# Unit 1(Real Numbers)

## Exercise 1.1 Multiple Choice Questions (MCQs)

**Question 1:**

For some integer m, every even integer is of the form

(a) m (b) m +1 (c) 2m +1 (d) 2m

**Solution:**

(c) We know that, even integers are 2, 4, 6,…

So, it can be written in the form of 2m.

where, $m = \text{Integer} = Z$ [since, integer is represented by $Z$]

or $m = \cdots, -1, 0, 1, 2, 3, \ldots$

$\therefore$ $2m = \cdots, -2, 0, 2, 4, 6 \ldots$

**Alternate Method**

Let 'a' be a positive integer. On dividing 'a' by 2, let $m$ be the quotient and $r$ be the remainder. Then, by Euclid's division algorithm, we have

$a = 2m + r$, where $a \le r < 2$ i.e., $r = 0$ and $r = 1$.

$\Rightarrow$ $a = 2m$ or $a = 2m + 1$

when, $a = 2m$ for some integer $m$, then clearly $a$ is even.

**Question 2:**

For some integer q, every odd integer is of the form

(a) q (b) q +1 (c) 2g (d) 2q +1

**Solution:**

***(d)*** We know that, odd integers are 1, 3, 5, ⋯

So, it can be written in the form of $2q + 1$.

where, $q = \text{integer} = Z$

or $q = \cdots, -1, 0, 1, 2, 3, \ldots$

$\therefore$ $2q + 1 = \cdots, -3, -1, 1, 3, 5, \ldots$

**Alternate Method**

Let 'a' be given positive integer. On dividing 'a' by 2, let $q$ be the quotient and $r$ be the remainder. Then, by Euclid's division algorithm, we have

$a = 2q + r$, where $0 \le r < 2$

$\Rightarrow$ $a = 2q + r$, where $r = 0$ or $r = 1$

$\Rightarrow$ $a = 2q$ or $2q + 1$

when $a = 2q + 1$ for some integer $q$, then clearly $a$ is odd.

**Question 3:**

$n^2 - 1$ is divisible by 8, if $n$ is

(a) an integer (b) a natural number

(c) an odd integer (d) an even integer

**Solution:**

Let $a = n^2 - 1$

Here $n$ can be ever or odd.

**Case I** $n$ = Even *i.e.*, $n = 2k$, where $k$ is an integer.

$\Rightarrow \quad a = (2k)^2 - 1$

$\Rightarrow \quad a = 4k^2 - 1$

At $k = -1, = 4(-1)^2 - 1 = 4 - 1 = 3$, which is not divisible by 8.

At $k = 0, a = 4(0)^2 - 1 = 0 - 1 = -1$, which is not divisible by 8, which is not

**Case II** $n$ = Odd *i.e.*, $n = 2k + 1$, where $k$ is an odd integer.

$\Rightarrow \quad a = 2k + 1$

$\Rightarrow \quad a = (2k + 1)^2 - 1$

$\Rightarrow \quad a = 4K^2 + 4k + 1 - 1$

$\Rightarrow \quad a = 4k^2 + 4k$

$\Rightarrow \quad a = 4k(k + 1)$

At $k = -1, a = 4(-1)(-1 + 1) = 0$ which is divisible by 8.

At $k = 0, a = 4(0)(0 + 1) = 4$ which is divisible by 8.

At $k = 1, a = 4(1)(1 + 1) = 8$ which is divisible by 8.

Hence, we can conclude from above two cases, if $n$ is odd, then $n^2 - 1$ is divisible by 8.

**Question 4:**

If the HCF of 65 and 117 is expressible in the form 65m -117, then the value of m is

(a) 4 (b) 2 (c) 1 (d) 3

**Solution:**

***(b)*** By Euclid's division algorithm,

$b = aq + r, 0 \le r < a$ [$\because$ dividend = divisor × quotient + remainder]

$\Rightarrow \quad 117 = 65 \times 1 + 52$

$\Rightarrow \quad 65 = 52 \times 1 + 13$

$\Rightarrow \quad 52 = 13 \times 4 + 0$

$\therefore \quad$ HCF (65, 117) = 13 ...(i)

Also, given that, HCF (65, 117) = $65m - 117$ ...(ii)

From Eqs. (i) and (ii),

$65m - 117 = 13$

$\Rightarrow \quad 65m = 130$

$\Rightarrow \quad m = 2$

**Question 5:**

The largest number which divides 70 and 125, leaving remainders respectively, is

(a) 13 (b) 65 (c) 875 (d) 1750

**Solution:**

(a) Since, 5 and 8 are the remainders of 70 and 125, respectively. Thus, after subtracting these remainders from the numbers, we have the numbers 65 = (70-5), 117 = (125 – 8), which is divisible by the required number.

Now, required number = HCF of 65,117 [for the largest number]

For this, $117 = 65 \times 1 + 52$ [$\because$ dividend = divisor × quotient + remainder]

$\Rightarrow \quad 65 = 52 \times 1 + 13$

$\Rightarrow \quad 52 = 13 \times 4 + 0$

$\therefore \quad$ HCF = 13

Hence, 13 is the largest number which divides 70 and 125, leaving remainders 5 and 8.

