Unit 10 (Construction)

Exercise 10.1 Multiple Choice Questions (MCQs)

Question 1:

To divide a line segment AB in the ratio 5 : 7, first a ray AX is drawn, so that ∠BAX is an acute angle and then at equal distances points are marked on the ray AX such that the minimum number of these points is

(a) 8 (b) 10 (c) 11 (d) 12

Solution:

(d) We know that, to divide a line segment AB in the ratio m: n, first draw a ray AX which makes an acute angle ∠BAX, then marked m + n points at equal distance.

Here, m = 5, n = 7

So, minimum number of these points = m+n = 5 + 7 = 12.

Question 2:

To divide a line segment AB in the ratio 4 : 7, a ray AX is drawn first such that ∠BAX is an acute angle and then points A_1 A_2, A_3,… are located at equal distances on the ray AY and the point B is joined to

(a) A_{12} (b) A_{11} (c) A_{12} (d) A_9

Solution:

(b) Here, minimum 4+7 = 11 points are located at equal distances on the ray AX, and then B is joined to last point is A_{11}

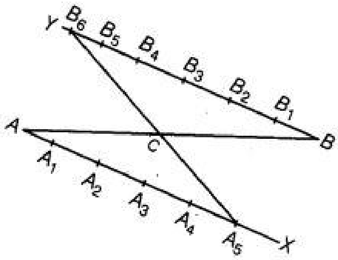
Question 3:

To divide a line segment AB in the ratio 5 : 6, draw a ray AY such that ∠BAX is an acute angle, then draw a ray BY parallel to AY and the points A_1, A_2, A_3,… and B_1, B_2, B_3,… are located to equal distances on ray AY and BY, respectively. Then, the points joined are

(a) A_5 and A_6 (b) A_6 and B_5 (c) A_4 and B_5 (d) A_5 and B_4

Solution:

(a) Given a line segment AB and we have to divide it in the ratio 5:6.



Steps of construction

1. Draw a ray AX making an acute ∠BAX.
2. Draw a ray BY parallel to AX by making ∠ABY equal to ∠BAX.
3. Now, locate the points A_1, A_2, A_3, A_4 and A_5 (m= 5) on AX and B_1, B_2, B_3, B_4, B_5 and B_6 (n = 6) such that all the points are at equal distance from each other.
4. Join B_6A_5. Let it intersect AB at a point C.
   Then, AC:BC = 5:6

Question 4:

To construct a triangle similar to a given ΔABC with its sides $\frac{3}{7}$ of the corresponding sides of ΔABC, first draw a ray BX such that ∠CBX is an acute angle and X lies on the opposite side of A with respect to BC. Then, locate points B_1, B_2, B_3,… on BX at equal distances and next step is to join

(a) B_{10} to C (b) B_{13} to C (c) B_7 to C (d) B_4 to C

Solution:

(c) Here, we locate points B_1, B_2, B_3, B_4, B_5, B_6 and B_7 on BX at equal distance and in next step join the last points is B_7 to C.

Question 5:

To construct a triangle similar to a given ΔABC with its sides $\frac{8}{5}$ of the corresponding sides of ΔABC draw a ray BX such that ∠ CBX is an acute angle and X is on the opposite side of A with respect to BC. The minimum number of points to be located at equal distances on ray BX is

(a) 5 (b) 8 (c)13 (d) 3

Solution:

(b) To construct a triangle similar to a given triangle, with its sides $\frac{m}{n}$ of the corresponding sides of given triangle the minimum number of points to be located at equal distance is equal to the greater of m and n is $\frac{8}{5}$

Hence, $\frac{m}{n}=\frac{8}{5}$

So, the minimum number of point to be located at equal distance on ray BX is 8.

Question 6:

To draw a pair of tangents to a circle which are inclined to each other at an angle of 60°, it is required to draw tangents at end points of those two radii of the circle, the angle between them should be

(a) 135° (b) 90° (c) 60° (d) 120°

Solution:

(d) The angle between them should be 120° because in that case the figure formed by the intersection point of pair of tangent, the two end points of those-two radii tangents are drawn) and the centre of the circle is a quadrilateral.

From figure it is quadrilateral,

∠POQ + ∠PRQ = 180° [∴ sum of opposite angles are 180°]

60°+ θ = 180°

θ=120

Hence, the required angle between them is 120°.

