-cream $= 392.85 - 392.85 \times \frac{1}{6} = 392.85 - 65.475$

$= 327.4 \text{ cm}^3$

Question 10:

Marbles of diameter 1.4 cm are dropped into a cylindrical beaker of diameter 7 cm containing some water. Find the number of marbles that should be dropped into the beaker, so that the water level rises by 5.6 cm.

Solution:

Given, diameter of a marble = 1.4 cm

$\therefore$ Radius of marble $= \frac{1.4}{2} = 0.7$ cm

So, volume of one marble $= \frac{4}{3}\pi(0.7)^3$

$= \frac{4}{3}\pi \times 0.343 = \frac{1.372}{3}\pi \text{ cm}^3$

Also, given diameter of beaker = 7 cm

$\therefore$ Radius of beaker $= \frac{7}{2} = 3.5$ cm

Height of water level raised = 5.6 cm

$\therefore$ Volume of the raised water in beaker $= \pi(3.5)^2 \times 5.6 = 68.6\pi \text{ cm}^3$

Now, required number of marbles $= \frac{\text{Volume of the raised water in beaker}}{\text{Volume of one spherical marble}}$

$= \frac{68.6\pi}{1.372\pi} \times 3 = 150$

Question 11:

How many spherical lead shots each of diameter 4.2 cm can be obtained from a solid rectangular lead piece with dimensions 66 cm, 42 cm and 21 cm?

Solution:

Given that, lots of spherical lead shots made from a solid rectangular lead piece.

$\therefore$ Number of spherical lead shots

∴ Number of spherical lead shots

$$= \frac{\text{Volume of solid rectangular lead piece}}{\text{Volume of a spherical lead shot}} \quad \ldots(i)$$

Also, given that diameter of a spherical lead shot *i.e.*, sphere = 4.2 cm

∴ Radius of a spherical lead shot, $r = \frac{42}{2} = 2.1 \text{ cm}$ $\left[\because \text{ radius} = \frac{1}{2} \text{ diameter}\right]$

So, volume of a spherical lead shot *i.e.*, sphere

$$= \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times \frac{22}{7} \times (2.1)^3$$
$$= \frac{4}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 2.1$$
$$= \frac{4 \times 22 \times 21 \times 21 \times 21}{3 \times 7 \times 1000}$$

Now, length of rectangular lead piece, $l = 66$ cm

Breadth of rectangular lead piece, $b = 42$ cm

Height of rectangular lead piece, $h = 21$ cm

∴ Volume of a solid rectangular lead piece *i.e.*, cuboid $= l \times b \times h = 66 \times 42 \times 21$

From Eq. (i),

$$\text{Number of spherical lead shots} = \frac{66 \times 42 \times 21}{4 \times 22 \times 21 \times 21 \times 21} \times 3 \times 7 \times 1000$$
$$= \frac{3 \times 22 \times 21 \times 2 \times 21 \times 21 \times 1000}{4 \times 22 \times 21 \times 21 \times 21}$$
$$= 3 \times 2 \times 250$$
$$= 6 \times 250 = 1500$$

Hence, the required number of special lead shots is 1500.

Question 12:

How many spherical lead shots of diameter 4 cm can be made out of a solid cube of lead whose edge
measures 44 cm.

Solution:

Given that, lots of spherical lead shots made out of a solid cube of lead.

∴ Number of spherical lead shots

$$= \frac{\text{Volume of a solid cube of lead}}{\text{Volume of a spherical lead shot}} \quad \ldots(i)$$

Given that, diameter of a spherical lead shot i.e., sphere = 4cm

⇒ Radius of a spherical lead shot $(r) = \frac{4}{2}$

$r = 2$ cm $[\because \text{diameter} = 2 \times \text{radius}]$

So, volume of a spherical lead shot *i.e.*, sphere

$$= \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times \frac{22}{7} \times (2)^3$$
$$= \frac{4 \times 22 \times 8}{21} \text{ cm}^3$$

Now, since edge of a solid cube $(a) = 44$ cm

So, volume of a solid cube $= (a)^3 = (44)^3 = 44 \times 44 \times 44 \text{ cm}^3$

From Eq. (i),

$$\text{Number of spherical lead shots} = \frac{44 \times 44 \times 44}{4 \times 22 \times 8} \times 21$$
$$= 11 \times 21 \times 11 = 121 \times 21$$
$$= 2541$$

Hence, the required number of spherical lead shots is 2541.

Question 13:

A wall 24 m long, 0.4 m thick and 6 m high is constructed with the bricks each of dimensions 25 cm x 16 cm x 10 cm. If the mortar occupies $\frac{1}{6}$ th of the volume of the wall, then find the number of bricks used in constructing the wall.