**Question 6:**

If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$, wfiere x, y are prime numbers, then HCF (a, b) is

(a) xy (b) $xy^2$ (c)$x^3y^3$ (d) $xy^2$

**Solution:**

**(b)** Given that, $a = x^3 y^2 = x \times x \times x \times y \times y$

and $b = xy^3 = x \times y \times y \times y$

$\therefore$ HCF of $a$ and $b$ $= \text{HCF}(x^3y^2, xy^3) = x \times y \times y = xy^2$

[since, HCF is the product of the smallest power of each common prime facter involved in the numbers]

**Question 7:**

If two positive integers p and q can be expressed' as $p=ab^2$ and $q = a^3b$; where 0, b being prime numbers, then LCM (p, q) is equal to

(a) ab (b) $a^2b^2$ (c) $a^3b^2$ (d) $a^3b^3$

**Solution:**

**(c)** Given that, $p = ab^2 = a \times b \times b$

and $q = a^3b = a \times a \times a \times b$

$\therefore$ LCM of $p$ and $q$ $= \text{LCM}(ab^2, a^3b) = a \times b \times b \times a \times a = a^3b^2$

[since, LCM is the product of the greatest power of each prime factor involved in the numbers]

**Question 8:**

The product of a non-zero rational and an irrational number is

(a) always irrational (b) always rational

(c) rational or irrational (d) one

**Solution:**

**(a)** Product of a non-zero rational and an irrational number is always irrational *i.e.*, $\frac{3}{4} \times \sqrt{2} = \frac{3\sqrt{2}}{4}$ (irrational).

**Question 9:**

The least number that is divisible by all the numbers from 1 to 10 (both inclusive)

(a) 10 (b) 100 (c) 504 (d) 2520

**Solution:**

(d) Factors of 1 to 10 numbers

$$1 = 1$$
$$2 = 1 \times 2$$
$$3 = 1 \times 3$$
$$4 = 1 \times 2 \times 2$$
$$5 = 1 \times 5$$
$$6 = 1 \times 2 \times 3$$
$$7 = 1 \times 7$$
$$8 = 1 \times 2 \times 2 \times 2$$
$$9 = 1 \times 3 \times 3$$
$$10 = 1 \times 2 \times 5$$

$\therefore$ LCM of number 1 to 10 $= \text{LCM}(1, 2, 3, 4, 5, 6, 7, 8, 9, 10)$

$= 1 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 2520$

**Question 10:**

The decimal expansion of the rational number $\frac{14587}{1250}$ will terminate after

(a) one decimal place (b) two decimal places

(c) three decimal places (d) four decimal places

**Solution:**

***(d)*** Rational number $= \frac{14587}{1250} = \frac{14587}{2^1 \times 5^4}$

$$= \frac{14587}{10 \times 5^3} \times \frac{(2)^3}{(2)^3}$$

$$= \frac{14587 \times 8}{10 \times 1000}$$

$$= \frac{116696}{10000} = 11.6696$$

| | |
|---|---|
| 2 | 1250 |
| 5 | 625 |
| 5 | 125 |
| 5 | 25 |
| 5 | 5 |
| | 1 |

Hence, given rational number will terminate after four decimal places.

## Exercise 1.2 Very Short Answer Type Questions

**Question 1:**

Write whether every positive integer can be of the form 4q + 2, where q is an integer. Justify your answer.

**Solution:**

No, by Euclid's Lemma, b = aq + r,0<,r<a [-.-dividend = divisor x quotient + remainder] Here, b is any positive integer a = 4, b = Aq + r for 0 ^ r < 4 i.e., r = 0,1,2, 3 So, this must be in the form Aq, 4q + 1, Aq + 2 or 4q + 3.

**Question 2:**

The product of two consecutive positive integers is divisible by 2′. Is this statement true or false? Give reasons.

**Solution:**

yes, two consecutive integers can be n, (n +1). So, one number of these two must be divisible by 2. Hence, product of numbers is divisible by 2.

**Question 3:**

The product of three consecutive positive integers is divisible by 6′ Is this statement true or false? Justify your answer.

**Solution:**

yes, three consecutive integers can be n, (n + 1)and (n + 2).

So, one number of these three must be divisible by 2 and another one must be divisible by 3. Hence,

product of numbers is divisible by 6.

**Question 4:**

Write whether the square of any positive integer can be of the form 3m + 2, where m is a natural number. Justify your answer.

**Solution:**

*No*, by Euclid's lemma, $b = aq + r, 0 \le r \le a$ Here, $b$ is any positive integer, $a = 3, b = 3q + r$ for $0 \le r \le 2$

So, any positive integer is of the form $3k$, $3k + 1$ or $3k + 2$.

Now, $(3k)^2 = 9k^2 = 3m$ [where, $m = 3k^2$]

and $(3k + 1)^2 = 9k^2 + 6k + 1$

$= 3(3k^2 + 2k) + 1 = 3m + 1$ [where, $m = 3k^2 + 2k$]

Also, $(3k + 2)^2 = 9k^2 + 12k + 4$ [$\because (a + b)^2 = a^2 + 2ab + b^2$]

$= 9k^2 + 12k + 3 + 1$

$= 3(3k^2 + 4k + 1) + 1$

$= 3m + 1$ [where, $m = 3k^2 + 4k + 1$]

which is in the form of $3m$ and $3m+1$. Hence, square of any positive number cannot be of the form $3m+2$.

**Question 5:**

A positive integer is of the form 3q +1, q being a natural number. Can you write its square in any form other than 3m+1, i.e., 3m or 3m + 2 for some integer ml Justify your answer.

**Solution:**

*No*, by Euclid's Lemma, $b = aq + r, 0 \le r < a$

Here, $b$ is any positive integer $a = 3, b = 3q + r$ for $0 \le r < 3$

So, this must be in the form $3q$, $3q + 1$ or $3q + 2$.