Exercise 10.2 Very Short Answer Type Questions

Question 1:

By geometrical construction, it is possible to divide a line segment in the ratio √3 $\frac{1}{\sqrt{3}}$.

Solution:

True

Given, ratio $= \sqrt{3} : \frac{1}{\sqrt{3}}$

$\therefore$ Required ratio = 3 : 1 [multiply $\sqrt{3}$ in each term]

So, $\sqrt{3} : \frac{1}{\sqrt{3}}$ can be simplified as 3 : 1 and 3 as well as 1 both are positive integer.

Hence, the geometrical constrution is possible to divide a line segment in the ratio 3 : 1

Question 2:

To construct a triangle similar to a given ΔABC with its sides $\frac{7}{3}$ of the corresponding sides of ΔABC, draw a ray BX making acute angle with BC and X lies on the opposite side of A with respect of BC. The points B_1, B_2, …, B_7 are located at equal distances on BX, B_3 is joined to C and then a line segment B_6C' is drawn parallel to B_3C, where C' lines on BC produced. Finally line segment A'C' is drawn parallel to AC.

Solution:

False

Steps of construction

1. Draw a line segment BC with suitable length.
2. Taking B and C as centres draw two arcs of suitable radii intersecting each other at A
3. Join BA and CA ΔABC is the required triangle.
4. From B draw any ray BX downwards making an acute angle CBX.
5. Locate seven points B_1, B_2, …, B_7 on SX, such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6 = B_6B_7$.
6. Join B_3C and from B_7 draw a line $B_7C' \parallel B_3C$ intersecting the extended line segment BC at C'.
7. From point C' draw C'A'|| CA intersecting the extended line segment BA at A'.Then, ΔA'BC' is the required triangle whose sides are $\frac{7}{3}$ of the corresponding sides of ΔABC.

Given that, segment B_6C' is drawn parallel to B_3C. But from our construction is never possible that segment B_6C' is parallel to B_3C because the similar triangle A'BC' has its sides $\frac{7}{3}$ of the corresponding sides of triangle ABC. So, B_7C' is parallel to B_3C.

Question 3:

A pair of tangents can be constructed from a point P to a circle of radius 3.5 cm situated at a distance of 3 cm from the centre.

Solution:

False

Since, the radius of the circle is 3.5 cm i.e., r = 3.5 cm and a point P situated at a distance of 3 cm from the centre i.e.,d= 3 cm

We see that, $r > d$

i.e., a point P lies inside the circle. So, no tangent can be drawn to a circle from a point lying inside it. '

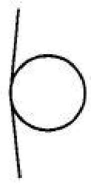
Question 4:

A pair of tangents can be constructed to a circle inclined at an angle of 170°.

Solution:

True

If the angle between the pair of tangents is always greater than 0 or less than 180°, then we can construct a pair of tangents to a circle.
Hence, we can drawn a pair of tangents to a circle inclined at an angle of 170°.



Exercise 10.3 Short Answer Type Questions

Question 1:

Draw a line segment of length 7 cm. Find a point P on it which divides it in the ratio 3:5.

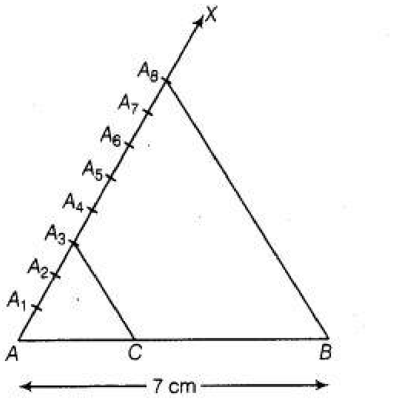
Solution:

Steps of construction

1. Draw a line segment AB = 7 cm.
2. Draw a ray AX, making an acute ∠BAX
3. Along AX, mark 3+ 5= 8 points
   A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8 such that
   $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7 = A_7A_8$
4. Join A_8B
5. From A_3, draw $A_3C \parallel A_8B$ meeting AB at C.
   [by making an angle equal to $\angle BA_8A$ at A_3]