Solution:

Given that, a wall is constructed with the help of bricks and mortar.

$$\therefore \quad \text{Number of bricks} = \frac{(\text{Volume of wall}) - \left(\frac{1}{10}\text{th volume of wall}\right)}{\text{Volume of a brick}} \quad ...(i)$$

Also, given that
Length of a wall $(l) = 24$ m,
Thickness of a wall $(b) = 0.4$ m,
Height of a wall $(h) = 6$ m
So, volume of a wall constructed with the bricks $= l \times b \times h$

$$= 24 \times 0.4 \times 6 = \frac{24 \times 4 \times 6}{10} \text{ m}^3$$

Now, $\frac{1}{10}$th volume of a wall $= \frac{1}{10} \times \frac{24 \times 4 \times 6}{10} = \frac{24 \times 4 \times 6}{10^2} \text{ m}^3$

and Length of a brick $(l_1) = 25 \text{ cm} = \frac{25}{100}$ m

$$\text{Breadth of a brick } (b_1) = 16 \text{ cm} = \frac{16}{100} \text{ m}$$

$$\text{Height of a brick } (h_1) = 10 \text{ cm} = \frac{10}{100} \text{ m}$$

So, volume of a brick $= l_1 \times b_1 \times h_1$

$$= \frac{25}{100} \times \frac{16}{100} \times \frac{10}{100} = \frac{25 \times 16}{10^5} \text{ m}^3$$

From Eq. (i),

$$\text{Number of bricks} = \frac{\left(\frac{24 \times 4 \times 6}{10} - \frac{24 \times 4 \times 6}{100}\right)}{\left(\frac{25 \times 16}{10^5}\right)}$$

$$= \frac{24 \times 4 \times 6}{100} \times 9 \times \frac{10^5}{25 \times 16}$$

$$= \frac{24 \times 4 \times 6 \times 9 \times 1000}{25 \times 16}$$

$$= 24 \times 6 \times 9 \times 10 = 12960$$

Hence, the required number of bricks used in constructing the wall is 12960.

Question 14:

Find the number of metallic circular disc with 1.5 cm base diameter and of height 0.2 cm to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

Solution:

Given that, lots of metallic circular disc to be melted to form a right circular cylinder. Here, a circular disc work as a circular cylinder.

Base diameter of metallic circular disc = 1.5 cm

$\therefore$ Radius of metallic circular disc $= \frac{1.5}{2}$ cm [$\because$ diameter $= 2 \times$ radius]

and height of metallic circular disc *i.e.*, $= 0.2$ cm

$\therefore$ Volume of a circular disc $= \pi \times (\text{Radius})^2 \times \text{Height}$

$$= \pi \times \left(\frac{1.5}{2}\right)^2 \times 0.2$$

$$= \frac{\pi}{4} \times 1.5 \times 1.5 \times 0.2$$

Now, height of a right circular cylinder $(h) = 10$ cm
and diameter of a right circular cylinder $= 4.5$ cm
$\Rightarrow$ Radius of a right circular cylinder $(r) = \frac{4.5}{2}$ cm

$\therefore$ Volume of right circular cylinder $= \pi r^2 h$

$$= \pi\left(\frac{4.5}{2}\right)^2 \times 10 = \frac{\pi}{4} \times 4.5 \times 4.5 \times 10$$

$$\therefore \text{Number of metallic circular disc} = \frac{\text{Volume of a right circular cylinder}}{\text{Volume of a metallic circular disc}}$$

$$= \frac{\frac{\pi}{4} \times 4.5 \times 4.5 \times 10}{\frac{\pi}{4} \times 1.5 \times 1.5 \times 0.2}$$

$$= \frac{3 \times 3 \times 10}{0.2} = \frac{900}{2} = 450$$

Hence, the required number of metallic circular disc is 450.

Exercise 12.4 Long Answer Type Questions

Question 1:

A solid metallic hemisphere of radius 8 cm is melted and recasted into a right circular cone of base radius 6 cm. Determine the height of the cone.

Solution:

Let height of the cone be h.

Given, radius of the base of the cone = 6 cm

$\therefore$ Volume of circular cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (6)^2 h = \frac{36\pi h}{3} = 12\pi h \text{ cm}^3$

Also, given radius of the hemisphere = 8 cm

$\therefore$ Volume of the hemisphere $= \frac{2}{3}\pi r^3 = \frac{2}{3}\pi (8)^3 = \frac{512 \times 2\pi}{3} \text{ cm}^3$

According to the question,

Volume of the cone = Volume of the hemisphere

$$\Rightarrow \quad 12\pi h = \frac{512 \times 2\pi}{3}$$

$$\therefore \quad h = \frac{512 \times 2\pi}{12 \times 3\pi}$$

$$= \frac{256}{9} = 28.44 \text{ cm}$$

Question 2:

A rectangular water tank of base 11 m x 6 m contains water upto a height of 5 m. If the water in the tank is transferred to a cylindrical tank of radius 3.5 m, find the height of the water level in the tank.