Now, $(3q)^2 = 9q^2 = 3m$ [here, $m = 3q^2$]

and $(3q + 1)^2 = 9q^2 + 6q + 1$

$= 3(3q^2 + 2q) + 1 = 3m + 1$ [where, $m = 3q^2 + 2q$]

Also, $(3q + 2)^2 = 9q^2 + 12q + 4$

$= 9q^2 + 12q + 3 + 1$

$= 3(3q^2 + 4q + 1) + 1$

$= 3m + 1$ [here, $m = 3q^2 + 4q + 1$]

Hence, square of a positive integer is of the form 3g + 1 is always in the form 3 m + 1 for some integer m.

**Question 6:**

The numbers 525 and 3000 are both divisible only by 3, 5, 15, 25 and 75. What is HCF (525, 3000)? Justify your answer.

**Solution:**

Since, the HCF (525, 3000) = 75

By Euclid's Lemma, $3000 = 525 \times 5 + 375$ [$\because$ dividend = divisor × quotient + remainder]

$525 = 375 \times 1 + 150$

$375 = 150 \times 2 + 75$

$150 = 75 \times 2 + 0$

and the numbers 3, 5, 15, 25 and 75 divides the numbers 525 and 3000 that mean these terms are common in both 525 and 3000. So, the highest common factor among these is 75.

**Question 7:**

Explain why3 x 5 x 7 + 7 is a composite number,

**Solution:**

We have, $3 \times 5 \times 7 + 7 = 105 + 7 = 112$

Now, $112 = 2 \times 2 \times 2 \times 2 \times 7 = 2^4 \times 7$

So, it is the product of prime factors 2 and 7. i.e., it has more than two factors.

Hence, it is a composite number.

**Question 8:**

Can two numbers have 18 as their HCF and 380 as their LCM? Give reasons.

**Solution:**

No, because HCF is always a factor of LCM but here 18 is not a factor of 380.

**Question 9:**

Without actually performing the long division, find if $\frac{987}{10500}$ will have terminating or non-terminating (repeating) decimal expansion. Give reasons for your answer.

**Solution:**

Yes, after simplification denominator has factor $5^3 \cdot 2^2$ and which is of the type $2^m \cdot 5^n$. So, this is terminating decimal.

$$\because \quad \frac{987}{10500} = \frac{47}{500} = \frac{47}{5^3 \cdot 2^2} \times \frac{2}{2}$$

$$= \frac{94}{5^3 \times 2^3} = \frac{94}{(10)^3} = \frac{94}{1000} = 0.094$$

**Question 10:**

A rational number in its decimal expansion is 327.7081. What can you say about the prime factors of q, when this number is expressed in the form p/q ? Give reasons.

**Solution:**

327.7081 is terminating decimal number. So, it represents a rational number and also its denominator must have the form $2^m \times 5^n$ .

Thus, $$327.7081 = \frac{3277081}{10000} = \frac{p}{q}$$

$$\therefore \quad q = 10^4 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$$

$$= 2^4 \times 5^4 = (2 \times 5)^4$$

Hence, the prime factors of q is 2 and 5.

## Exercise 1.3 Short Answer Type Questions

**Question 1:**

Show that the square of any positive integer is either of the form 4**q** or 4g + 1 for some integer **q.**

**Solution:**

**Let a be an arbitrary positive integer. Then, by, Euclid's division algorithm, corresponding**
**to the positive integers a and 4, there exist non-negative integers m and r, such that**

$$a = 4m + r, \text{ where } 0 \le r < 4$$

$\Rightarrow$ $$a^2 = 16m^2 + r^2 + 8mr \quad \ldots(i)$$

where, $0 \le r < 4$ $\quad [\because (a+b)^2 = a^2 + 2ab + b^2]$

**Case I** When $r = 0$, then putting $r = 0$ in Eq. (i), we get

$$a^2 = 16m^2 = 4(4m^2) = 4q$$

where, $q = 4m^2$ is an integer.

**Case II** When $r = 1$, then putting $r = 1$ in Eq. (i), we get

$$a^2 = 16m^2 + 1 + 8m$$
$$= 4(4m^2 + 2m) + 1 = 4q + 1$$

where, $q = (4m^2 + 2m)$ is an integer.

**Case III** When $r = 2$, then putting $r = 2$ in Eq. (i), we get

$$a^2 = 16m^2 + 4 + 16m$$
$$= 4(4m^2 + 4m + 1) = 4q$$

where, $q = (4m^2 + 4m + 1)$ is an integer.

**Case IV** When $r = 3$, then putting $r = 3$ in Eq. (i), we get

$$a^2 = 16m^2 + 9 + 24m = 16m^2 + 24m + 8 + 1$$
$$= 4(4m^2 + 6m + 2) + 1 = 4q + 1$$

where, $q = (4m^2 + 6m + 2)$ is an integer.

**Hence, the square of any positive integer is either of the form 4 q or 4 q + 1 for some integer q.**

**Question 2:**

**Show that cube of any positive integer is of the form 4m, 4m + 1 or Am + 3, for some integer m.**

**Solution:**

**Let a be an arbitrary positive integer. Then, by Euclid's division algorithm, corresponding to the positive integers a and 4, there exist non-negative integers q and r such that a = 4q + r, where 0< r< 4**

$$a = 4q + r, \text{ where } 0 \le r < 4$$

$\Rightarrow$ $$a^3 = (4q + r)^3 = 64q^3 + r^3 + 12qr^2 + 48q^2r$$

$$[\because (a+b)^3 = a^3 + b^3 + 3ab^2 + 3a^2b]$$

$\Rightarrow$ $$a^3 = (64q^2 + 48q^2r + 12qr^2) + r^3 \quad \ldots(i)$$

where, $0 \le r < 4$

**Case I** When $r = 0$,

Putting $r = 0$ in Eq. (i), we get

$$a^3 = 64q^3 = 4(16q^3)$$

$\Rightarrow a^3 + 4m$ where $m = 16q^3$ is an integer.