Then, C is the point on AB which divides it in the ratio 3 : 5,

Thus, $AC : CB = 3 : 5$



Justification

Let $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = \dots = A_7A_8 = x$

In ΔABA_8, we have

$$A_3C \parallel A_8B$$

∴ $$\frac{AC}{CB} = \frac{AA_3}{A_3A_8} = \frac{3x}{5x} = \frac{3}{5}$$

Hence, $AC : CB = 3 : 5$

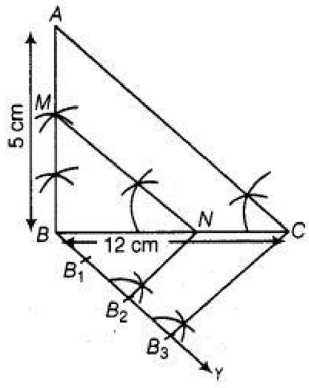
Question 2:

Draw a right ΔABC in which BC = 12 cm, AB = 5 cm and ∠B = 90°.Construct a triangle similar to it and of scale factor Is the new triangle also a right triangle?

Solution:

Steps of construction

1. Draw a line segment BC = 12 cm,
2. From 6 draw a line AB = 5 cm which makes right angle at B.



3. Join AC, ΔABC is the given right triangle.
4. From B draw an acute ∠CBY downwards.
5. On ray BY, mark three points B_1, $B_2$and B_3, such that $BB_1 = B_1B_2 = B_2B_3$.
6. Join B_3 C.
7. From point B_2 draw B_2N || B_3C intersect BC at N.
8. From point N draw NM || CA intersect BA at M. ΔMBN is the required triangle.
   ΔMBN is also a right angled triangle at B.

Question 3:

Draw a ΔABC in which BC = 6 cm, CA = 5 cm and AB = 4 cm. Construct a triangle similar to it and of scale factor $\frac{5}{3}$

Solution:

Steps of construction

1. Draw a line segment BC = 6 cm.
2. Taking Sand C as centres, draw two arcs of radii 4 cm and 5 cm intersecting each other at A.
3. Join BA and CA. ΔABC is the required triangle.
4. From B, draw any ray BX downwards making at acute angle.
5. Mark five points B_1, B_2,B_3, B_4 and B_5 on BX, such that
   BB, = B,B_2 = $B_2B_3 = B_3B_4 = B_4B_5$.
6. Join B_3C and from B_5 draw B_5M || B_3C intersecting the extended line segment BC at
7. From point M draw MN || CA intersecting the extended line segment BA at N.
   Then, ΔNBM is the required triangle whose sides is equal to $\frac{5}{3}$ of the corresponding sides of the ΔABC.
   Hence, ΔNBM is the required triangle.

Question 4:

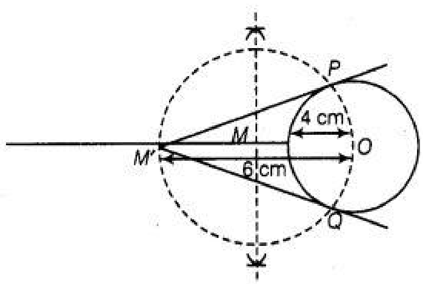
Construct a tangent to a circle of radius 4 cm from a point which is at a distance of 6 cm from its centre.

Solution:

Given, a point M' is at a distance of 6 cm from the centre of a circle of radius 4 cm.

Steps of construction

1. Draw a circle of radius 4 cm. Let centre of this circle is *O*.
2. Join *OM'* and bisect it. Let *M* be mid-point of *OM'*.
3. Taking *M* as centre and *MO* as radius draw a circle to intersect circle (0, 4) at two points, *P* and *Q*.
4. Join *PM'* and *QM'*. *PM'* and *QM'* are the required tangents from *M'* to circle *C* (0, 4).



Exercise 10.4 Long Answer Type Questions

Question 1:

Two line segments AB and AC include an angle of 60°, where AB = 5 cm and AC = 7 cm. Locate points P and Q on AB and AC, respectively such that AP = $\frac{3}{4}$AB and AQ = $\frac{1}{4}$AC. Join P and Q and measure the length PQ.

Solution:

Given that, AB = 5 cm and AC = 7 cm

Also, $AP = \frac{3}{4}AB$ and $AQ = \frac{1}{4}AC$ ...(i)

From Eq. (i), $AP = \frac{3}{4} \cdot AB = \frac{3}{4} \times 5 = \frac{15}{4}$ cm

Then, $PB = AB - AP = 5 - \frac{15}{4} = \frac{20 - 15}{4} = \frac{5}{4}$ cm [$\because$ P is any point on the AB]

$\therefore$ $AP : PB = \frac{15}{4} : \frac{5}{4} \Rightarrow AP : PB = 3 : 1$

i.e., scale factor of line segment AB is $\frac{3}{1}$.