Solution:

Given, dimensions of base of rectangular tank = 11 m x 6 m and height of water = 5 m

Volume of the water in rectangular tank = 11 x 6 x 5 = 330 m^3

Also, given radius of the cylindrical tank = 3.5 m

Let height of water level in cylindrical tank be h.

Then, volume of the water in cylindrical tank $= \pi r^2 h = \pi (3.5)^2 \times h$

$$= \frac{22}{7} \times 3.5 \times 3.5 \times h$$

$$= 11.0 \times 3.5 \times h = 38.5\, h \text{ m}^3$$

According to the question,

$$330 = 38.5\, h \quad \text{[since, volume of water is same in both tanks]}$$

$$\therefore \quad h = \frac{330}{38.5} = \frac{3300}{385}$$

$$\therefore \quad = 8.57 \text{ m or } 8.6 \text{ m}$$

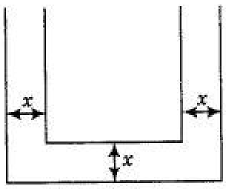
Hence, the height of water level in cylindrical tank is 8.6 m.

Question 3:

How many cubic centimetres of iron is required to construct an open box whose external dimensions are 36 cm, 25 cm and 16.5 cm provided the thickness of the iron is 1.5 cm. If one cubic centimetre of iron weights 7.5 g, then find the weight of the box.

Solution:

Let the length(l), breadth (b) and height (h) be the external dimension of an open box and thickness be x.



Given that,

external length of an open box (l) = 36 cm

external breadth of an open box (b) = 25 cm

and external height of an open box (h) = 16.5 cm

$\therefore$ External volume of an open box = lbh

$= 36 \times 25 \times 16.5$

$= 14850 \text{ cm}^3$

Since, the thickness of the iron (x) = 1.5 cm

So, internal length of an open box (l_1) = $l - 2x$

$= 36 \times 2 \times 1.5$

$= 36 - 3 = 33 \text{ cm}$

Therefore, internal breadth of an open box (b_2) = $b - 2x$

$= 25 - 2 \times 1.5 = 25 - 3 = 22 \text{ cm}$

and internal height of an open box (h_2) = $(h - x)$

$= 16.5 - 1.5 = 15 \text{ cm}$

So, internal volume of an open box = $(l - 2x) \cdot (b - 2x) \cdot (h - x)$

$= 33 \times 22 \times 15 = 10890 \text{ cm}^3$

Therefore, required iron to construct an open box

= External volume of an open box − Internal volume of an open box

$= 14850 - 10890 = 3960 \text{ cm}^3$

Hence, required iron to construct an open box is 3960 cm^3.

Given that, 1 cm^3 of iron weights = $7.5 \text{ g} = \frac{7.5}{1000} \text{ kg} = 0.0075 \text{ kg}$

$\therefore$ 3960 cm^3 of iron weights = $3960 \times 0.0075 = 29.7 \text{ kg}$

Question 4:

The barrel of a fountain pen, cylindrical in shape, is 7 cm long and 5 mm in diameter. A full barrel of ink in the pin is used up on writing 3300 words on an average. How many words can be written in a bottle of ink containing one-fifth of a litre?

Solution:

Given, length of the barrel of a fountain pen = 7 cm

and diameter = $5 \text{ mm} = \frac{5}{10} \text{ cm} = \frac{1}{2} \text{ cm}$

$\therefore$ Radius of the barrel = $\frac{1}{2 \times 2} = 0.25 \text{ cm}$

Volume of the barrel = $\pi r^2 h$ [since, its shape is cylindrical]

$= \frac{22}{7} \times (0.25)^2 \times 7$

$= 22 \times 0.0625 = 1.375 \text{ cm}^3$

Also, given volume of ink in the bottle = $\frac{1}{5}$ of litre = $\frac{1}{5} \times 1000 \text{ cm}^3 = 200 \text{ cm}^3$

Now, 1.375 cm^3 ink is used for writing number of words = 3300

$\therefore$ 1 cm^3 ink is used for writing number of words = $\frac{3300}{1.375}$

$\therefore$ 200 cm^3 ink is used for writing number of words = $\frac{3300}{1.375} \times 200 = 480000$

Question 5:

Water flows at the rate of 10 m min^{-1} through a cylindrical pipe 5 mm in diameter. How long would it take to fill a conical vessel whose diameter at the base is 40 cm and depth 24 cm?