**Case II** When $r = 1$, then putting $r = 1$ in Eq. (i), we get

$$a^3 = 64q^3 + 48q^2 + 12q + 1$$
$$= 4(16q^3 + 12q^2 + 3q) + 1$$
$$= 4m + 1$$

where, $m = (16q^2 + 12q^2 + 3q)$ is an integer.

**Case III** When $r = 2$, then putting $r = 2$ in Eq. (i), we get

$$a^3 = 64q^3 + 144q^2 + 108q + 27$$
$$= 64q^3 + 144q^2 + 108q + 24 + 3$$
$$= 4(16q^3 + 36q^2 + 27q + 6) + 3 = 4m + 3$$

where, $m = (16q^3 + 36q^2 + 27q + 6)$ is an integer.

**Hence, the cube of any positive integer is of the form 4m, 4m + 1 or 4m + 3 for some integer m.**

**Question 3:**

**Show that the square of any positive integer cannot be of the form 5q + 2 or 5q + 3 for any integer q.**

**Solution:**

**Let a be an arbitrary positive integer.**

**Then, by Euclid's divisions Algorithm, corresponding to the positive integers a and 5, there exist non-negative integers m and r such that**

$$a = 5m + r, \text{ where } 0 \le r < 5$$

$\Rightarrow$ $a^2 = (5m + r)^2 = 25m^2 + r^2 + 10mr$ $[\because (a + b)^2 = a^2 + 2ab + b^2]$

$\Rightarrow$ $a^2 = 5(5m^2 + 2mr) + r^2$ ...(i)

where, $0 \le r < 5$

**Case I** When $r = 0$, then putting $r = 0$ in Eq. (i), we get

$$a^2 = 5(5m^2) = 5q$$

where, $q = 5m^2$ is an integer.

**Case II** When $r = 1$, then putting $r = 1$ is Eq. (i), we get

$$a^2 = 5(5m^2 + 2m) + 1$$

$\Rightarrow$ $q = 5q + 1$

where, $q = (5m^2 + 2m)$ is an integer.

**Case III** When $r = 2$, then putting $r = 2$ in Eq. (i), we get

$$a^2 = 5(5m^2 + 4m) + 4 = 5q + 4$$

where, $q = (5m^2 + 4m)$ is an integer.

**Case IV** When $r = 3$, then putting $r = 3$ in Eq. (i), we get

$$a^2 = 5(5m^2 + 6m) + 9 = 5(5m^2 + 6m) + 5 + 4$$

$$= 5(5m^2 + 6m + 1) + 4 = 5q + 4$$

where, $q = (5m^2 + 6m + 1)$ is an integer.

**Case V** When $r = 4$, then putting $r = 4$ in Eq. (i), we get

$$a^2 = 5(5m^2 + 8m) + 16 = 5(5m^2 + 8m) + 15 + 1$$

$\Rightarrow$ $a^2 = 5(5m^2 + 8m + 3) + 1 = 5q + 1$

where, $q = (5m^2 + 8m + 3)$ is an integer.

**Hence, the square of any positive integer cannot be of the form 5q + 2 or 5q + 3 for any integer q.**

**Question 4:**

**Show that the square of any positive integer cannot be of the form 6m+ 2 or 6m + 5 for any integer m.**

**Solution:**

**Let a be an arbitrary positive integer, then by Euclid's division algorithm, corresponding to the positive integers a and 6, there exist non-negative integers q and r such that a = 6q + r, where 0< r< 6**

$$a = 6q + r, \text{ where } 0 \le r < 6$$

$\Rightarrow$ $$a^2 = (6q + r)^2 = 36q^2 + r^2 + 12qr \quad [\because (a+b)^2 = a^2 + 2ab + b^2]$$

$\Rightarrow$ $$a^2 = 6(6q^2 + 2qr) + r^2 \quad \ldots(i)$$

where, $0 \le r < 6$

**Case I** When $r = 0$, then putting $r = 0$ in Eq. (i), we get

$$a^2 = 6(6q^2) = 6m$$

where, $m = 6q^2$ is an integer.

**Case II** When $r = 1$, then putting $r = 1$ in Eq. (i), we get

$$a^2 = 6(6q^2 + 2q) + 1 = 6m + 1$$

where, $m = (6q^2 + 2q)$ is an integer.

**Case III** When $r = 2$, then putting $r = 2$ in Eq (i), we get

$$a^2 = 6(6q^2 + 4q) + 4 = 6m + 4$$

where, $m = (6q^2 + 4q)$ is an integer.

**Case IV** When $r = 3$, then putting $r = 3$ in Eq. (i), we get

$$a^2 = 6(6q^2 + 6q) + 9$$

$$= 6(6q^2 + 6a) + 6 + 3$$

$\Rightarrow$ $$a^2 = 6(6q^2 + 6q + 1) + 3 = 6m + 3$$

where, $m = (6q + 6q + 1)$ is an integer.

**Case V** When $r = 4$, then putting $r = 4$ in Eq. (i), we get

$$a^2 = 6(6q^2 + 8q) + 16$$

$$= 6(6q^2 + 8q) + 12 + 4$$

$\Rightarrow$ $$a^2 = 6(6q^2 + 8q + 2) + 4 = 6m + 4$$

where, $m = (6q^2 + 8q + 2)$ is an integer.