Again from Eq. (i), $AQ = \frac{1}{4}AC = \frac{1}{4} \times 7 = \frac{7}{4}$ cm

Then, $QC = AC - AQ = 7 - \frac{7}{4}$

$= \frac{28 - 7}{4} = \frac{21}{4}$ cm [$\because$ Q is any point on the AC]

$\therefore$ $AQ : QC = \frac{7}{4} : \frac{21}{4} = 1 : 3$

$\Rightarrow$ $AQ : QC = 1 : 3$

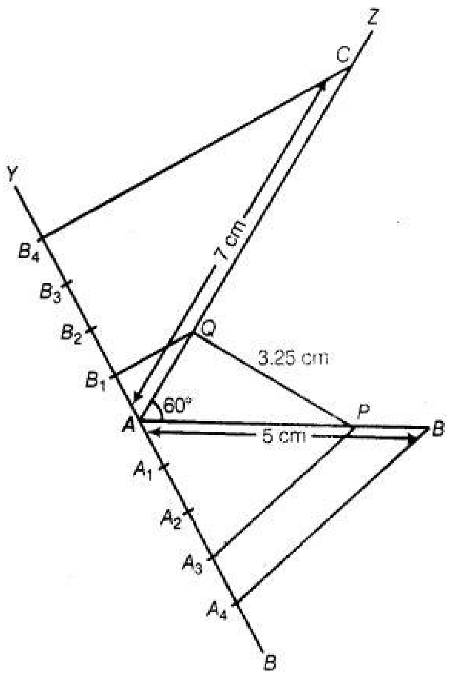
i.e., scale factor of line segment AQ is $\frac{1}{3}$.

Steps of construction

1. Draw a line segment AB = 5 cm.
2. Now draw a ray AZ making an acute ∠BAZ = 60°.
3. With A as centre and radius equal to 7 cm draw an arc cutting the line AZ at C.
4. Draw a ray AX, making an acute ∠BAX
5. Along AX, mark 1+3 = 4 points A_1, A_2, A_3, and A_4 Such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4$
6. Join A_4B
7. From A_3 draw $A_3P \parallel A_4B$ meeting AB at P [by making an angle equal to $\angle AA_4B$]

   Then, Pis the point on AB which divides it in the ratio 3:1.

   So, AP: PB = 3:1
8. Draw a ray AY, making an acute ∠CAY



9. Along AY, mark 3+1 = 4 points B_1, B_2, B_3 and B_4.
   Such that $AB_1 = B_1B_2 = B_2B_3 = B_3B_4$
10. Join B_4C
11. From B_1 draw $B_1Q \parallel B_4C$ meeting AC atQ. [by making an angle equal to $\angle AB_4C$]
    Then, Q is the point on AC which divides it in the ratio 1 : 3.
    So AQ:OC = 1:3
12. Finally, join PQ and its measurment is 3.25 cm.

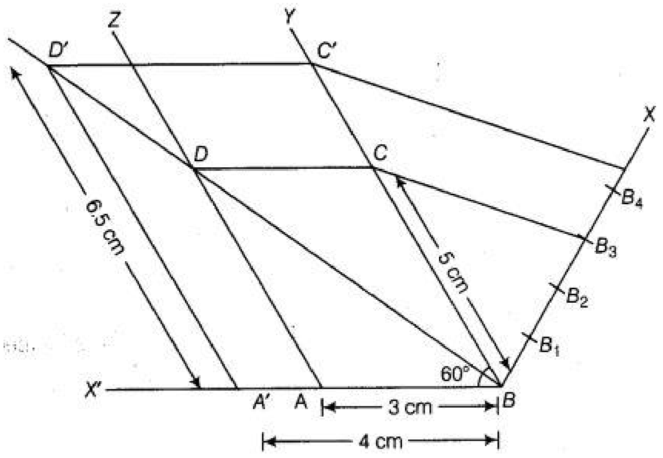
Question 2:

Draw a parallelogram ABCD in which BC = 5 cm, AB = 3 cm and ∠ABC = 60°, divide it into triangles BCD and ABD by the diagonal BD. Construct the triangles BD'C' similar to ΔBDC with Scale factor $\frac{1}{4}$. Draw the line segment D'A' parallel to DA, where A' lies on extended side BA. Is A'BC'D' a parallelogram?