Solution:

Given, speed of water flow = 10 m min^1 = 1000 cm/min and diameter of the pipe = 5 mm = $\frac{5}{10}$ cm

$\therefore$ Radius of the pipe = $\frac{5}{10 \times 2} = 0.25$ cm

$\therefore$ Area of the face of pipe = $\pi r^2 = \frac{22}{7} \times (0.25)^2 = 0.1964\ cm^2$

Also, given diameter of the conical vessel = 40 cm

$\therefore$ Radius of the conical vessel = $\frac{40}{2} = 20$ cm

and depth of the conical vessel = 24 cm

$$\therefore \quad \text{Volume of conical vessel} = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (20)^2 \times 24$$

$$= \frac{211200}{21} = 10057.14 cm^3$$

$$\therefore \quad \text{Required time} = \frac{\text{Volume of the conical vessel}}{\text{Area of the face of pipe} \times \text{Speed of water}}$$

$$= \frac{10057.14}{0.1964 \times 10 \times 100}$$

$$= 51.20 \text{ min} = 51 \text{ min} \frac{20}{100} \times 60 \text{ s} = 51 \text{ min } 12 \text{ s}$$

Question 6:

A heap of rice is in the form of a cone of diameter 9 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover heap?

Solution:

Given that, a heap of rice is in the form of a cone.

Height of a heap of rice i.e., cone (h) = 3.5 m

and diameter of a heap of rice i.e., cone = 9 m

Radius of a heap of rice *i.e.*, cone (r) = $\frac{9}{2}$ m

$$\text{So,} \quad \text{volume of rice} = \frac{1}{3}\pi \times r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{9}{2} \times \frac{9}{2} \times 3.5$$

$$= \frac{6237}{84} = 74.25\ m^3$$

Now, canvas cloth required to just cover heap of rice

$$= \text{Surface area of a heap of rice}$$

$$= \pi r l$$

$$= \frac{22}{7} \times r \times \sqrt{r^2 + h^2}$$

$$= \frac{22}{7} \times \frac{9}{2} \times \sqrt{\left(\frac{9}{2}\right)^2 + (3.5)^2}$$

$$= \frac{11 \times 9}{7} \times \sqrt{\frac{81}{4} + 12.25}$$

$$= \frac{99}{7} \times \sqrt{\frac{130}{4}} = \frac{99}{7} \times \sqrt{32.5}$$

$$= 14.142 \times 5.7$$

$$= 80.61\ m^2$$

Hence, 80.61 m^2 canvas cloth is required to just cover heap.

Question 7:

A factory manufactures 120000 pencils daily. The pencils are cylindrical in shape each of length 25 cm and circumference of base as 1.5 cm. Determine the cost of colouring the curved surfaces of the pencils manufactured in one day at ₹ 0.05 per dm^2.

Solution:

Given, pencils are cylindrical in shape.

Length of one pencil = 25 cm

and circumference of base = 1.5 cm

$$\Rightarrow \quad r = \frac{1.5 \times 7}{22 \times 2} = 0.2386 \text{ cm}$$

Now, curved surface area of one pencil $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 0.2386 \times 25$$

$$= \frac{262.46}{7} = 37.49 \text{ cm}^2$$

$$= \frac{37.49}{100} \text{ dm}^2 \qquad \left[\because 1 \text{ cm} = \frac{1}{10} \text{ dm}\right]$$

$$= 0.375 \text{ dm}^2$$

$\therefore$ Curved surface area of 120000 pencils $= 0.375 \times 120000 = 45000 \text{ dm}^2$

Now, cost of colouring 1 dm^2 curved surface of the pencils manufactured in one day

$$= ₹\ 0.05$$

Cost of colouring 45000 dm^2 curved surface = ₹ 2250

Question 8:

Water is flowing at the rate of 15 kmh^{-1} through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in pond rise by 21 cm?

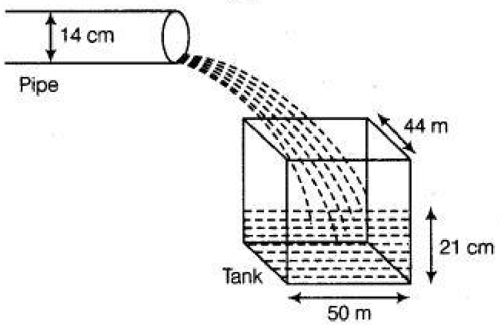
Solution:

Given, length of the pond= 50 m and width of the pond = 44 m

$$\text{Depth required of water} = 21 \text{ cm} = \frac{21}{100} \text{ m}$$

$$\therefore \quad \text{Volume of water in the pond} = \left(50 \times 44 \times \frac{21}{100}\right)^3 = 462 \text{ m}^3$$

Also, given radius of the pipe $= 7 \text{ cm} = \frac{7}{100}$ m



and speed of water flowing through the pipe $= (15 \times 1000) = 15000 \text{ mh}^{-1}$

Now, volume of water flow in 1 h $= \pi R^2 H$

$$= \left(\frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 15000\right)$$

$$= 231 \text{ m}^3$$

Since, 231 m^3 of water falls in the pond in 1 h.

So, 1 m^3 water falls in the pond in $\frac{1}{231}$ h.

Also, 462 m^3 of water falls in the pond in $\left(\frac{1}{231} \times 462\right)$ h $= 2$ h

Hence, the required time is 2 h.