**Case VI** When $r = 5$, then putting $r = 5$ in Eq. (i), we get

$$a^2 = 6(6q^2 + 10q) + 25$$

$$= 6(6q^2 + 10q) + 24 + 1$$

$\Rightarrow$ $$a^2 = 6(6q^2 + 10q + 4) + 1 = 6m + 1$$

where, $m = (6q^2 + 10q + 1)$ is an integer.

**Hence, the square of any positive integer cannot be of the form 6m + 2 or 6m + 5 for any integer m.**

**Question 5:**

**Show that the square of any odd integer is of the form 4m + 1, for some integerm**

**Solution:**

**By Euclid's division algorithm, we have a = bq + r, where 0 <r< 4 …(i)**

**On putting b = 4 in Eq. (i), we get**

$$a = 4q + r, \text{ where } 0 \le r < 4 \text{ i.e., } r = 0, 1, 2, 3 \quad \ldots(ii)$$

If $r = 0 \Rightarrow a = 4q$, $4q$ is divisible by $2 \Rightarrow 4q$ is even.

If $r = 1 \Rightarrow a = 4q + 1$, $(4q + 1)$ is not divisible by 2.

If $r = 2 \Rightarrow a = 4q + 2$, $2(2q + 1)$ is divisible by $2 \Rightarrow 2(2q + 1)$ is even.

If $r = 3 \Rightarrow a = 4q + 3$, $(4q + 3)$ is not divisible by 2.

So, for any positive integer $q$, $(4q + 1)$ and $(4q + 3)$ are odd integers.

Now, $$a^2 = (4q + 1)^2 = 16q^2 + 1 + 8q = 4(4q^2 + 2q) + 1$$

$$[\because (a+b)^2 = a^2 + 2ab + b^2]$$

is a square which is of the form $4m + 1$, where $m = (4q^2 + 2q)$ is an integer.

and $a^2 = (4q + 3)^2 = 16q^2 + 9 + 24q = 4(4q^2 + 6q + 2) + 1$ is a square

$$[\because (a+b)^2 = a^2 + 2ab + b^2]$$

**which is of the form 4m + 1, where m = $(4q^2 + 6q + 2)$ is an integer.**

**Hence, for some integer m, the square of any odd integer is of the form 4m + 1.**

**Question 6:**

**If n is an odd integer, then show that $n^2 - 1$ is divisible by 8.**

**Solution:**

Let $a = n^2 - 1$ ...(i)

Given that, $n$ is an odd integer.

$\therefore$ $n = 1, 3, 5, \ldots$

From Eq. (i), at $n = 1$, $a = (1)^2 - 1 = 1 - 1 = 0$,

which is divisible by 8.

From Eq. (i), at $n = 3$, $a = (3)^2 - 1 = 9 - 1 = 8$,

which is divisible by 8.

From Eq. (i), at $n = 5$, $a = (5)^2 - 1 = 25 - 1 = 24 = 3 \times 8$,

which is divisible by 8.

From Eq. (i), at $n = 7$, $a = (7)^2 - 1 = 49 - 1 = 48 = 6 \times 8$,

which is divisible by 8.

Hence, $(n^2 - 1)$ is divisible by 8, where $n$ is an odd integer.

**Alternate Method**

We know that an odd integer $n$ is of the from $(4q + 1)$ or $(4q + 3)$ for some integer $q$.

***Case I*** When $n = 4q + 1$

In this case, we have

$$(n^2 - 1) = (4q + 1)^2 - 1$$
$$= 16q^2 + 1 + 8q - 1 \quad [\because (a + b)^2 = a^2 + 2ab + b^2]$$
$$= 16q^2 + 8q = 8q\,(2q + 1)$$
$$= 16q^2 + 8q = 8q\,(2q + 1)$$

which is clearly, divisible by 8.

***Case II*** When $n = 4q + 3$

In this case, we have

$$(n^2 - 1) = (4q + 3)^2 - 1$$
$$= 16q^2 + 9 + 24q - 1 \quad [\because (a + b)^2 = a^2 + 2ab + b^2]$$
$$= 16q^2 + 24q + 8$$
$$= 8(2q^2 + 3q + 1)$$

which is clearly divisible by 8.

Hence, $(n^2 - 1)$ is divisible by 8.

**Question 7:**

**Prove that, if x and y are both odd positive integers, then $x^2 + y^z$ is even but not divisible by 4.**

**Solution:**

**Let x = 2m + 1 and y = 2m + 3 are odd positive integers, for every positive integer m.**

Then, $$x^2 + y^2 = (2m + 1)^2 + (2m + 3)^2$$
$$= 4m^2 + 1 + 4m + 4m^2 + 9 + 12m \quad [\because (a + b)^2 = a^2 + 2ab + b^2]$$
$$= 8m^2 + 16m + 10 = \text{even}$$
$$= 2(4m^2 + 8m + 5) \text{ or } 4(2m^2 + 4m + 2) + 1$$

**Hence, $x^2$ +' $y^2$ is even for every positive integer m but not divisible by 4.**

**Question 8:**

**Use Euclid's division algorithm to find the HCF of 441, 567 and 693.**

**Solution:**

**Let a = 693, b = 567 and c = 441 By Euclid's division algorithms,**

By Euclid's division algorithms,

$$a = bq + r \quad \ldots(i)$$

[$\because$ dividend = divisor × quotient + remainder]

First we take, $a = 693$ and $b = 567$ and find their HCF.

$$693 = 567 \times 1 + 126$$
$$567 = 126 \times 4 + 63$$
$$126 = 63 \times 2 + 0$$

$\therefore$ HCF (693, 567) = 63

Now, we take $c = 441$ and say $d = 63$, then find their HCF.