Solution:

Steps of construction

1. Draw a line segment AB = 3 cm.
2. Now, draw a ray BY making an acute ∠ABY = 60°.
3. With B as centre and radius equal to 5 cm draw an arc cut the point C on
4. Again draw a ray AZ making an acute ∠ZAX' = 60°. [∵ BY || AZ, ∴ ∠YBX' = TAX' = 60°]
5. With A as centre and radius equal to 5 cm draw an arc cut the point D on AZ.



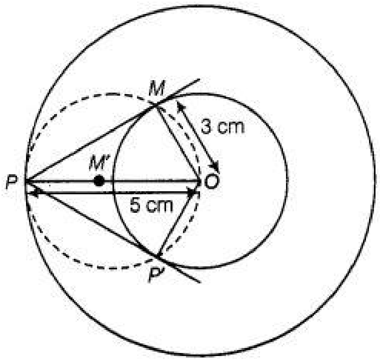
6. Now, join CD and finally make a parallelogram ABCD
7. Join BD, which is a diagonal of parallelogram ABCD
8. From B draw any ray BX downwards making an acute ∠CBX.
9. Locate 4 points B_1, B_2, B_3, B_4 on BX, such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$,
10. Join B_4C and from B_3C draw a line B_4C' || B_3C intersecting the extended line segment BC at C'.
11. From point C' draw C'D'|| CD intersecting the extended line segment BD at D'. Then, AD'BC' is the required triangle whose sides are $\frac{4}{3}$ of the corresponding sides of ΔDBC
12. Now draw a line segment D'A' parallel to DA, where A' lies on extended side BA i.e ray BX'.
13. Finally, we observe that A'BCD' is a parallelogram in which A'D' = 6.5,cm A'B = 4 cm and ∠A'BD' = 60° divide it into triangles BCD' and A'BD' by the diagonal BD.

Question 3:

Draw two concentric circles of radii 3 cm and 5 cm. Taking a point on outer circle construct the pair of tangents to the other. Measure the length of a tangent and verify it by actual calculation.

Solution:

Given, two concentric circles of radii 3 cm and 5 cm with centre 0. We have to draw pair of tangents from point P on outer circle to the other.



Steps of construction

1. Draw two concentric circles with centre 0 and radii 3 cm and 5 cm.
2. Taking any point P on outer circle. Join OP.
3. Bisect OP, let M' be the mid-point of .
   Taking M' as centre and OM' as radius draw a circle dotted which cuts the inner circle at M and P'.
4. Join P M and PP'. Thus, PM and PP' are the required tangents.
5. On measuring PM and PP', we find that PM = PP' = 4 cm.

Actual calculation

In right angle ΔOMP, $\angle PMO = 90°$

∴ $PM^2 = OP^2 - OM^2$

[by Pythagoras theorem *i.e.* $(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$]

⇒ $PM^2 = (5)^2 - (3)^2 = 25 - 9 = 16$

⇒ $PM = 4 \text{cm}$

Hence, the length of both tangents is 4 cm.

Question 4:

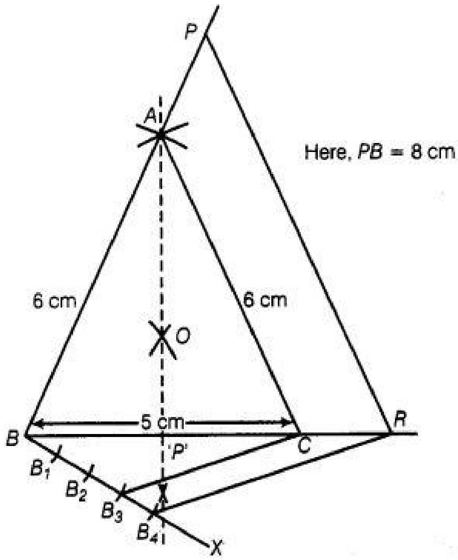
Draw an isosceles triangle ABC in which AB = AC = 6 cm and BC = 5 cm. Construct a triangle PQR similar to AABC in which PQ = 8 cm. Also justify the construction.