Question 9:

A solid iron cuboidal block of dimensions 4.4 m x 2.6m x lm is recast into a hollow cylindrical pipe of

internal radius 30 cm and thickness 5 cm. Find the length of the pipe.

Solution:

Given that, a solid iron cuboidal block is recast into a hollow cylindrical pipe,

Length of cuboidal pipe (l) = 4.4 m

Breadth of cuboidal pipe (b) = 2.6 m and height of cuboidal pipe (h) = 1m

So, volume of a solid iron cuboidal block = $l \cdot b \cdot h$

$$= 4.4 \times 2.6 \times 1 = 11.44 \text{ m}^3$$

Also, internal radius of hollow cylindrical pipe (r_i) = 30 cm = 0.3 m

and thickness of hollow cylindrical pipe = 5 cm = 0.05 m

So, external radius of hollow cylindrical pipe $(r_e) = r_i +$ Thickness

$$= 0.3 + 0.05$$
$$= 0.35 \text{ m}$$

$\therefore$ Volume of hollow cylindrical pipe = Volume of cylindrical pipe with external radius − Volume of cylindrical pipe with internal radius

$$= \pi r_e^2 h_1 - \pi r_i^2 h_1 = \pi(r_e^2 - r_i^2) h_1$$
$$= \frac{22}{7}[(0.35)^2 - (0.3)^2] \cdot h_1$$
$$= \frac{22}{7} \times 0.65 \times 0.05 \times h_1 = 0.715 \times h_1 / 7$$

where, h_1 be the length of the hollow cylindrical pipe.

Now, by given condition,

Volume of solid iron cuboidal block = Volume of hollow cylindrical pipe

$\Rightarrow$ $$11.44 = 0.715 \times h / 7$$

$\therefore$ $$h = \frac{11.44 \times 7}{0.715} = 112 \text{ m}$$

Hence, required length of pipe is 112 m.

Question 10:

500 persons are taking a dip into a cuboidal pond which is 80 m long and 50 m broad. What is the rise of water level in the pond, if the average displacement of the water by a person is 0.04 m^3?

Solution:

Let the rise of water level in the pond be hm, when 500 persons are taking a dip into a cuboidal pond.

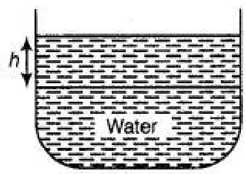
Given that,

Length of the cuboidal pond = 80 m

Breadth of the cuboidal pond = 50 m

Now, volume for the rise of water level in the pond

$$= \text{Length} \times \text{Breadth} \times \text{Height}$$
$$= 80 \times 50 \times h = 4000 h \text{ m}^3$$



Pond

and the average displacement of the water by a person = 0.04 m^3

So, the average displacement of the water by 500 persons = $500 \times 0.04 \text{ m}^3$

Now, by given condition,

Volume for the rise of water level in the pond = Average displacement of the water by 500 persons

$\Rightarrow$ $$4000 h = 500 \times 0.04$$

$\therefore$ $$h = \frac{500 \times 0.04}{4000} = \frac{20}{4000} = \frac{1}{200} \text{ m}$$
$$= 0.005 \text{ m}$$
$$= 0.005 \times 100 \text{ cm}$$
$$= 0.5 \text{ cm}$$

Hence, the required rise of water level in the pond is 0.5 cm.

Question 11:

glass spheres each of radius 2 cm are packed into a cuboidal box of internal dimensions 16 cm x 8 cm x 8 cm and then the box is filled with water. Find the volume of water filled in the box.

Solution:

Given, dimensions of the cuboidal = 16 cm x 8 cm x 8 cm

Volume of the cuboidal = 16 x 8 x 8 = 1024 cm^3

Also, given radius of one glass sphere = 2 cm

$\therefore$ Volume of one glass sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (2)^3$

$= \frac{704}{21} = 33.523 \text{ cm}^3$

Now, volume of 16 glass spheres $= 16 \times 33.523 = 536.37 \text{ cm}^3$

$\therefore$ Required volume of water = Volume of cuboidal – Volume of 16 glass spheres

$= 1024 - 536.37 = 487.6 \text{ cm}^3$

Question 12:

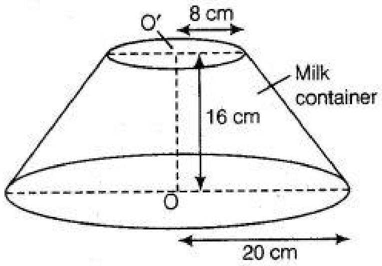
A milk container of height 16 cm is made of metal sheet in the form of a frustum of a cone with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of milk at the rate of ? 22 per L which the container can hold.