Again, using Euclid's division algorithm,

$$c = dq + r$$

$\Rightarrow$ $441 = 63 \times 7 + 0$

$\therefore$ HCF (693, 567, 441) = 63

**Question 9:**

**Using Euclid's division algorithm, find the largest number that divides 1251, 9377 and 15628 leaving remainders 1, 2 and 3, respectively.**

**Solution:**

**Since, 1, 2 and 3 are the remainders of 1251, 9377 and 15628, respectively. Thus, after subtracting these remainders from the numbers.**

**We have the numbers, 1251-1 = 1250,9377-2 = 9375 and 15628-3 = 15625 which is divisible by the required number.**

**Now, required number = HCF of 1250,9375 and 15625 [for the largest number]**

**By Euclid's division algorithm,**

$$a = bq + r \quad \ldots(i)$$

[$\because$ dividend = divisor × quotient + remainder]

For largest number, put $a = 15625$ and $b = 9375$

$$15625 = 9375 \times 1 + 6250 \quad \text{[from Eq. (i)]}$$

$\Rightarrow$ $9375 = 6250 \times 1 + 3125$

$\Rightarrow$ $6250 = 3125 \times 2 + 0$

$\therefore$ HCF (15625, 9375) = 3125

Now, we take $c = 1250$ and $d = 3125$, then again using Euclid's division algorithm,

$$d = cq + r \quad \text{[from Eq. (i)]}$$

$\Rightarrow$ $3125 = 1250 \times 2 + 625$

$\Rightarrow$ $1250 = 625 \times 2 + 0$

$\therefore$ HCF (1250, 9375, 15625) = 625

**Hence, 625 is the largest number which divides 1251,9377 and 15628 leaving remainder 1, 2 and 3, respectively.**

**Question 10:**

**Prove that√3 + √5 is irrational.**

**Solution:**

**Let us suppose that √3 + √5 is rational.**

**Let √3 + √5 = a, where a is rational.**

Therefore, $\sqrt{3} = a - \sqrt{5}$

On squaring both sides, we get

$$(\sqrt{3})^2 = (a - \sqrt{5})^2$$

$\Rightarrow$ $3 = a^2 + 5 - 2a\sqrt{5}$ [$\because (a-b)^2 = a^2 + b^2 - 2ab$]

$\Rightarrow$ $2a\sqrt{5} = a^2 + 2$

Therefore, $\sqrt{5} = \dfrac{a^2 + 2}{2a}$ which is contradiction.

**As the right hand side is rational number while V5 is irrational. Since, 3 and 5 are prime numbers. Hence, √3 + √5 is irrational.**

**Question 11:**

**Show that $12^n$ cannot end with the digit 0 or 5 for any natural number n.**

**Solution:**

**If any number ends with the digit 0 or 5, it is always divisible by 5.**

**If $12^n$ ends with the digit zero it must be divisible by 5.**

**This is possible only if prime factorisation of $12^n$ contains the prime number 5.**

This is possible only if prime factorisation of $12^n$ contains the prime number 5.

Now, $12 = 2 \times 2 \times 3 = 2^2 \times 3$

$\Rightarrow$ $12^n = (2^2 \times 3)^n = 2^{2n} \times 3^n$ [since, there is no term contains 5]

**Hence, there is no value of n e N for which $12^n$ ends with digit zero or five.**

**Question 12:**

**On a morning walk, three persons step off together and their steps measure 40 cm, 42 cm and 45 cm, respectively.What is the minimum distance each should walk, so that each can cover the same distance in complete steps?**

**Solution:**

**We have to find the LCM of 40 cm, 42 cm and 45 cm to get the required minimum distance.**

For this, $40 = 2 \times 2 \times 2 \times 5,$

$42 = 2 \times 3 \times 7$

and $45 = 3 \times 3 \times 5$

$\therefore$ $\text{LCM }(40, 42, 45) = 2 \times 3 \times 5 \times 2 \times 2 \times 3 \times 7$

$= 30 \times 12 \times 7 = 210 \times 12$

$= 2520$

**Minimum distance each should walk 2520 cm. So that, each can cover the same distance in**

**complete steps.**

**Question 13:**

**Write the denominator of rational number $\frac{257}{5000}$ in the form 2^m^ x$5^n$,where m, n are non-negative integers. Hence, write its decimal expansion, without actual division**

**Solution:**

**Denominator of the rational number $\frac{257}{5000}$ is 5000.**

Now, factors of $5000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5 = (2)^3 \times (5)^4$, which is of the type $2^m \times 5^n$, where $m = 3$ and $n = 4$ are non-negative integers.

$\therefore$ Rational number $= \frac{257}{5000} = \frac{257}{2^3 \times 5^4} \times \frac{2}{2}$

[since, multiplying numerator and denominater by 2]

$= \frac{514}{2^4 \times 5^4} = \frac{514}{(10)^4}$

$= \frac{514}{10000} = 0.0514$

**Hence, which is the required decimal expansion of the rational $\frac{257}{5000}$ and it is also a 5000 terminating decimal number.**

**Question 14:**

**Prove that √p + √q is irrational, where p and q are primes.**

**Solution:**

Let us suppose that $\sqrt{p} + \sqrt{q}$ is rational.