Solution:

Let ΔPQR and ΔABC are similar triangles, then its scale factor between the corresponding sides is $\frac{PQ}{AB} = \frac{8}{6} = \frac{4}{3}$

Steps of construction

1. Draw a line segment BC = 5 cm.
2. Construct OQ the perpendicular bisector of line segment BC meeting BC at P'.
3. Taking B and C as centres draw two arcs of equal radius 6 cm intersecting each other at A
4. Join BA and CA. So, ΔABC is the required isosceles triangle.



5. From B, draw any ray BX making an acute ∠CBX
6. Locate four points B_1, B_2, B_3 and B_4 on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$
7. Join B_3C and from B_4 draw a line $B_4R \parallel B_3C$ intersecting the extended line segment BC at R.
8. From point R, draw RP||CA meeting BA produced at P

   Then, ΔPBR is the required triangle.

Justification

∵ $B_4R \parallel B_3C$ (by construction)

∴ $\frac{BC}{CR} = \frac{3}{1}$

Now, $\frac{BR}{BC} = \frac{BC + CR}{BC}$

$= 1 + \frac{CR}{BC} = 1 + \frac{1}{3} = \frac{4}{3}$

Also, $RP \parallel CA$

∴ $\Delta ABC \sim \Delta PBR$

and $\frac{PB}{AB} = \frac{RP}{CA} = \frac{BR}{BC} = \frac{4}{3}$

Hence, the new triangle is similar to the given triangle whose sides are $\frac{4}{3}$ times of the corresponding sides of the isosceles ΔABC.

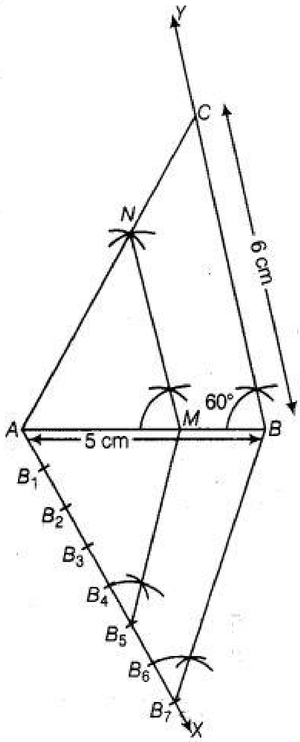
Question 5:

Draw a ΔABC in which AB = 5 cm, BC = 6 cm and ∠ABC = 60°. Construct a triangle similar to ABC with scale factor $\frac{5}{7}$ Justify the construction.

Solution:

Steps of construction

1. Draw a line segment $AB = 5$ cm.
2. From point B, draw $\angle ABY = 60°$ on which take $BC = 6$ cm.
3. Join AC, ΔABC is the required triangle.
4. From A, draw any ray AX downwards making an acute angle.
5. Mark 7 points $B_1, B_2, B_3, B_4, B_5, B_6$ and B_7 on AX, such that $AB_1 = B_1B_2 = B_2B_3$
   $= B_3B_4 = B_4B_5 = B_5B_6 = B_6B_7$.
6. Join B_7B and from B_5 draw $B_5M \parallel B_7B$ intersecting AB at M.
7. From point M draw $MN \parallel BC$ intersecting AC at N.
   Then, ΔAMN is the required triangle whose sides are equal to $\frac{5}{7}$ of the corresponding sides of the ΔABC.



Justification

Here, $B_5M \parallel B_7B$ (by construction)

$\therefore \quad \frac{AM}{MB} = \frac{5}{2}$

Now, $\frac{AB}{AM} = \frac{AM + MB}{AM}$

$= 1 + \frac{MB}{AM} = 1 + \frac{2}{5} = \frac{7}{5}$

Also, $MN \parallel BC$

$\therefore \quad \Delta AMN \sim \Delta ABC$

Therefore, $\frac{AM}{AB} = \frac{AN}{AC} = \frac{NM}{BC} = \frac{5}{7}$

Question 6:

Draw a circle of radius 4 cm. Construct a pair of tangents to it, the angle between which is 60°. Also justify the construction. Measure the distance between the centre of the circle and the point of intersection of tangents.

Solution:

In order to draw the pair of tangents, we follow the following steps

Steps of construction

1. Take a point 0 on the plane of the paper and draw a circle of radius OA = 4 cm.
2. Produce OA to B such that OA = AB = 4 cm.
3. Taking A as the centre draw a circle of radius AO = AB = 4 cm.
   Suppose it cuts the circle drawn in step 1 at P and Q.
4. Join BP and BQ to get desired tangents.