Solution:

Given that,height of milk container (h) = 16 cm,

Radius of lower end of milk container (r) = 8 cm

and radius of upper end of milk container (R) = 20 cm



$\therefore$ Volume of the milk container made of metal sheet in the form of a frustum of a cone

$= \frac{\pi h}{3}(R^2 + r^2 + Rr)$

$= \frac{22}{7} \times \frac{16}{3}[(20)^2 + (8)^2 + 20 \times 8]$

$= \frac{22 \times 16}{21}(400 + 64 + 160)$

$= \frac{22 \times 16 \times 624}{21} = \frac{219648}{21}$ $\quad [\because 1 \text{ L} = 1000 \text{ cm}^3]$

$= 10459.42 \text{ cm}^3 = 10.45942 \text{ L}$

So, volume of the milk container is 10459.42 cm^3.

$\because$ Cost of 1 L milk = ₹ 22

$\therefore$ Cost of 10.45942 L milk $= 22 \times 10.45942 =$ ₹ 230.12

Hence, the required cost of milk is ₹ 230.12

Question 13:

A cylindrical bucket of height 32 cm and base radius 18 cm is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

Solution:

Given, radius of the base of the bucket = 18 cm

Height of the bucket = 32 cm

So, volume of the sand in cylindrical bucket $= \pi r^2 h = \pi (18)^2 \times 32 = 10368\pi$

Also, given height of the conical heap (h) = 24 cm

Let radius of heap be r cm.

Then, volume of the sand in the heap $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi r^2 \times 24 = 8\pi r^2$

According to the question,

Volume of the sand in cylindrical bucket = Volume of the sand in conical heap

$$\Rightarrow \quad 10368\pi = 8\pi r^2$$
$$\Rightarrow \quad 10368 = 8r^2$$
$$\Rightarrow \quad r^2 = \frac{10368}{8} = 1296$$
$$\Rightarrow \quad r = 36 \text{ cm}$$

Again, let the slant height of the conical heap = l

Now,
$$l^2 = h^2 + r^2 = (24)^2 + (36)^2$$
$$= 576 + 1296 = 1872$$
$$\therefore \quad l = 43.267 \text{ cm}$$

Hence, radius of conical heap of sand = 36 cm

and slant height of conical heap = 43.267 cm

Question 14:

A rocket is in the form of a right circular cylinder closed at the lower end and surmounted by a cone with the same radius as that of the cylinder. The diameter and height of the cylinder are 6 cm and 12 cm, respectively. If the slant height of the conical portion is 5 cm, then find the total surface area and volume of the rocket, (use n = 3.14J)

Solution:

Since, rocket is the combination of a right circular cylinder and a cone.

Given, diameter of the cylinder = 6 cm

$\therefore$ Radius of the cylinder $= \frac{6}{2} = 3$ cm

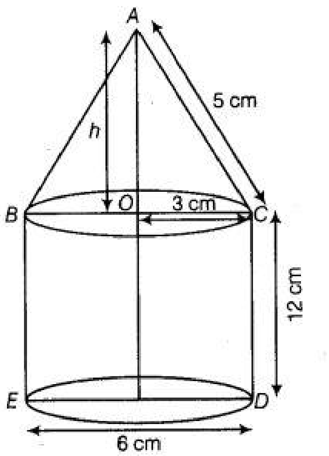
and height of the cylinder = 12 cm

$\therefore$ Volume of the cylinder $= \pi r^2 h = 3.14 \times (3)^2 \times 12$
$= 339.12 \text{ cm}^3$

and curved surface area $= 2\pi rh$
$= 2 \times 3.14 \times 3 \times 12 = 226.08$

Now, in right angled ΔAOC,

$$h = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$$



$\therefore$ Height of the cone, $h = 4$ cm

Radius of the cone, $r = 3$ cm

Now, volume of the cone

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times 3.14 \times (3)^2 \times 4$$
$$= \frac{113.04}{3} = 37.68 \text{ cm}^3$$

and curved surface area $= \pi rl = 3.14 \times 3 \times 5 = 47.1$

Hence, total volume of the rocket

$$= 339.12 + 37.68 = 376.8 \text{ cm}^3$$

and total surface area of the rocket = CSA of cone + CSA of cylinder + Area of base of cylinder

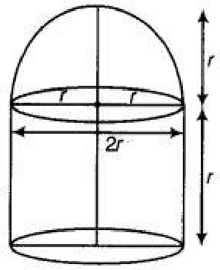
$$= 47.1 + 226.08 + 28.26$$
$$= 301.44 \text{ cm}^2$$

Question 15:

A building is in the form of a cylinder surmounted by a hemispherical vaulted dome and contains $41\frac{19}{21}$ m^3 of air. If the internal diameter of dome is equal to its total height above the floor, find the height of the building?