Again, let $\sqrt{p} + \sqrt{q} = a$, where a is rational.

Therefore, $\sqrt{q} = a - \sqrt{p}$

On squaring both sides, we get

$$q = a^2 + p - 2a\sqrt{p} \qquad [\because (a-b)^2 = a^2 + b^2 - 2ab]$$

Therefore, $\sqrt{p} = \frac{a^2 + p - q}{2a}$, which is a contradiction as the right hand side is rational number while $\sqrt{p}$ is irrational, since $p$ and $q$ are prime numbers.

Hence, $\sqrt{p} + \sqrt{q}$ is irrational.

## Exercise 1.4 Long Answer Type Questions

**Question 1:**

**Show that the cube of a positive integer of the form 6q + r,q is an integer and r = 0, 1, 2, 3, 4, 5 is also of the form 6m + r.**

**Solution:**

**Let a be an arbitrary positive integer. Then, by Euclid's division algorithm, corresponding to the positive integers 'a' and 6, there exist non-negative integersg and r such that a = 6 q + r, where, 0 < r < 6**

$$a = 6q + r, \text{ where, } 0 \le r < 6$$

$$\Rightarrow \quad a^3 = (6q + r)^3 = 216q^3 + r^3 + 3 \cdot 6q \cdot r(6q + r)$$

$$[\because (a+b)^3 = a^3 + b^3 + 3ab(a+b)]$$

$$\Rightarrow \quad a^3 = (216q^3 + 108q^2r + 18qr^2) + r^3 \qquad \ldots(i)$$

where, $0 \le r < 6$

**Case I** When $r = 0$, then putting $r = 0$ in Eq. (i), we get

$$a^3 = 216q^3 = 6(36q^3) = 6m$$

where, $m = 36q^3$ is an integer.

**Case II** When $r = 1$, then putting $r = 1$ in Eq. (i), we get

$$a^3 = (216q^3 + 108q^3 + 18q) + 1 = 6(36q^3 + 18q^3 + 3q) + 1$$

$$\Rightarrow \quad a^3 = 6m + 1, \text{ where } m = (36q^3 + 18q^3 + 3q) \text{ is an integer.}$$

**Case III** When $r = 2$, then putting $r = 2$ in Eq. (i), we get

$$a^3 = (216q^3 + 216q^2 + 72q) + 8$$

$$a^3 = (216q^3 + 216q^2 + 72q + 6) + 2$$

$$\Rightarrow \quad a^3 = 6(36q^3 + 36q^2 + 12q + 1) + 2 = 6m + 2$$

where, $m = (36q^2 + 36q^2 + 12q + 1)$ is an integer.

**Case IV** When $r = 3$, then putting $r = 3$ in Eq. (i), we get

$$a^3 = (216q^3 + 324q^2 + 162q) + 27 = (216q^3 + 324q^2 + 162q + 24) + 3$$

$$= 6(36q^3 + 54q^2 + 27q + 4) + 3 = 6m + 3$$

where, $m = (36q^2 + 54q^2 + 27q + 4)$ is an integer.

**Case V** When $r = 4$, then putting $r = 4$ in Eq. (i), we get

$$a^3 = (216q^2 + 432q^2 + 288q) + 64$$

$$= 6(36q^3 + 72q^2 + 48q) + 60 + 4$$

$$= a^3\ 6(36q^3 + 72q^2 + 48q + 10) + 4 = 6m + 4$$

where, $m = (36q^3 + 72q^2 + 48q + 10)$ is an integer.

**Case VI** When $r = 5$, then putting $r = 5$ in Eq. (i), we get

$$a^3 = (216q^3 + 540q^2 + 450q) + 125$$

$$\Rightarrow \quad a^3 = (216q^3 + 540q^2 + 450q) + 120 + 5$$

$$\Rightarrow \quad a^3 = 6(36q^3 + 90q^2 + 75q + 20) + 5$$

$$\Rightarrow \quad a^3 = 6m + 5$$

where, $m = (36q^3 + 90q^2 + 75q + 20)$ is an integer.

**Hence, the cube of a positive integer of the form 6q + r,q is an integer and r = 0,1,2, 3, 4,5 is also of the forms 6m, 6m + 1, 6m + 2, 6m + 3,6m + 4 and 6m + 5 i.e.,6m + r.**

**Question 2:**

**Prove that one and only one out of n, (n + 2) and (n + 4) is divisible by 3, where n is any positive integer,**

**Solution:**

Let a = n, b = n + 2 and c =n + 4

Order triplet is (a, b, c) = (n, n + 2, n + 4) … (i)

Where, n is any positive integer i.e.,n = 1,2, 3,…

At n = 1; (a, b, c) = (1,1 + 2,1 + 4) = (1, 3, 5)

At n = 2; (a, b, c) = (2,2 + 2,2 + 4)= (2, 4, 6)

At n = 3; (a, b,c) = (3, 3 + 2,3 + 4) = (3,5,7)

At n =4; (a,b, c) =(4, 4 + 2, 4 +4) = (4, 6, 8)

At n = 5; (a, b,c) = (5, 5 + 2, 5 +4)= (5,7,9)

At n = 6; (a,b, c) = (6, 6 + 2, 6 + 4)= (6,8,10) ‘

At n = 7; (a, b,c) = (7,7 + 2,7 + 4)= (7, 9,11)

At n = 8; (a, b,c) = (8,8+ 2, 8+ 4)= (8,10,12)

We observe that each triplet consist of one and only one number which is multiple of 3 i.e., divisible by 3.

Hence, one and only one out of n,(n + 2) and (n + 4) is divisible by 3, where, n is any positive integer.