Justification In ΔOAP, we have

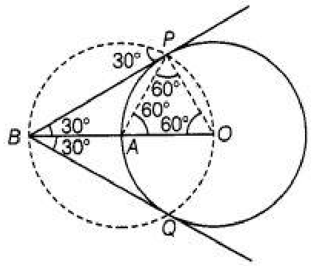
$OA = OP = 4\text{cm}$ (∵ Radius)

Also, $AP = 4\text{cm}$ (∵ Radius of circle with centre A)

∴ ΔOAP is equilateral

⇒ $\angle PAO = 60°$

⇒ $\angle BAP = 120°$



In ΔBAP, we have

$BA = AP$ and $\angle BAP = 120°$

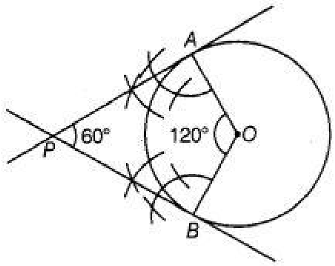
∴ $\angle ABP = \angle APB = 30°$

⇒ $\angle PBQ = 60°$

Alternate Method

Steps of construction

1. Take a point O on the plane of the paper and draw a circle with centre O and radius $OA = 4\text{cm}$.
2. At O construct radii OA and OB such that to $\angle AOB$ equal 120° *i.e.*, supplement of the angle between the tangents.
3. Draw perpendiculars to OA and OB at A and B, respectively. Suppose these perpendiculars intersect at P. Then, PA and PB are required tangents.



Justification

In quadrilateral $OAPB$, we have

$\angle OAP = \angle OBP = 90°$

and $\angle AOB = 120°$

∴ $\angle OAP + \angle OBP + \angle AOB + \angle APB = 360°$

⇒ $90° + 90° + 120° + \angle APB = 360°$

∴ $\angle APB = 360° - (90° + 90° + 120°)$

$= 360° - 300° = 60°$

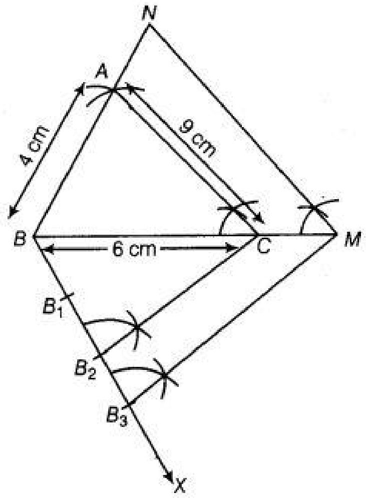
Question 7:

Draw a ΔABC in which AB = 4 cm, SC = 6 cm and AC = 9 cm. Construct a triangle similar to ΔABC with scale factor $\frac{1}{4}$ Justify the construction. Are the two triangles congruent? Note that, all the three angls and two sides of the two triangles are equal.

Solution:

Steps of construction

1. Draw a line segment BC = 6 cm.
2. Taking B and C as centres, draw two arcs of radii 4 cm and 9 cm intersecting each other at A.
3. Join BA and CA, ΔABC is the required triangle.
4. From B, draw any ray BX downwards making an acute angle.
5. Mark three points B_1, B_2, B_3 on BX, such that $BB_1 = B_1B_2 = B_2B_3$.



1. Join B_2C and from B_3 draw B_3M || B_2C intersecting the extended line segment BC at
2. From point M, draw MN||CA intersecting the extended line segment BA to N.
   Then,ΔNBM is the required triangle whose sides are equals to of the corresponding sides of the ΔABC

The two triangles are not congruent because, if two triangles are congruent, then they have same shape and same size. Here, all the three angles are same but three sides are not same one side is different.

Justification

Here, $B_3M \parallel B_2C$

$\therefore$ $\frac{BC}{CM} = \frac{2}{1}$

Now, $\frac{BM}{BC} = \frac{BC + CM}{BC}$

$= 1 + \frac{CM}{BC} = 1 + \frac{1}{2} = \frac{3}{2}$

Also, $MN \parallel CA$

$\therefore$ $\Delta ABC \sim \Delta NBM$

Therefore, $\frac{NB}{AB} = \frac{NM}{AC} = \frac{BM}{BC} = \frac{3}{2}$