Solution:

Let total height of the building = Internal diameter of the dome = 2r m



$\therefore$ Radius of building (or dome) $= \frac{2r}{2} = r$ m

Height of cylinder $= 2r - r = r$ m

$\therefore$ Volume of the cylinder $= \pi r^2 (r) = \pi r^3$ m^3

and volume of hemispherical dome cylinder $= \frac{2}{3} \pi r^3$ m^3

$\therefore$ Total volume of the building = Volume of the cylinder + Volume of hemispherical dome

$$= \left(\pi r^3 + \frac{2}{3}\pi r^3\right) \text{m}^3 = \frac{5}{3}\pi r^3 \text{ m}^3$$

According to the question,

Volume of the building = Volume of the air

$$\Rightarrow \frac{5}{3}\pi r^3 = 41\frac{19}{21}$$

$$\Rightarrow \frac{5}{3}\pi r^3 = \frac{880}{21}$$

$$\Rightarrow r^3 = \frac{880 \times 7 \times 3}{21 \times 22 \times 5} = \frac{40 \times 21}{21 \times 5} = 8$$

$$\Rightarrow r^3 = 8 \Rightarrow r = 2$$

$\therefore$ Height of the building $= 2r = 2 \times 2 = 4$ m

Question 16:

A hemispherical bowl of internal radius 9 cm is full of liquid. The liquid is to be filled into cylindrical shaped bottles each of radius 1.5 cm and height 4 cm. How many bottles are needed to empty the bowl?

Solution:

Given, radius of hemispherical bowl, r = 9 cm

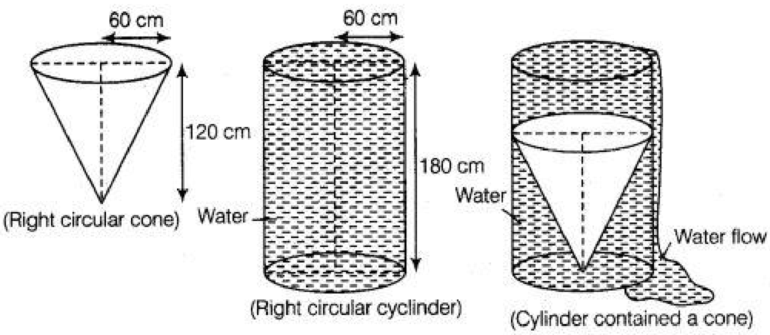
and radius of cylindrical bottles, R = 1.5 cm and height, h = 4 cm

$$\therefore \text{Number of required cylindrical bottles} = \frac{\text{Volume of hemispherical bowl}}{\text{Volume of one cylindrical bottle}}$$

$$= \frac{\frac{2}{3}\pi r^3}{\pi R^2 h} = \frac{\frac{2}{3} \times \pi \times 9 \times 9 \times 9}{\pi \times 1.5 \times 1.5 \times 4} = 54$$

Question 17:

A solid right circular cone of height 120 cm and radius 60 cm is placed in a right circular cylinder full of water of height 180 cm. Such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is equal to the radius to the cone.



Solution:

(i) Whenever we placed a solid right circular cone in a right circular cylinder with full of water, then volume of a solid right circular cone is equal to the volume of water failed from the cylinder.

(ii) Total volume of water in a cylinder is equal to the volume of the cylinder.

(iii) Volume of water left in the cylinder = Volume of the right circular cylinder – volume of a right circular cone.

Now, given that

Height of a right circular cone = 120 cm

Radius of a right circular cone = 60 cm

$\therefore$ Volume of a right circular cone $= \frac{1}{3}\pi r^2 \times h$

$$= \frac{1}{3} \times \frac{22}{7} \times 60 \times 60 \times 120$$
$$= \frac{22}{7} \times 20 \times 60 \times 120$$
$$= 144000\pi \text{ cm}^3$$

$\therefore$ Volume of a right circular cone = Volume of water falled from the cylinder

$= 144000\pi \text{ cm}^3$ [from point (i)]

Given that, height of a right circular cylinder = 180 cm

and radius of a right circular cylinder = Radius of a right circular cone = 60 cm

$\therefore$ Volume of a right circular cylinder $= \pi r^2 \times h$

$$= \pi \times 60 \times 60 \times 180$$
$$= 648000\,\pi \text{ cm}^3$$

So, volume of a right circular cylinder = Total volume of water in a cylinder

$= 648000\,\pi \text{ cm}^3$ [from point (ii)]

From point (iii),

Volume of water left in the cylinder

= Total volume of water in a cylinder – Volume of water falled from the cylinder when solid cone is placed in it

$$= 648000\,\pi - 144000\pi$$
$$= 504000\pi = 504000 \times \frac{22}{7} = 1584000 \text{ cm}^3$$
$$= \frac{1584000}{(10)^6} \text{ m}^3 = 1.584 \text{ m}^3$$

Hence, the required volume of water left in the cylinder is 1.584 m^3.

Question 18:

Water flows through a cylindrical pipe, whose inner radius is 1 cm, at the rate of 80 cms^{-1} in an empty cylindrical tank, the radius of whose base is 40 cm. What is the rise of water level in tank in half an hour?