Alternate Method

On dividing ‘n’ by 3, letg be the quotient and r be the remainder.

Then, n =,3 q + r, where, 0< r< 3

$\Rightarrow$ $n = 3q + r$, where, $r = 0, 1, 2$

$\Rightarrow$ $n = 3q$ or $n = 3q + 1$ or $n = 3q + 2$

***Case I*** If $n = 3q$, then $n$ is only divisible by 3.
but $n+2$ and $n+4$ are not divisible by 3.

***Case II*** If $n = 3q + 1$, then $(n+2) = 3q + 3 = 3(q+1)$, which is only divisible by 3,
but $n$ and $n+4$ are not divisible by 3.
So, in this case, $(n+2)$ is divisible by 3.

***Case III*** When $n = 3q + 2$, then $(n+4) = 3q + 6 = 3(q+2)$, which is only divisible by 3,
but $n$ and $(n+2)$ are not divisible by 3.
So, in this case, $(n+4)$ is divisible by 3.

Hence, one and only one out of n, (n + 2) and (n + 4) is divisible by 3.

**Question 3:**

**Prove that one of any three consecutive positive integers must be divisible by 3.**

**Solution:**

Any three consecutive positive integers must be of the form

n, (n + 1)and (n + 2), where n is any natural number, i.e., n = 1,2, 3…

Let, a = n,b = n+ 1 and c = n + 2

Order triplet is (a, b, c) = (n, n + 1, n + 2), where n = 1,2, 3,… …(i)

At n = 1; (a, b, c) = (1,1 + 1,1 + 2) = (1,2, 3)

At n = 2; (a,b,c) = (1,2 + 1,2 + 2) = (2, 3, 4)

At n = 3; (a, b, c) = (3, 3 + 1, 3 + 2) = (3, 4, 5)

At n = 4; (a, b, c) = (4, 4 + 1, 4 + 2) = (4, 5, 6)

At n = 5; (a, b, c) = (5, 5 + 1, 5 + 2) = (5, 6,7)

At n= 8 (a, b, c) = (6, 6+ 1, 6 + 2) = (6,7, 8)

Atn = 7; (a, b, c) = (7,7 + 1,7 + 2) = (7, 8, 9)

At n = 8; (a, b, c) = (8 8 + 1, 8 + 2) = (8, 9,10)

We observe that each triplet consist of one and only one number which is multiple of 3 i.e., divisible by 3.

Hence, one of any three consecutive positive integers must be divisible by 3.

**Question 4:**

**For any positive integer n, prove that $n^3$ – n is divisible by 6.**

**Solution:**

Let $a = n^3 - n \Rightarrow a = n \cdot (n^2 - 1)$

$\Rightarrow a = n \cdot (n - 1) \cdot (n + 1)$ $[\because (a^2 - b^2) = (a - b)(a + b)]$

$\Rightarrow a = (n - 1) \cdot n \cdot (n + 1)$ ...(i)

We know that,

1. If a number is completely divisible by 2 and 3, then it is also divisible by 6.
2. If the sum of digits of any number is divisible by 3, then it is also divisible by 3.
3. If one of the factor of any number is an even number, then it is also divisible by 2.

∴ a = (n – 1) . n . (n + 1) [from Eq. (i)]

Now, sum of the digits =n-1+n+n+1=3n

= multiple of 3, where n is any positive integer,

and (n -1)- n- (n + 1) will always be even, as one out of (n -1) or n or (n + 1) must of even. Since, conditions II and III is completely satisfy the Eq. (i).

Hence, by condition I the number $n^3$ – n is always divisible by 6, where n is any positive integer. Hence proved.

**Question 5:**

**Show that one and only one out of n, n + 4 , n + 8, n + 12 and n + 16 is divisible by 5, where n is any positive integer.**

**Solution:**

Given numbers are n, (n+ 4), (n + 8), (n + 12) and (n + 16), where n is any positive integer.

Then, let n = 5q, 5g + 1, 5g + 2, 5g + 3, 5q + 4 for q∈ N [by Euclid's algorithm]

Then, in each case if we put the different values of n in the given numbers. We definitely get one and only one of given numbers is divisible by 5.

Hence, one and only one out of n, n+ 4, n+ 8, n+12 and n+16 is divisible by 5.

Alternate Method

On dividing on n by 5, letq be the quotient and r be the remainder.

Thenn=5q + r, where 0<r< 5.

$\Rightarrow n = 5q + r$, where $r = 0, 1, 2, 3, 4$.

$\Rightarrow n = 5q$ or $5q + 1$ or $5q + 2$ or $5q + 3$ or $5q + 4$.

**Case I** If $n = 5q$, then $n$ is only divisible by 5.

**Case II** If $n = 5q + 1$, then $n + 4 = 5q + 1 + 4 = 5q + 5 = 5(q + 1)$, which is only divisible by 5.

So, in this case, $(n + 4)$ is divisible by 5.

**Case III** If $n = 5q + 3$, then $n + 2 = 5q + 3 + 12 = 5q + 15 = 5(q + 3)$, which is divisible by 5.

So, in this case $(n + 12)$ is only divisible by 5.

**Case IV** If $n = 5q + 4$, then $n + 16 = 5q + 4 + 16 = 5q + 20 = 5(q + 4)$, which is divisible by 5.

So, in this case, $(n + 16)$ is only divisible by 5.

Hence, one and only one out of $n$, $n + 4$, $n + 8$, $n + 12$ and $n + 16$ is divisible by 5, where $n$ is any positive integer.