Solution:

Given, radius of tank, r_1 = 40 cm

Let height of water level in tank in half an hour = 1 cm.

Also, given internal radius of cylindrical pipe, r_2 = 1 cm

and speed of water = 80 cm/s i.e., in 1 water flow = 80 cm

In 30 (min) water flow = 80x 60 x 30 = 144000 cm According to the question,

Volume of water in cylindrical tank = Volume of water flow from the circular pipe in half an hour

$$\Rightarrow \quad \pi r_1^2 h_1 = \pi r_2^2 h_2$$
$$\Rightarrow \quad 40 \times 40 \times h_1 = 1 \times 1 \times 144000$$
$$\therefore \quad h_1 = \frac{144000}{40 \times 40} = 90 \text{ cm}$$

Hence, the level of water in cylindrical tank rises 90 cm in half an hour.

Question 19:

The rain water from a roof of dimensions 22 m x 20 m drains into a cylindrical vessel having diameter of base 2 m and height 3.5 m. If the rain water collected from the roof just fill the cylindrical vessel, then find the rainfall (in cm).

Solution:

Given, length of roof = 22 m and breadth of roof = 20 m

Let the rainfall be a cm.

$\therefore$ Volume of water on the roof $= 22 \times 20 \times \frac{a}{100} = \frac{22a}{5} \text{ m}^3$

Also, we have radius of base of the cylindrical vessel = 1 m
and height of the cylindrical vessel = 3.5 m
$\therefore$ Volume of water in the cylindrical vessel when it is just full

$$= \left(\frac{22}{7} \times 1 \times 1 \times \frac{7}{2}\right) = 11 \text{ m}^3$$

Now, volume of water on the roof = Volume of water in the vessel

$$\Rightarrow \quad \frac{22a}{5} = 11$$

$$\therefore \quad a = \frac{11 \times 5}{22} = 2.5 \qquad [\because \text{volume of cylinder} = \pi \times (\text{radius})^2 \times \text{height}]$$

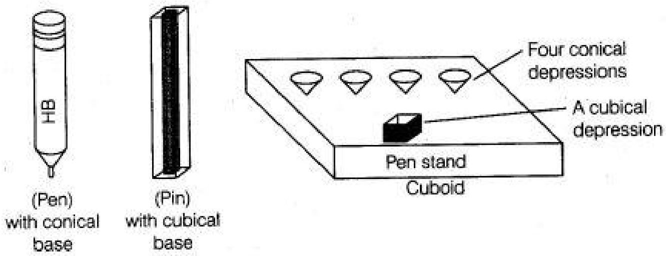
Hence, the rainfall is 2.5 cm

Question 20:

A pen stand made of wood is in the shape of a cuboid with four conical depressions and a cubical depression to hold the pens and pins, respectively. The dimensions of cubiod are 10 cm, 5 cm and 4 cm. The radius of each of the conical depressions is 0.5 cm and the depth is 2.1 cm. The edge of the cubical depression is 3 cm. Find the volume of the wood in the entire stand.

Solution:

Given that, length of cuboid pen stand (l) = 10 cm
Breadth of cubiod pen stand (b) = 5 cm
and height of cuboid pen stand (h) = 4 cm



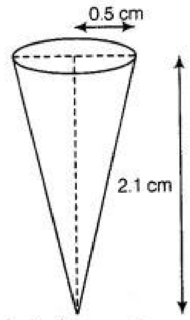
$\therefore$ Volume of cuboid pen stand $= l \times b \times h = 10 \times 5 \times 4 = 200 \text{ cm}^3$

Also, radius of conical depression $(r) = 0.5$ cm
and height (depth) of a conical depression $(h_1) = 2.1$ cm

$\therefore$ Volume of a conical depression $= \frac{1}{3}\pi r^2 h_1$

$$= \frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 2.1$$

$$= \frac{22 \times 5 \times 5}{1000} = \frac{22}{40} = \frac{11}{20} = 0.55 \text{ cm}^3$$



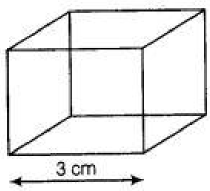
Conical depression

Also, given
Edge of cubical depression $(a) = 3$ cm
$\therefore$ Volume of cubical depression $= (a)^3 = (3)^3 = 27 \text{ cm}^3$

So, volume of 4 conical depressions

$$= 4 \times \text{Volume of a conical depression}$$

$$= 4 \times \frac{11}{20} = \frac{11}{5} \text{ cm}^3$$



Cubical depression

Hence, the volume of wood in the entire pen stand
= Volume of cuboid pen stand − Volume of 4 conical depressions − volume of a cubical depressions

$$= 200 - \frac{11}{5} - 27 = 200 - \frac{146}{5}$$

$$= 200 - 29.2 = 170.8 \text{ cm}^3$$

So, the required volume of the wood in the entire stand is 170.8 cm^3.